PADERBORN UNIVERSITY

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Quasiprobabilities — the art of erroneously determining chances

Jan Sperling (he/him)

Theoretical Quantum Science

Institute for Photonic Quantum Systems (PhoQS)

Paderborn University

before we begin, a few remarks ...

- ✓ special thanks to organizers
- × disclaimer: high-level presentation (sorry for skipping interesting details)

PhoQS

- interdisciplinary department for photonic quantum technologies
  - ✓ experimental physics
  - ✓ theoretical physics
  - ✓ mathematics
  - ✓ computer science
  - ✓ electrical engineering
- topics: quantum computation, quantum communication, quantum sensing, quantum simulation, photonic quantum systems, ...

speaker

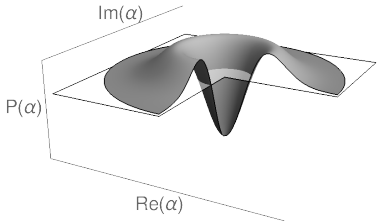
Christine Silberhorn

Motivation: Quasiprobabilities

- early example: Glauber–Sudarshan P distribution [1,2]

$$\hat{\rho} = \int d^2\alpha \, P(\alpha) |\alpha\rangle\langle\alpha|$$

- P fails as a classical probability
  $\rightarrow$ nonclassical/quantum



single-photon-added thermal state [3]

chances, e.g., $\overbrace{-21\%}^{\text{neg. part}}$: $\overbrace{121\%}^{\text{pos. part}}$

$<0\%$ $>100\%$

[1] Glauber, Phys. Rev. 131, 2766 (1963)

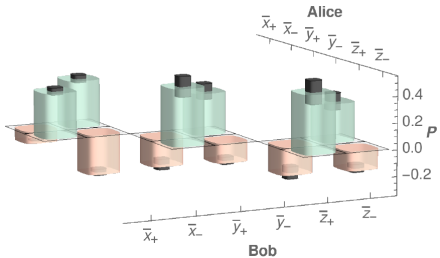
[2] Sudarshan, Phys. Rev. Lett. 10, 277 (1963)

[3] Kiesel, Vogel, Parigi, Zavatta, Bellini, Phys. Rev. A 78, 021804(R) (2008)

Other notions of quantumness example: entanglement

- similar quasiprobability decomposition for general kinds of quantumness
- other classical reference states

- e.g., entanglement [1]
  $\hat{\rho} = \sum_{a,b} P(a,b)|a\rangle\langle a| \otimes |b\rangle\langle b|$
- formulation as optimization problem w/ min. negativity [2]

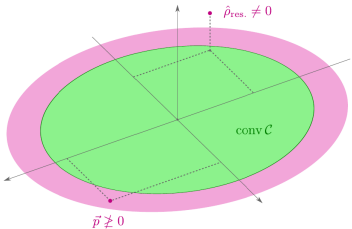


experiment [3] for
target state $\propto |0\rangle \otimes |1\rangle - |1\rangle \otimes |0\rangle$

[1] Sanpera, Tarrach, Phys. Rev. A 58, 826 (1998)
[2] JS, Walmsley, Phys. Rev. A 97, 062327 (2018)
[3] JS, Meyer-Scott, Barkhofen, Brecht, Silberhorn, Phys. Rev. Lett. 122, 053602 (2019)

Non-decomposability

- classical states: convex combinations of classical reference states, $P \geq 0$
- quasiprobability representation: linear combinations of classical reference states, $P \ngeq 0$
- states outside of linear span
  $\rightarrow$ non-decomposable



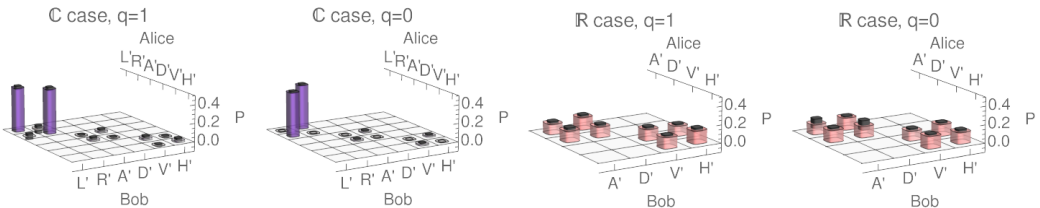
residual component perpendicular to linear span [1]

[1] JS, Walmsley, Phys. Rev. A 97, 062327 (2018)

Rebit entanglement

- state [1]: non-entangled w.r.t. products of $\mathbb{C}$-vectors, yet entangled w.r.t. products of $\mathbb{R}$-vectors
- for $0 \leq q \leq 1$,
$$\hat{\rho} = \tfrac{1}{4}\hat{\sigma}_0 \otimes \hat{\sigma}_0 + \tfrac{2q-1}{4}\hat{\sigma}_y \otimes \hat{\sigma}_y$$

- no residual component: $\|\hat{\rho} - \hat{\rho}_{\mathbb{C}\text{-dec.}}\| = \|\hat{\rho}_{\text{res.}}\| = 0$
- max. residual component ($q = 0, 1$): $\|\hat{\rho} - \hat{\rho}_{\mathbb{R}\text{-dec.}}\| = 1/2$
- experimental verification of non-decomposability [2]



[1] Caves, Fuchs, Rungta, Found. Phys. Lett. 14, 199 (2001)

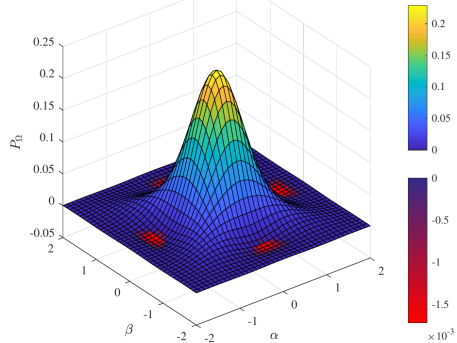
[2] Prasannan, De, Barkhofen, Brecht, Silberhorn, JS, Phys. Rev. A 103, L040402 (2021)

Convolutions
example: nonclassical correlations

- convolution of quasiprobability [1]
  $P_\Omega(\alpha') = \int d^2\alpha\, \Omega(\alpha', \alpha) P(\alpha)$
- Gaussian Ω: Wigner, Husimi

  better: non-Gaussian kernel [2]
  → even squeezed states with regular P_Ω and negativities
- example: state [3,4] without discord and entanglement, but nonclassical correlations

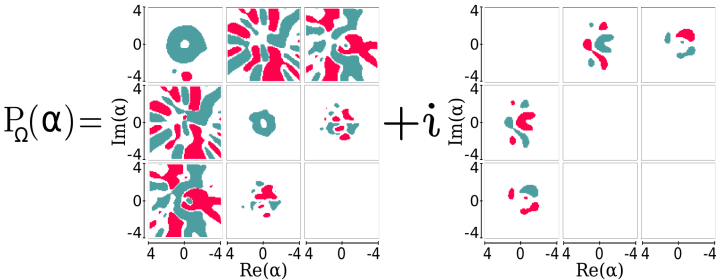
  neg. $(-1.570 \pm 0.010) \times 10^{-3}$



experiment [5] for phase-randomized two-mode squeezed vacuum

[1] Agarwal, Wolf, Phys. Rev. D 2, 2161 (1970); Agarwal, Wolf, Phys. Rev. D 2, 2187 (1970)
[2] Kiesel, Vogel, Phys. Rev. A 82, 032107 (2010)
[3] Ferraro, Paris, Phys. Rev. Lett. 108, 260403 (2012)
[4] Agudelo, JS, Vogel, Phys. Rev. A 87, 033811 (2013)
[5] Köhnke, Agudelo, Schünemann, Schlettwein, Vogel, JS, Hage, Phys. Rev. Lett. 126, 170404 (2021)

Hybrid correlations



- experiment [1] + regularization
- state of form $|\Psi\rangle \propto \sum_n \lambda_n |\alpha_n\rangle \otimes |n\rangle$

- discrete-continuous hybrid state $\rightarrow \hat{\rho} = \sum_{m,n} \int d^2\alpha \, P_{m,n}(\alpha) |\alpha\rangle\langle\alpha| \otimes |m\rangle\langle n|$
- classical if matrix of quasiprobabilities pos.-semidefinite $\rightarrow [P_{m,n}]_{m,n} \geq 0$

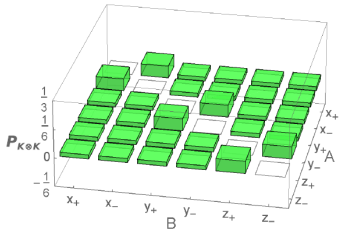
[1] Agudelo, JS, Costanzo, Bellini, Zavatta, Vogel, Phys. Rev. Lett. 119, 120403 (2017)

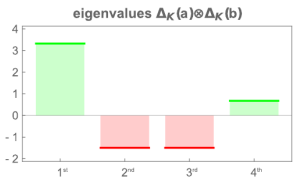
Quasistates
example: entanglement Husimi-like function

- effect of convolution on reference states [1,2]
- quasiprobability P_Ω convoluted,
  then deconvoluted $\hat{\Delta}_\Omega(\beta) = \int d^2\alpha\, \Omega^{-1}(\beta,\alpha)|\alpha\rangle\langle\alpha|$
  $\rightarrow$ quasistates
- e.g., Wigner function

$$\hat{\rho} \propto \int d^2\beta\, W(\beta)\, \underbrace{(-1)^{\hat{n}(\beta)}}_{\not\geq 0}$$

- left: Bell state with joint $P_\Omega \geq 0$ ($\Omega = K \otimes K$)





[1] Agarwal, Wolf, Phys. Rev. D 2, 2161 (1970); Agarwal, Wolf, Phys. Rev. D 2, 2187 (1970)

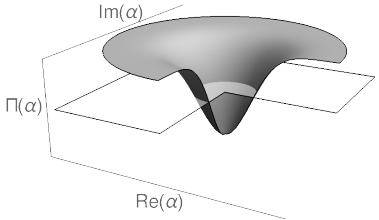
[2] JS, Walmsley, Phys. Rev. A 98, 042122 (2018)

Detector nonclassicality

- analogously to state $\hat{\rho}$ characterization, nonclassical detector features

- POVM element

$$\hat{\Pi} = \int d^2\alpha \, \Pi(\alpha) |\alpha\rangle\langle\alpha|$$



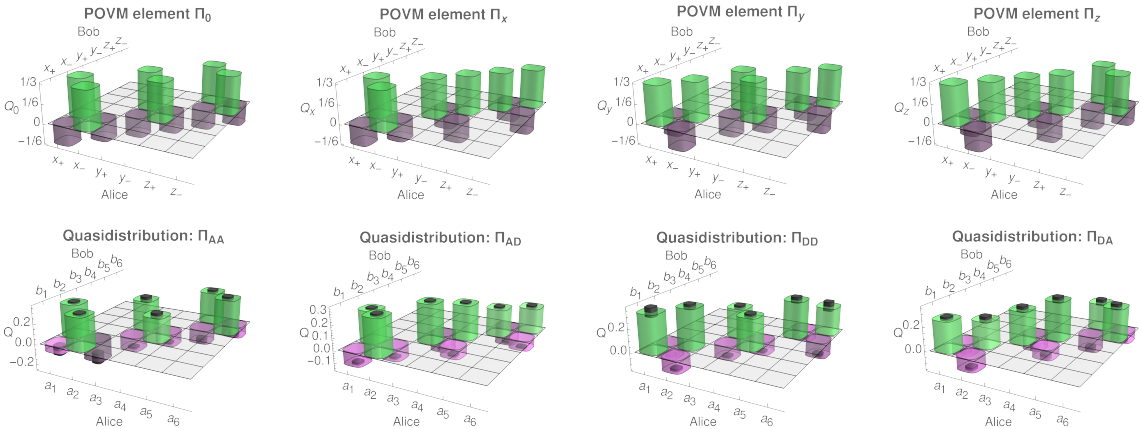
single-photon detector [1]

- similar to state, Π fails as a non-neg. distribution $\rightarrow$ nonclassical/quantum detection

[1] Lundeen, Feito, Coldenstrodt-Ronge, Pregnell, Silberhorn, Ralph, Eisert, Plenio, Walmsley, Nat. Phys. 5, 27 (2009)

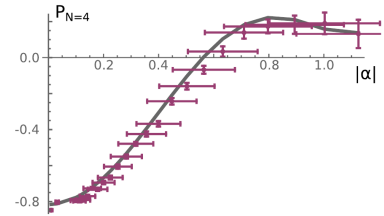
Bell measurements

- detection ideally based on Bell-state projections
- entanglement quasidistributions Q: theory top, experiment bottom [1]

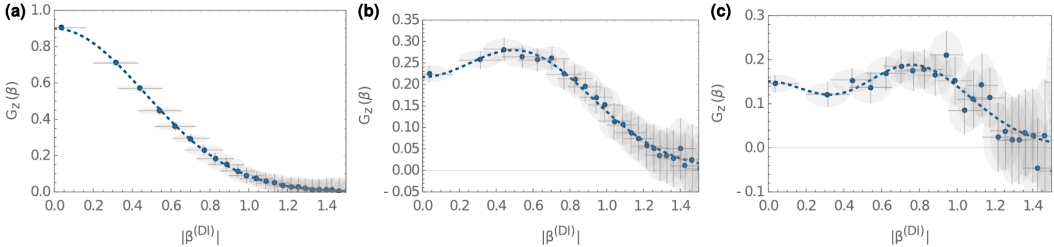


[1] JS, Gianani, Barbieri, Agudelo, Phys. Rev. A 107, 012426 (2023)

Detector-agnostic reconstructions



- reconstruction of quasiprobabilities with informationally incomplete detectors and/or unknown absorption properties [1,3]
- experiments: left [2] & bottom [3]



[1] Luis, JS, Vogel, Phys. Rev. Lett. 120, 063607 (2018)

[2] Bohmann, Tiedau, Bartley, JS, Silberhorn, Vogel, Phys. Rev. Lett. 120, 063607 (2018)

[3] JS, Phillips, Bulmer, Thekkadath, Eckstein, Wolterink, Lugani, Nam, Lita, Gerrits, Vogel, Agarwal, Silberhorn, Walmsley, Phys. Rev. Lett. 124, 013605 (2020)

Phase-space inequalities

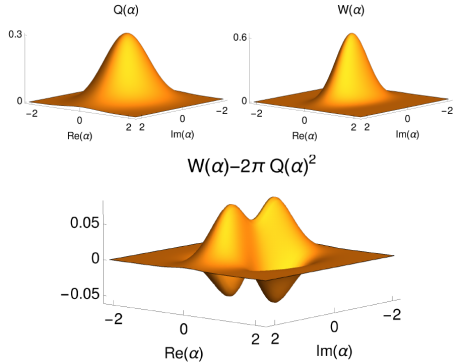
- correlating quasiprobabilities
  $\rightarrow$ insight beyond negativities [1]

  e.g., $W(\alpha) - 2\pi Q(\alpha)^2 \overset{\text{cl.}}{\geq} 0$

  (incl. Wigner and Husimi fcts.)

- generalization to higher-order correlations and multimode nonclassicality [2]

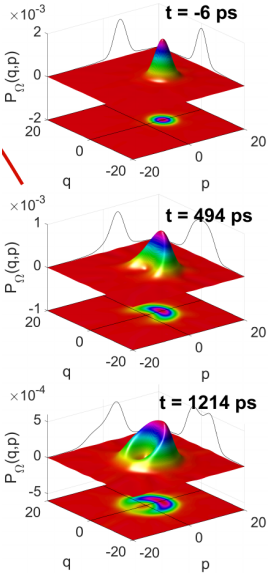
- experimentally demonstrated [3]



non-neg. distributions fot detecting nonclassicality of squeezed state [1]

[1] Bohmann, Agudelo, Phys. Rev. Lett. 124, 133601 (2020)
[2] Bohmann, Agudelo, JS, Quantum 4, 343 (2020)
[3] Biagi, Bohmann, Agudelo, Bellini, Zavatta, Phys. Rev. Lett. 126, 023605 (2021)

Decoherence of Bose-Einstein condensates

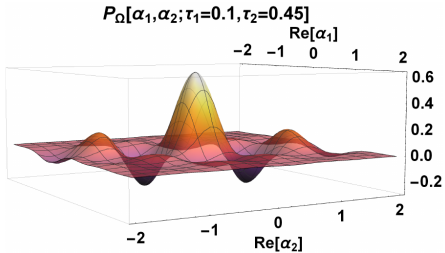


- study decoherence of polariton condensates [1]
- again: useful information
  despite (regularized) $P \geq 0$
- here: decoherence over time
  through circular (i.e., phase) variance
- application: detemination of decay rates

[1] Lüders, Pukrop, Barkhausen, Rozas, Schneider, Höfling, JS, Schumacher, Aßmann, Phys. Rev. Lett. 130, 113601 (2023)

Quasiprobabilities for stochastic quantum processes

- quantum processes via multi-time P functions [1]
- random variables $\{\alpha_t\}_t$ for time t
- quantumness of process, $P \not\geq 0$
- outlook: similar for other notions of quantumness?



simulation of two-time, regularized P function for nonlinear process [2]

[1] Vogel, Phys. Rev. Lett. 100, 013605 (2008)

[2] Krumm, Vogel, JS, Phys. Rev. A 95, 063805 (2017)

Summary

- ✓ quasiprobability representation for general quantum effects (e.g., entanglement)
- ✓ non-decomposable representations (e.g., entanglement over $\mathbb{R}$ and $\mathbb{C}$)
- ✓ convolution of quasiprobability representation (e.g., non-Gaussian kernels, quasistates)
- ✓ quasiprobabilities for measurements (e.g., Bell-measurement POVMs)
- ✓ value of non-negative representations (e.g., phase-space inequalities, condensate decoherence)
- ✓ ... (e.g., nonclassical processes, hybrid systems, correlations, detector-agnostic quasiprobabilities)

[review] JS, Vogel, Phys. Scr. 95, 034007 (2020)

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Many thanks — questions?

jan.sperling@upb.de

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