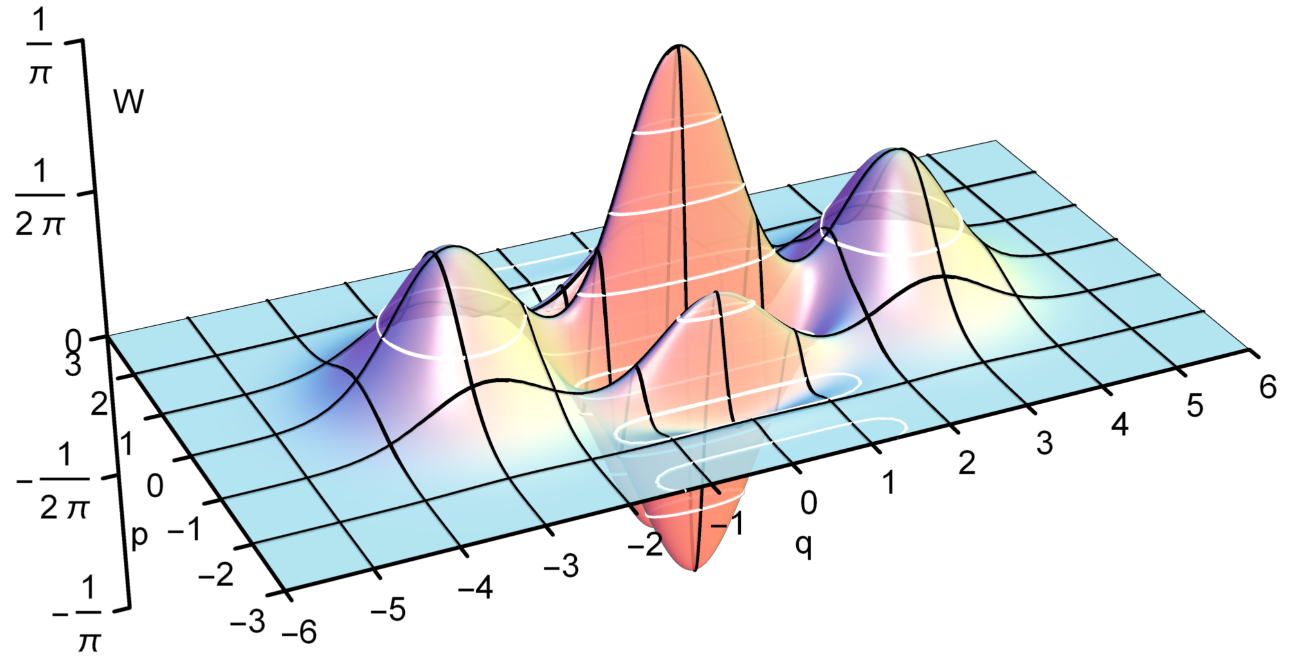

QuiDiQua

Contextuality and Wigner negativity are equivalent for continuous-variable quantum measurements

Robert I. Booth, Ulysse Chabaud, Pierre-Emmanuel Emeriau

Physical Review Letters 129 (23), 230401 (2022)
arXiv: 2111.13218

Outline of the talk

I. Context

II. Wigner negativity and CV contextuality

III. The equivalence

It's a bug

[EPR 1935]

EPR paradox

Wigner function

[Wigner 1932]

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EPR paradox

Quantum mechanics has **nonlocal effects** and needs hidden variables

Wigner function

The statistical description of quantum mechanics has **negative probabilities**

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Quantum mechanics is incompatible with hidden variable theories without:

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Otherwise, we could simulate it efficiently classically

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What about the original Wigner function? → this work

Wigner negativity and CV contextuality

Discrete (DV)	Continuous (CV)

Discrete (DV)	Continuous (CV)
Finite-dimensional Hilbert spaces	Separable, infinite-dimensional Hilbert spaces
$\lvert\psi\rangle = \alpha\lvert 0\rangle + \beta\lvert 1\rangle$	$\lvert\psi\rangle = \sum_{n=0}^{+\infty} \psi_n \lvert n\rangle$

Discrete (DV)	Continuous (CV)
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Wigner negativity

Quadrature operators

Position-like and momentum-like operators (quadrature operators):

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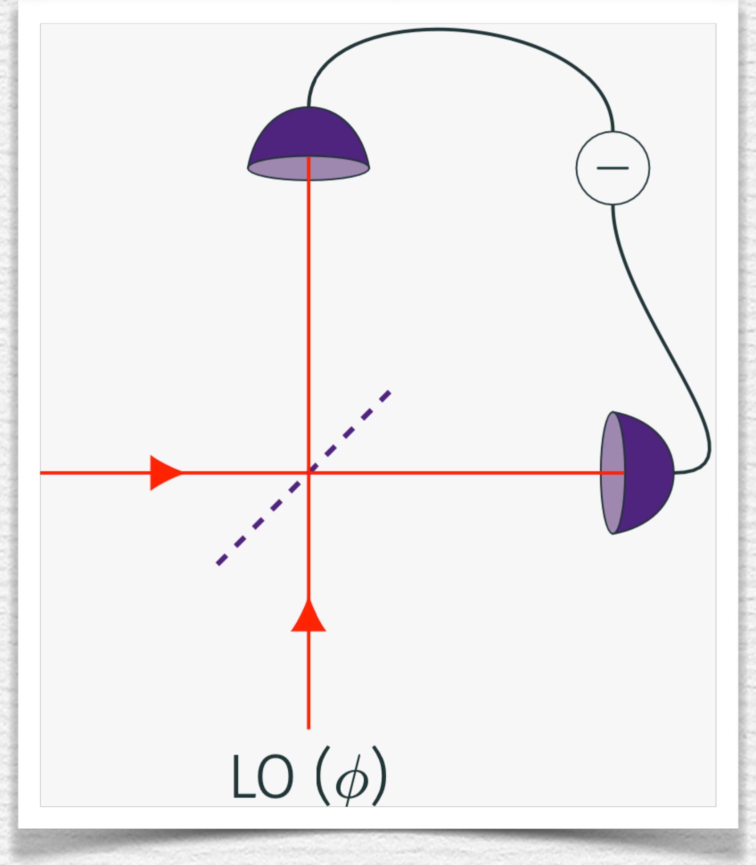
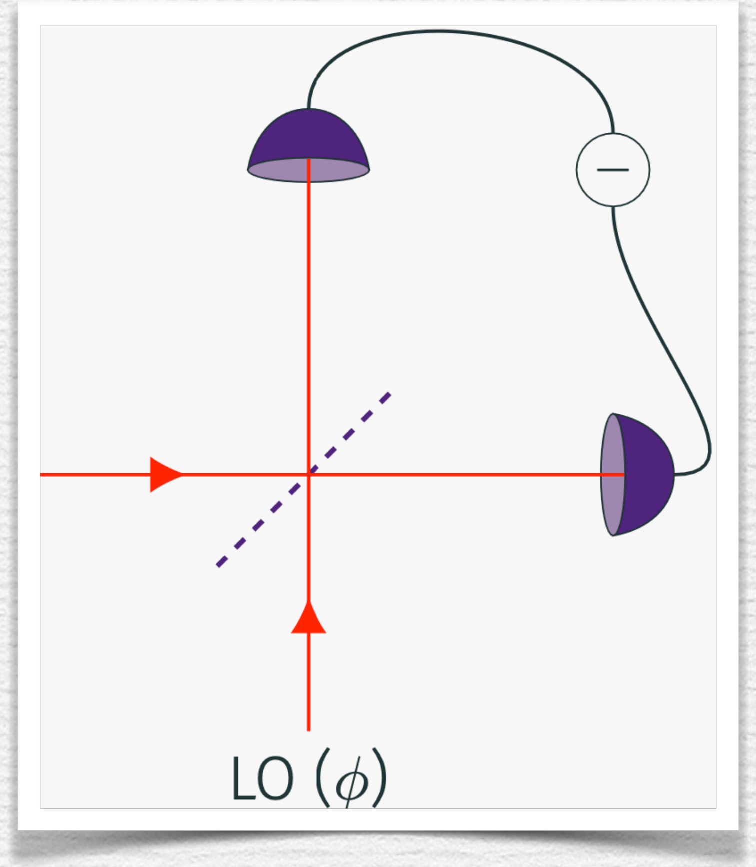


Homodyne

Quadrature operators



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Multimode quadrature operators:

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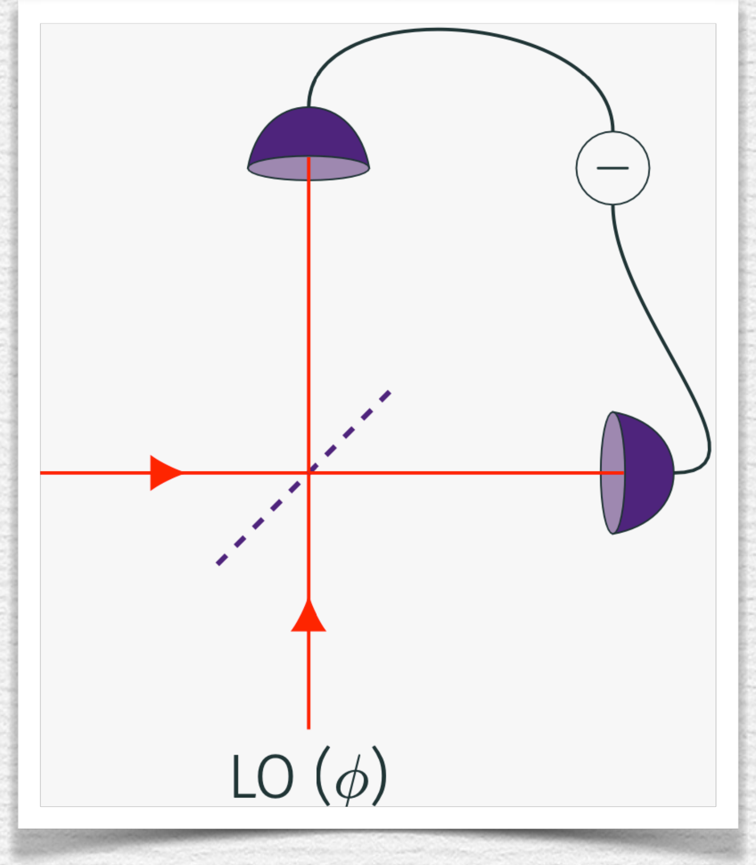

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Symplectic form: $\Omega(\mathbf{x}, \mathbf{y}) := \mathbf{x}^T J \mathbf{y}$ $\qquad J = \begin{pmatrix} 0_M & I_M \\ -I_M & 0_M \end{pmatrix}$

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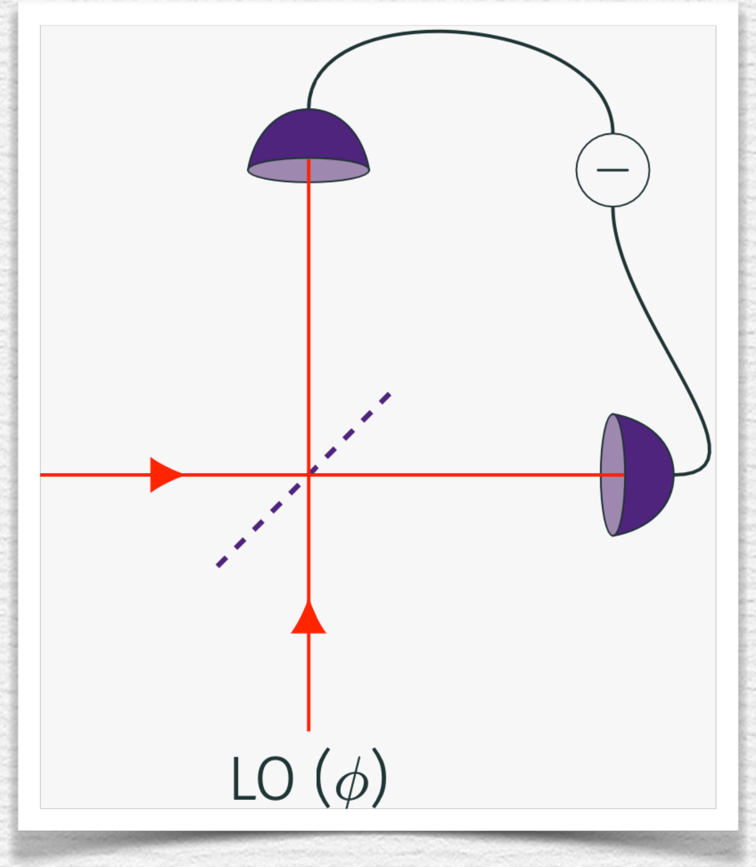

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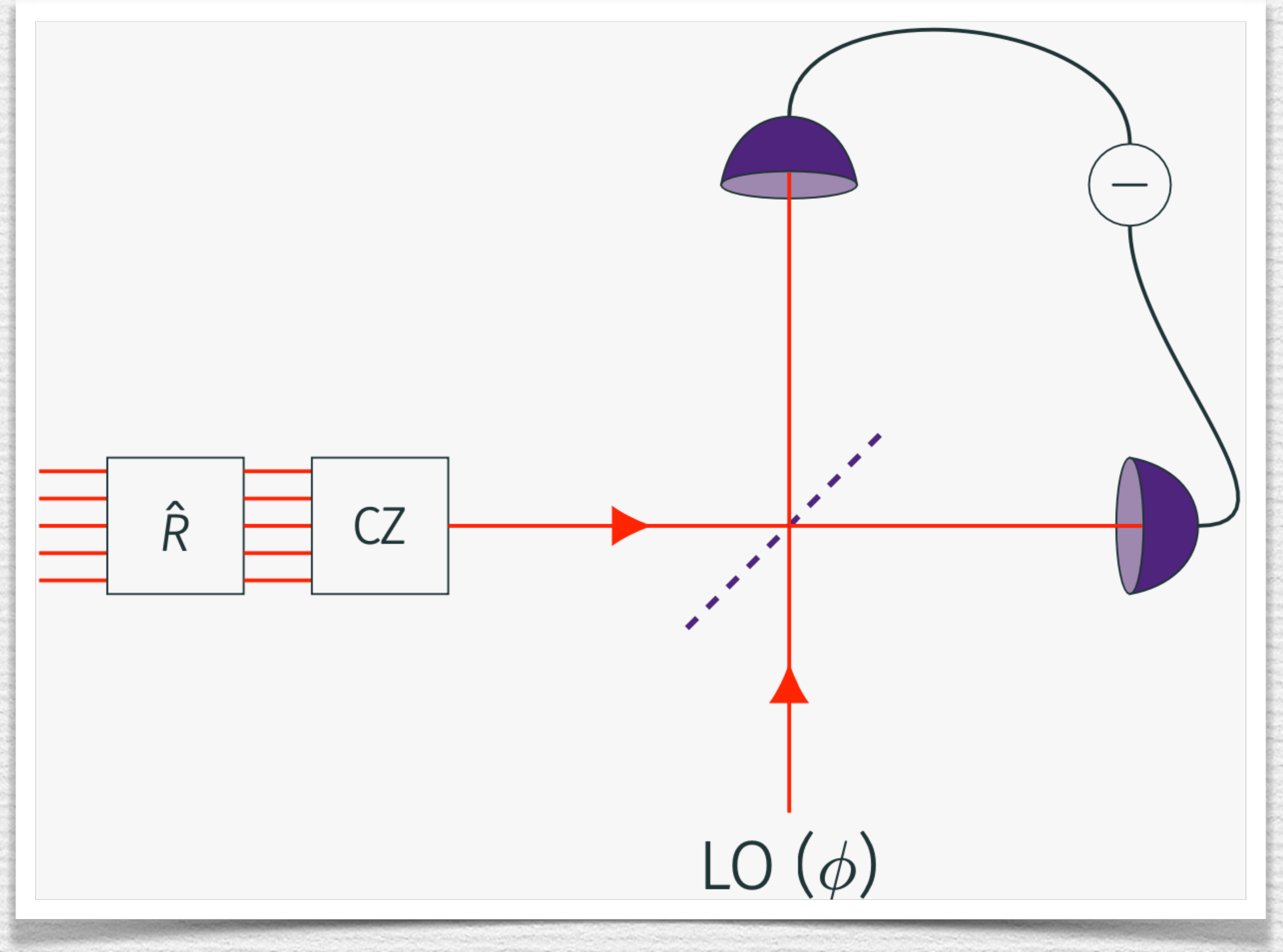
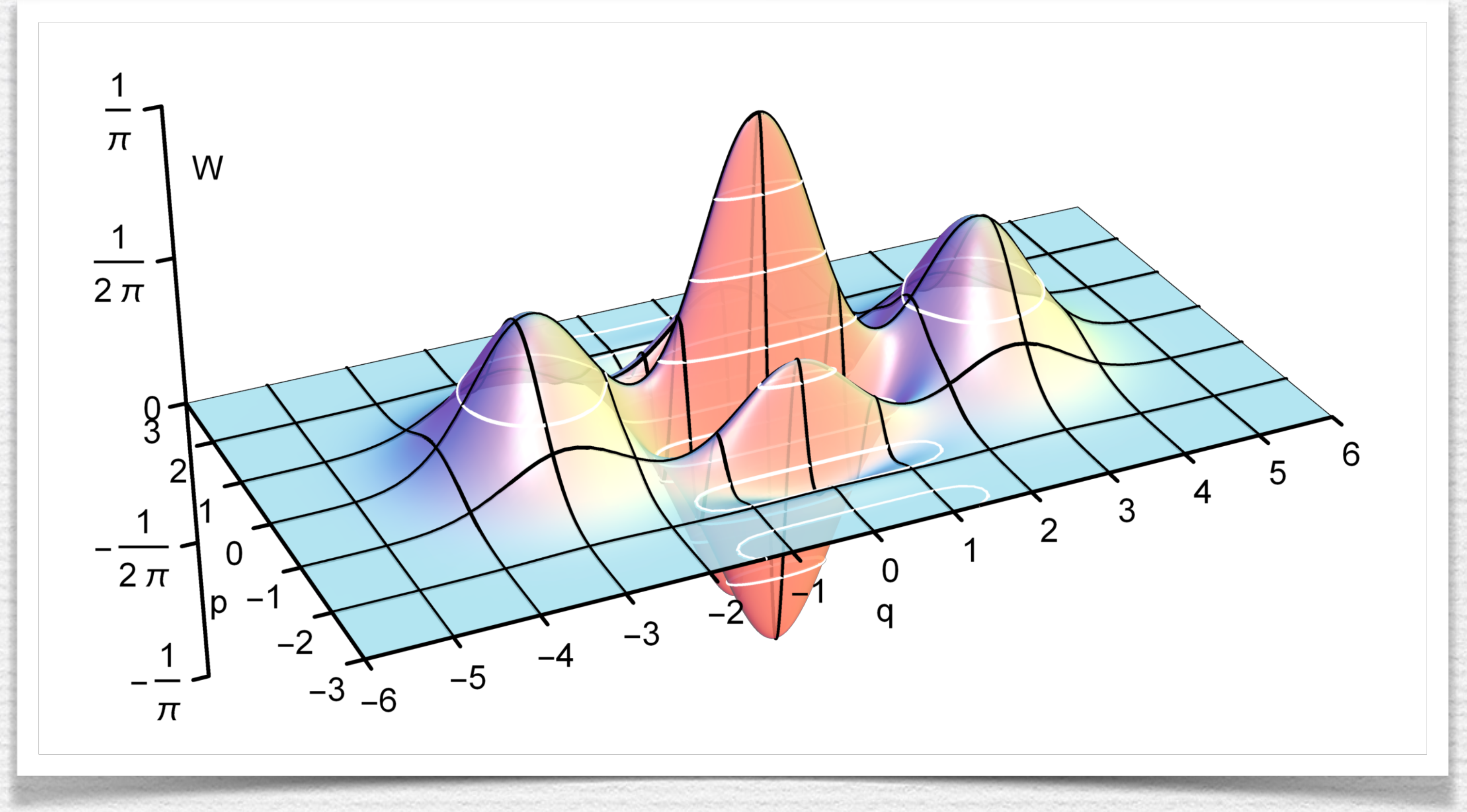


Generalised homodyne

Wigner negativity

The Wigner function:

$$W_\rho(q,p) = \frac{1}{\pi\hbar}\int_{-\infty}^{+\infty} {}_{\hat{q}}\langle q+x|\rho|q-x\rangle_{\hat{q}}\, e^{2ipx/\hbar} dx$$



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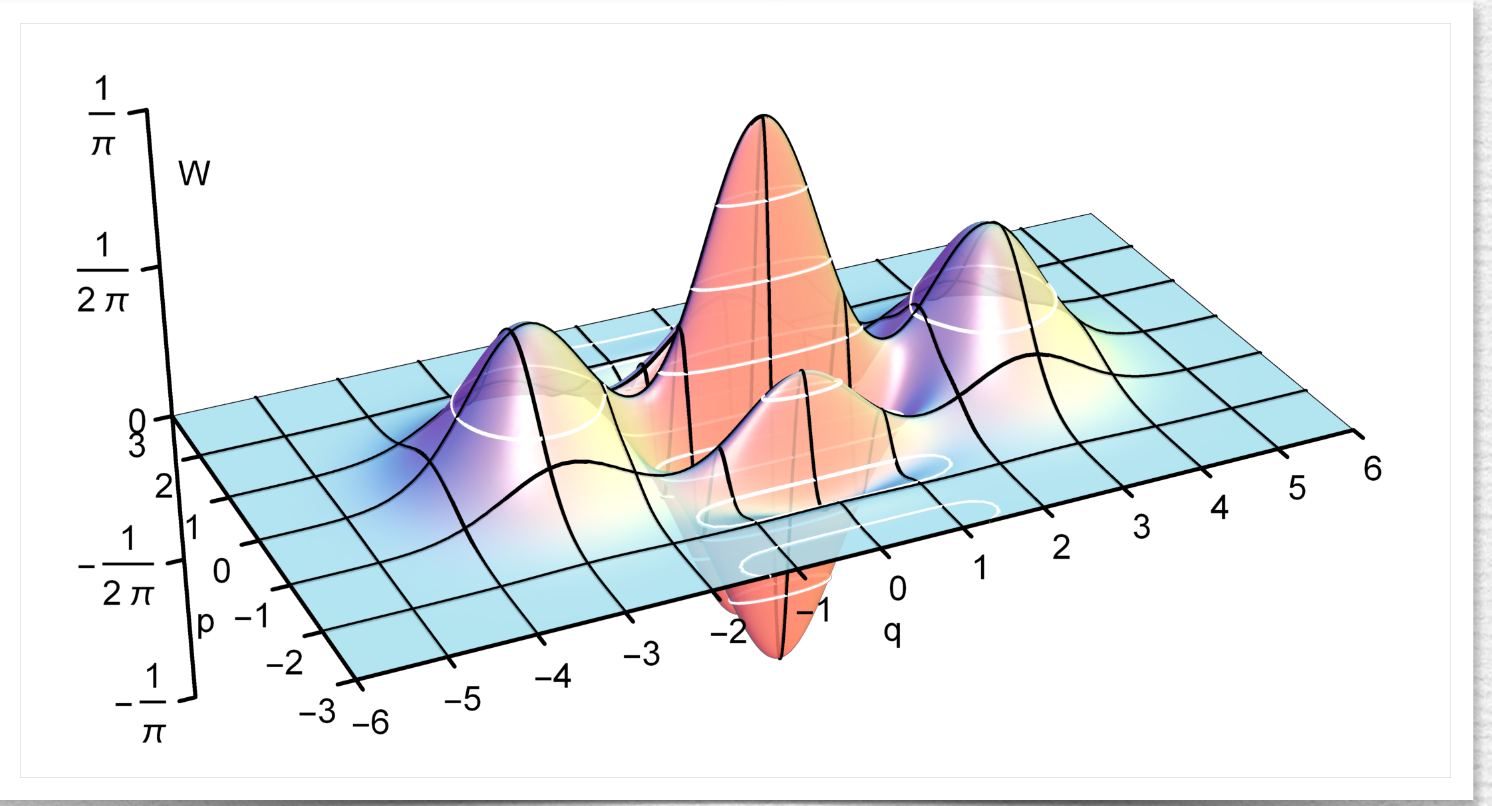
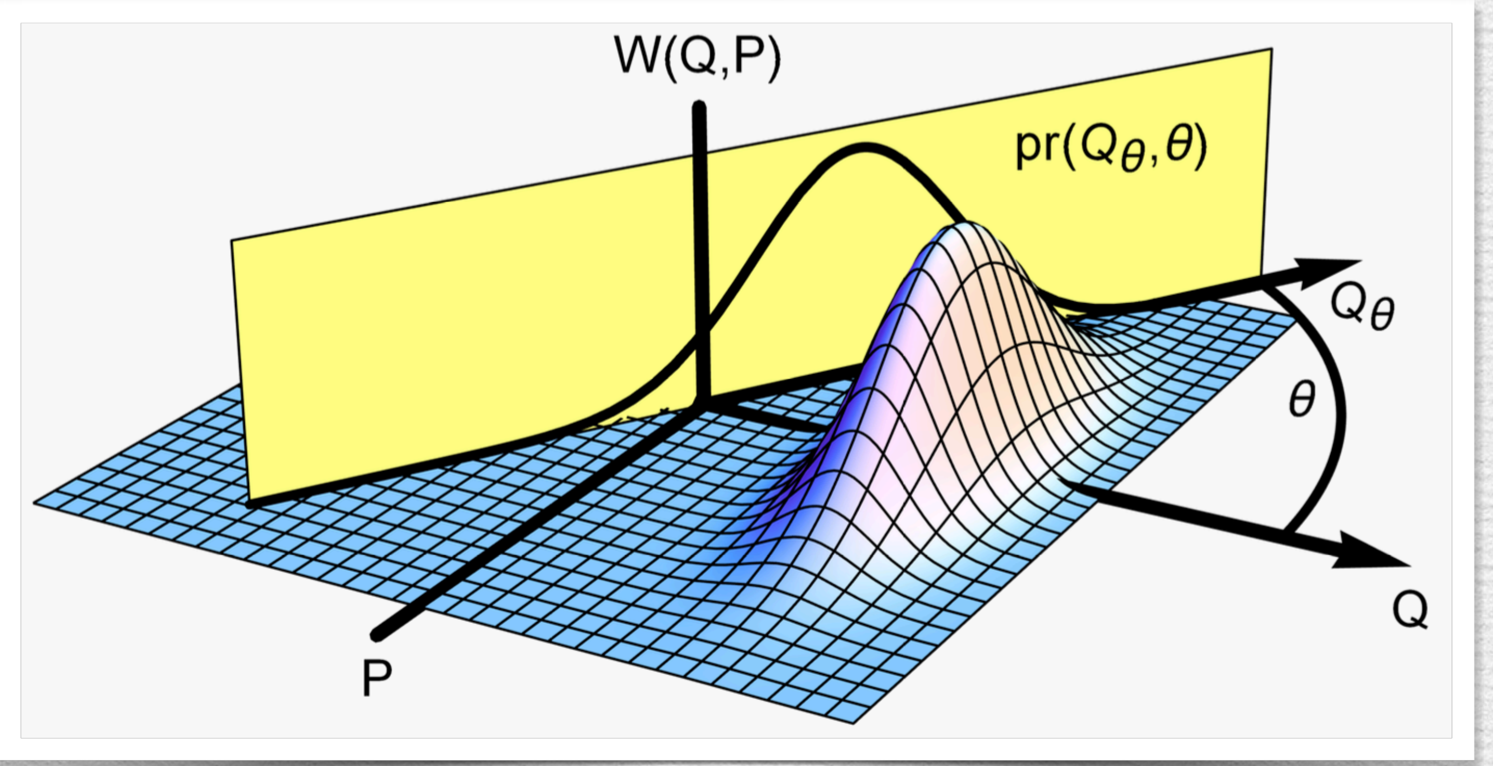


The marginals of the Wigner function are the quadrature probability distributions:

$$\int_{-\infty}^{+\infty} W_\rho(q,p)dp = {}_{\hat{q}}\langle q|\rho|q\rangle_{\hat{q}}$$



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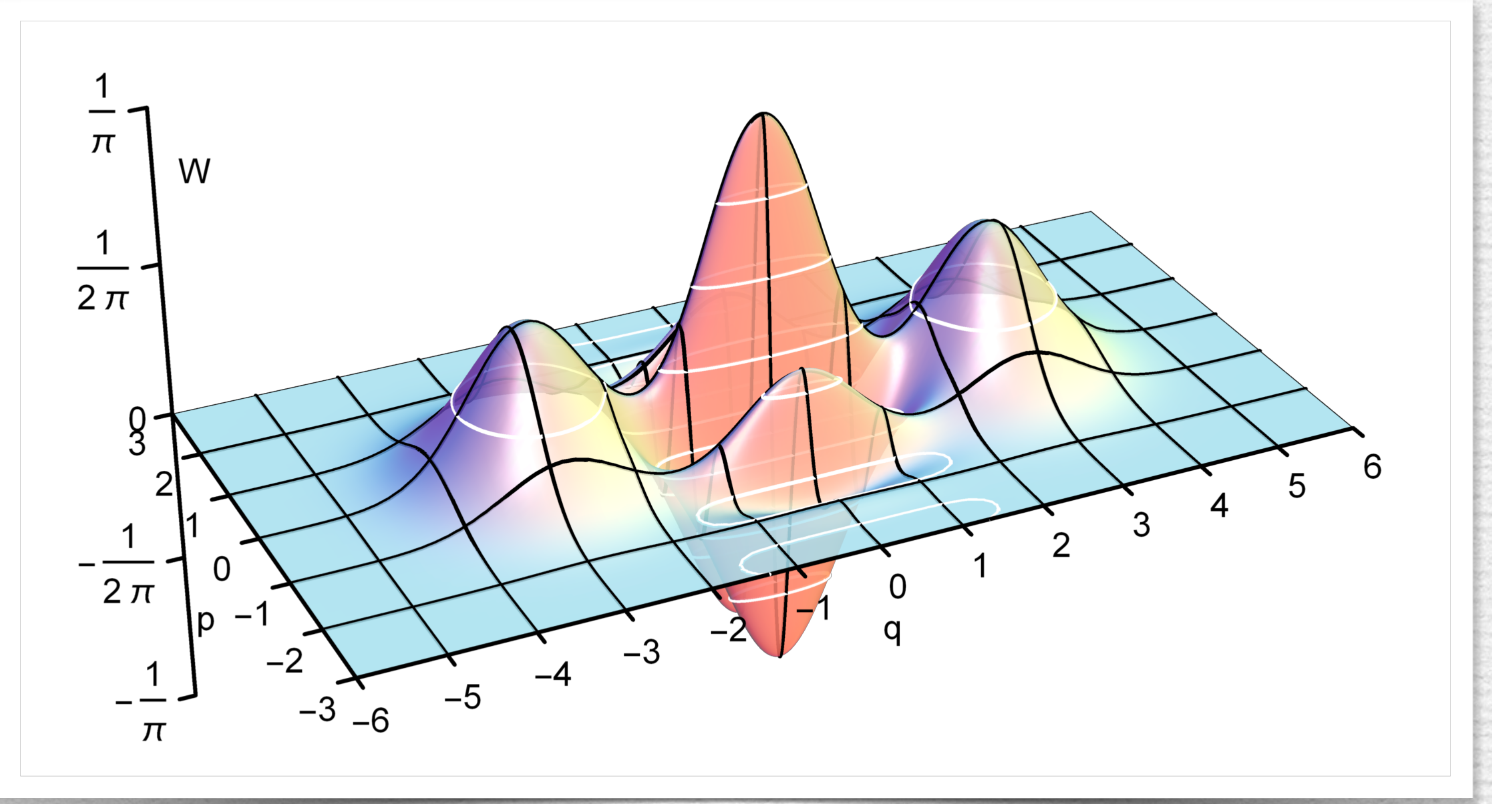

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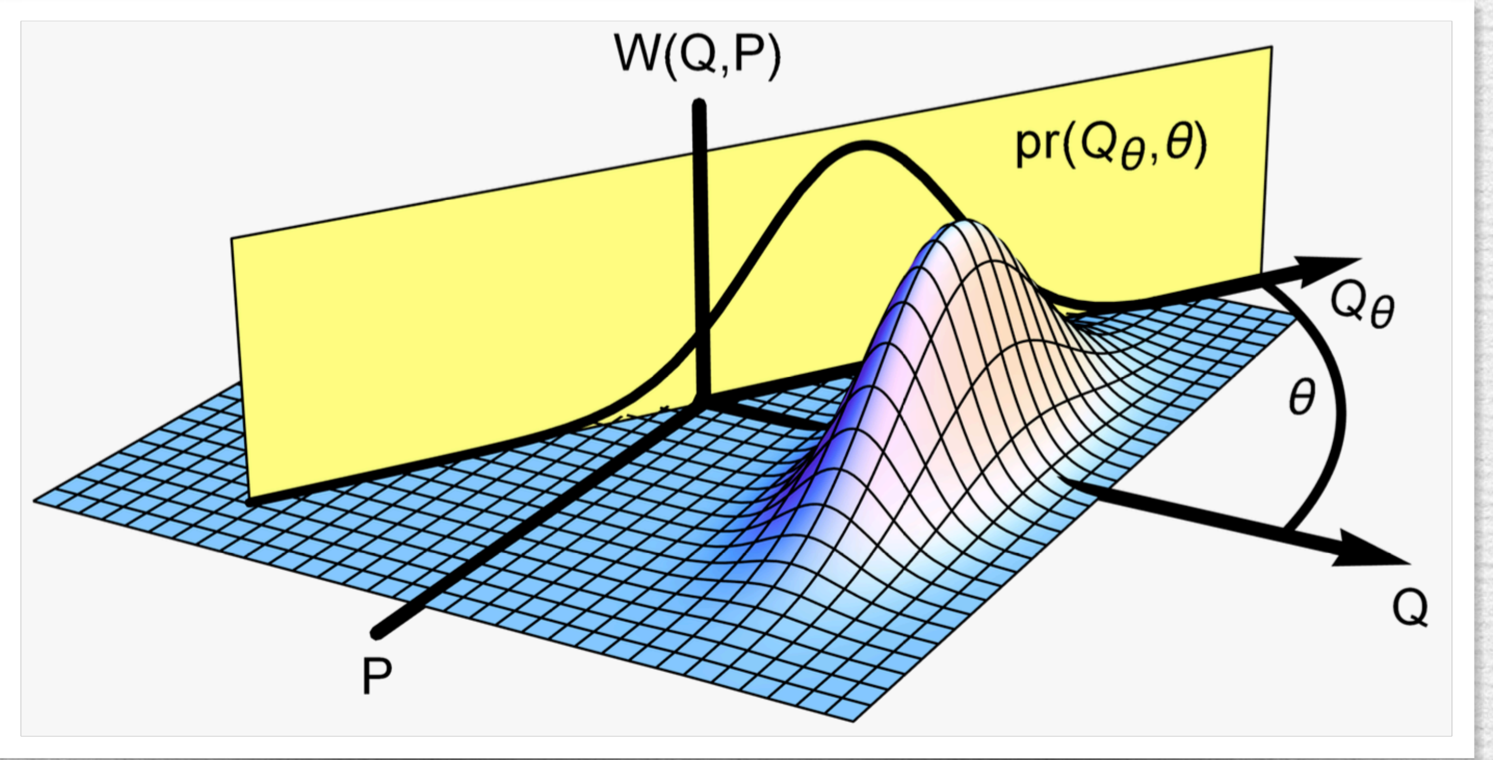

Wigner negativity → Nonexistence of a joint probability distribution

Contextuality

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Correlations are consistent within all contexts,
but cannot be explained globally

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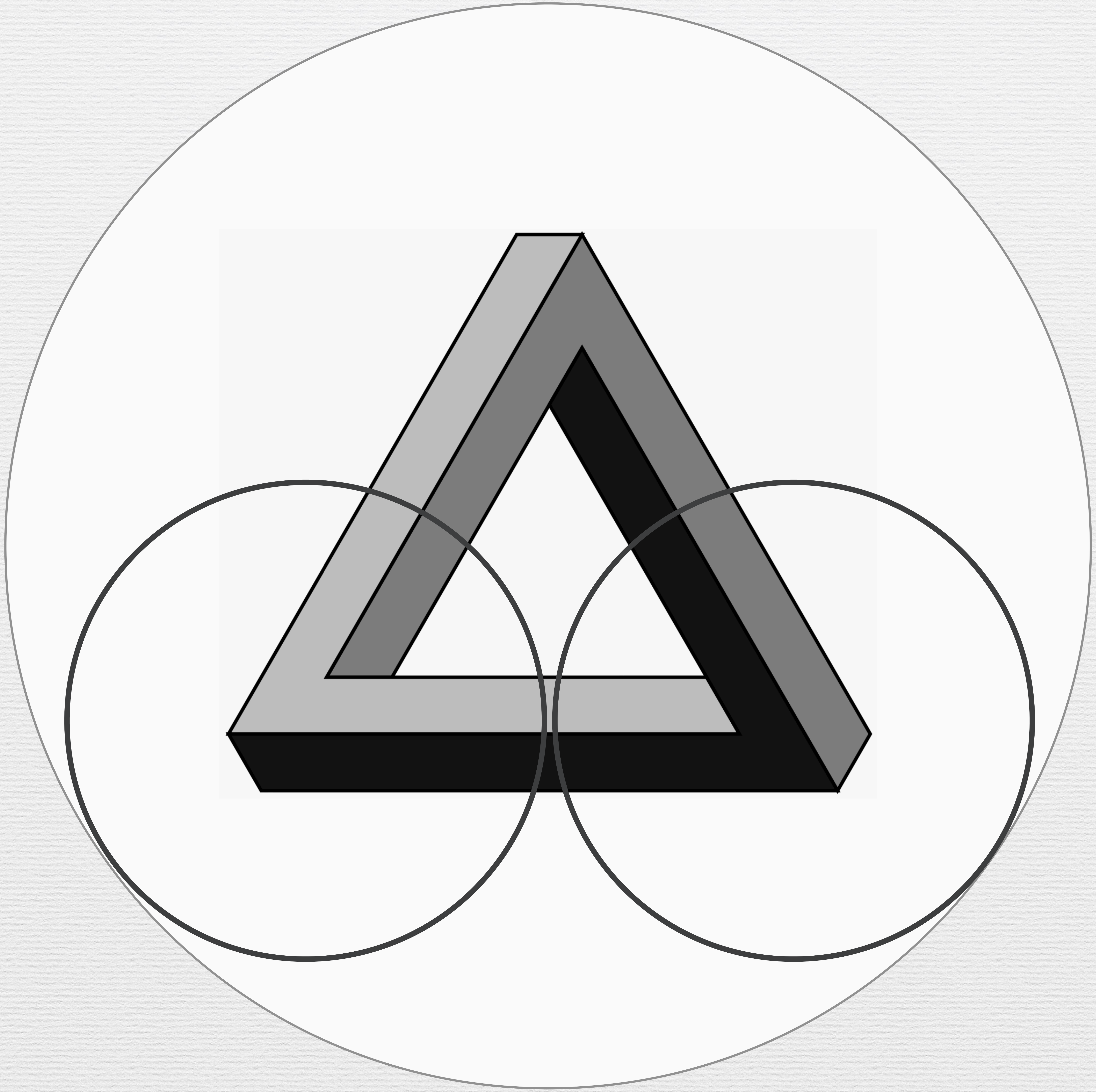

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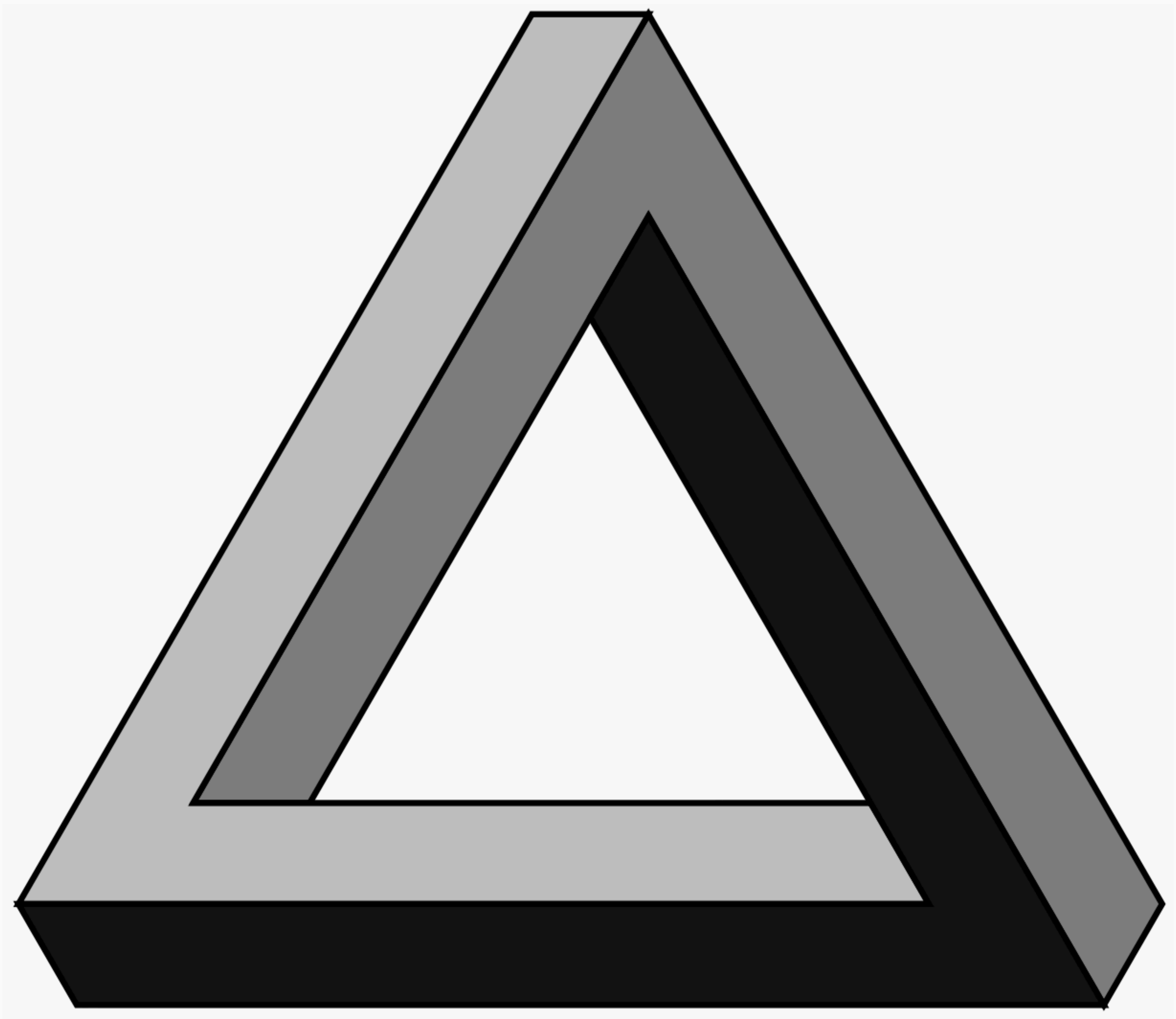

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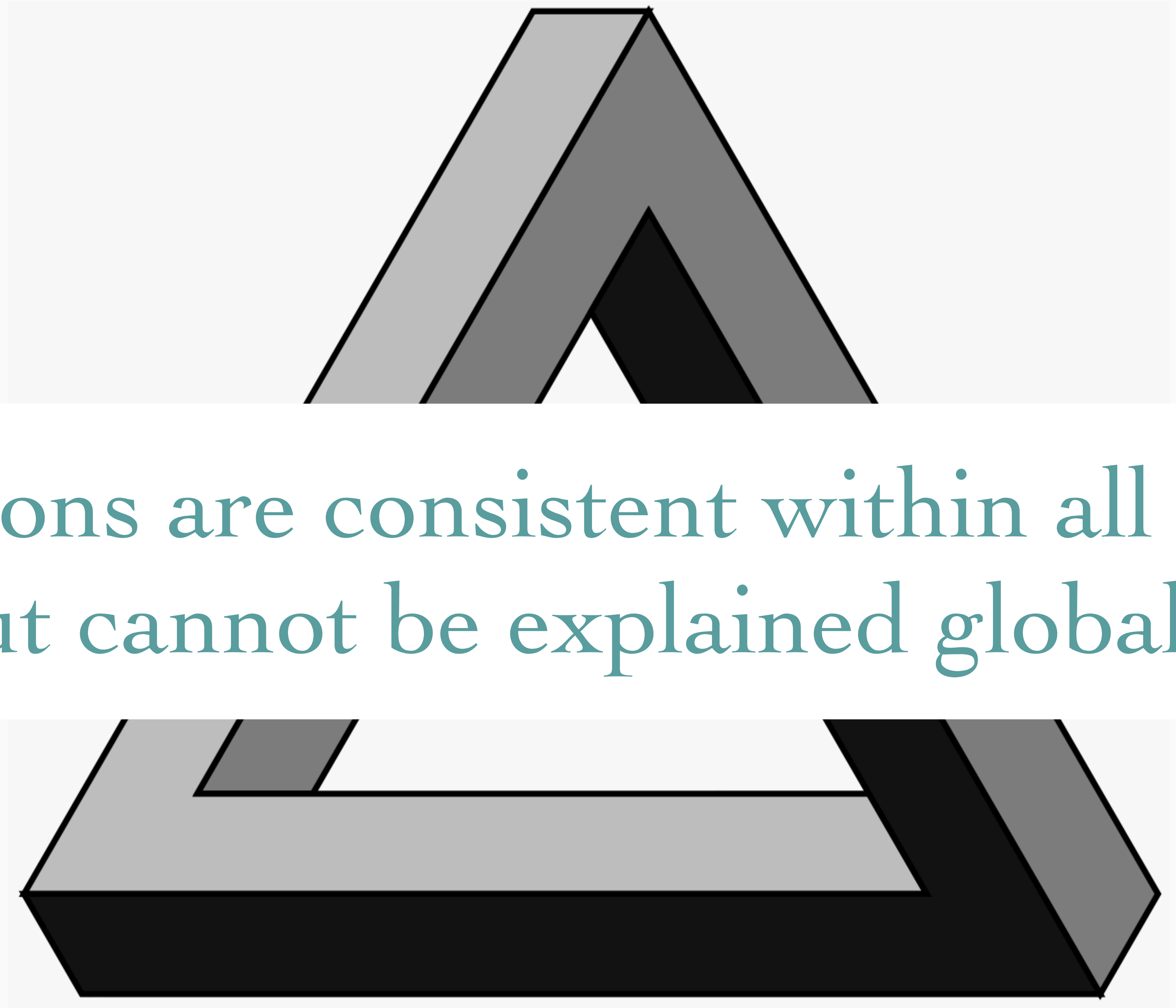

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Quantum (CV) setting

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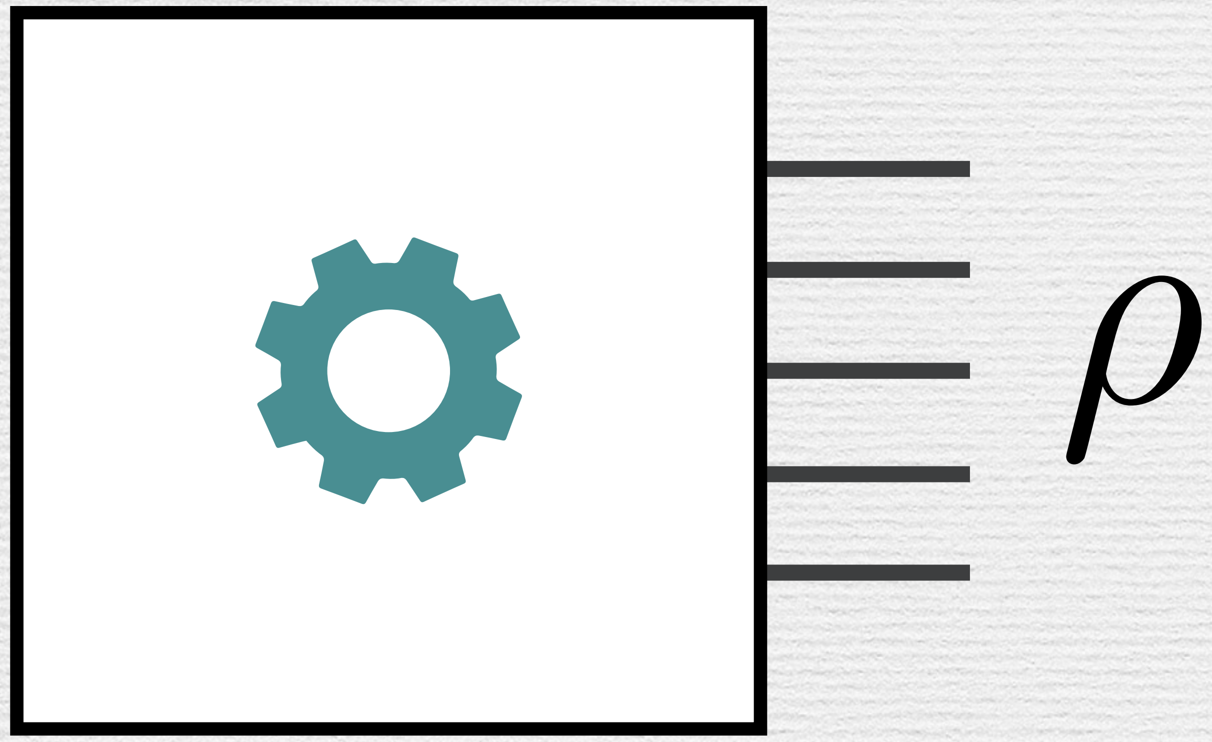
State:
A physical system
with multiple subsystems



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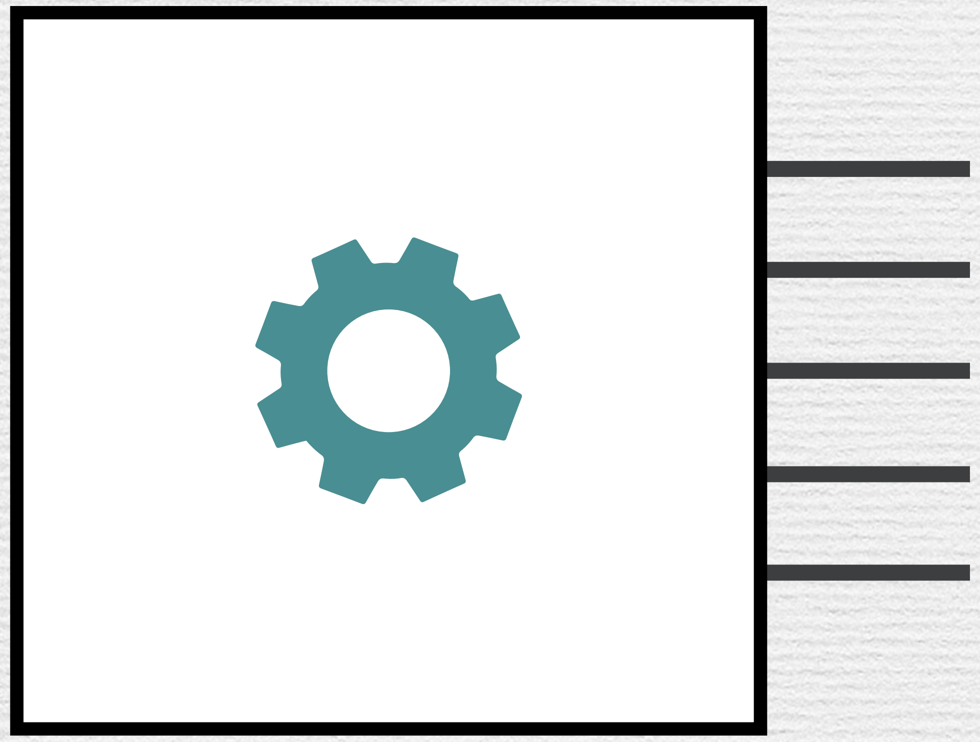

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Measurement scenario:

A set of (CV) observables,
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$\hat{q}_1 \quad \hat{q}_2 \quad \hat{p}_1 \quad \hat{p}_1 - \hat{p}_3 \quad \ldots$

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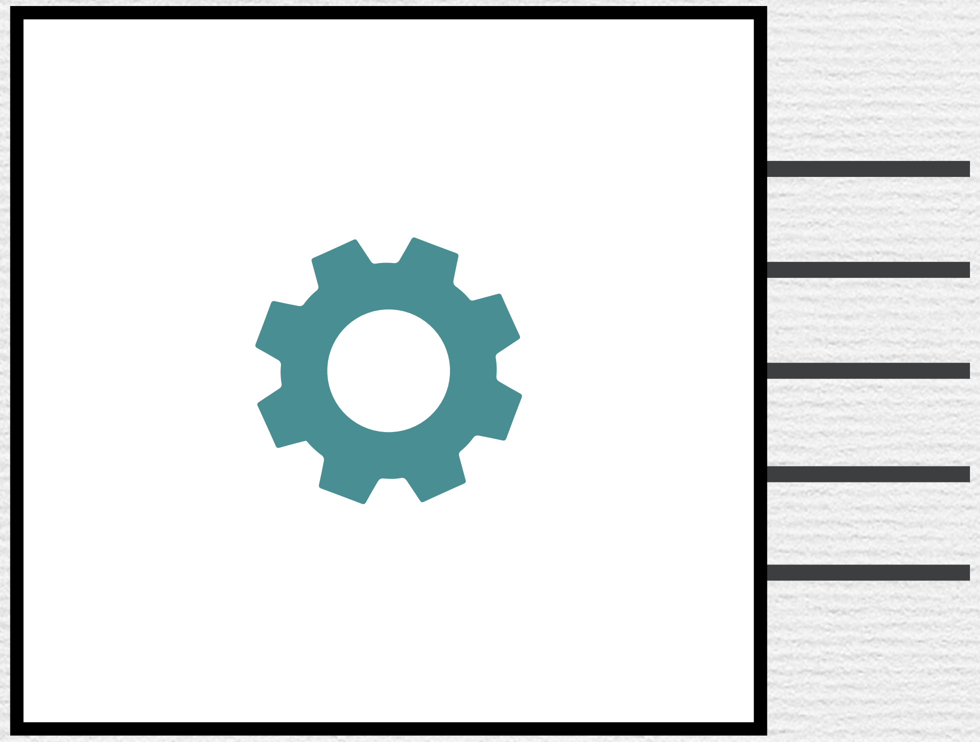

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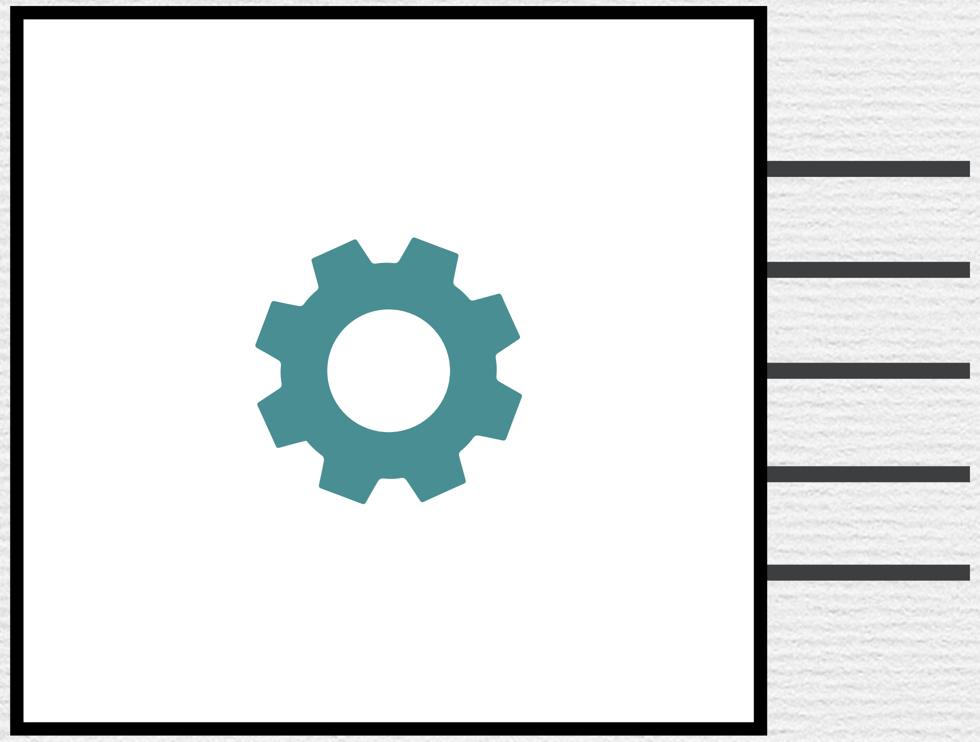

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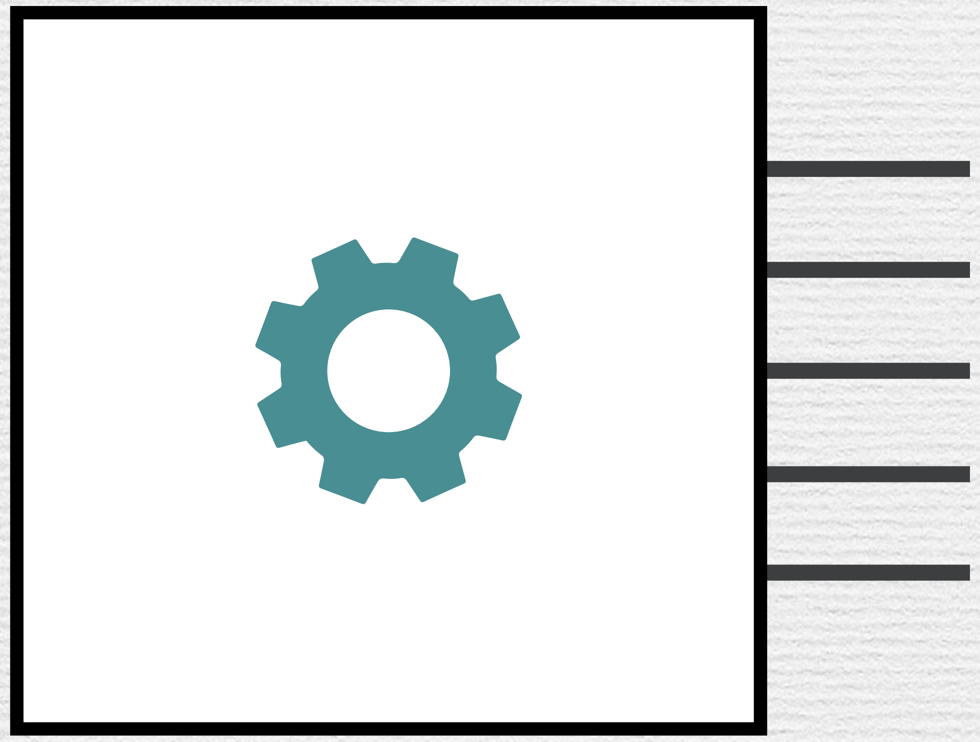

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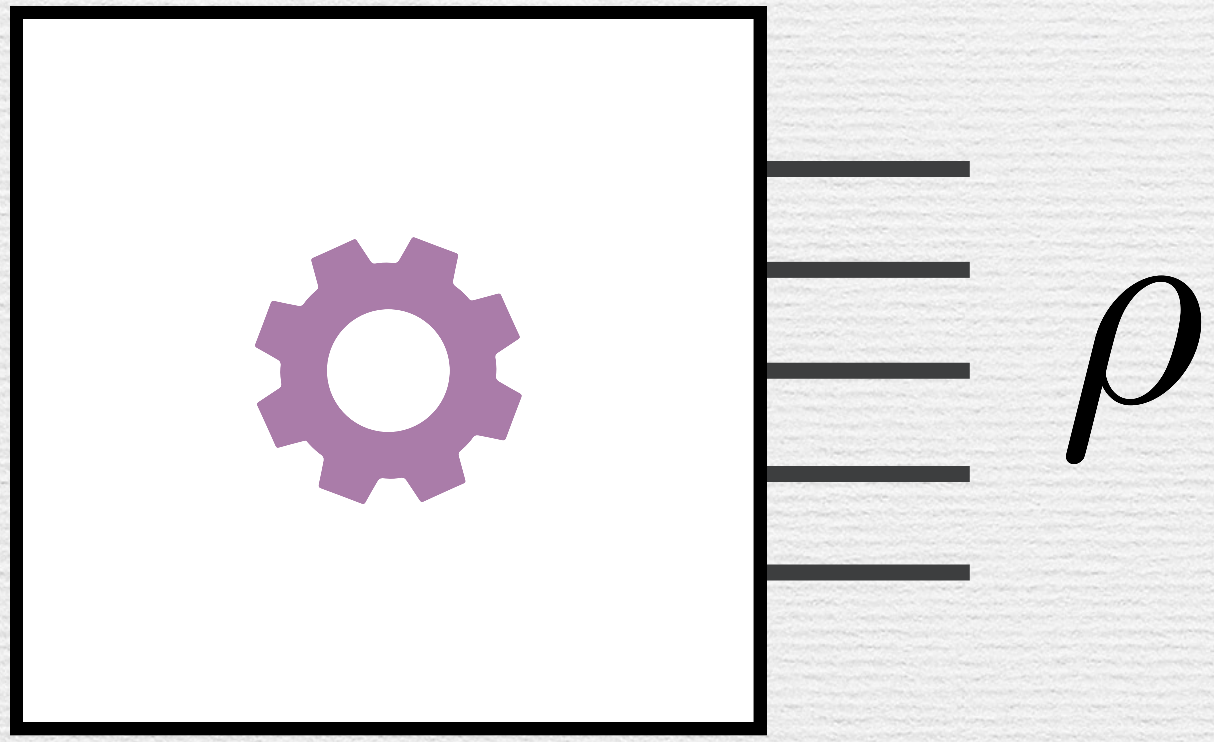
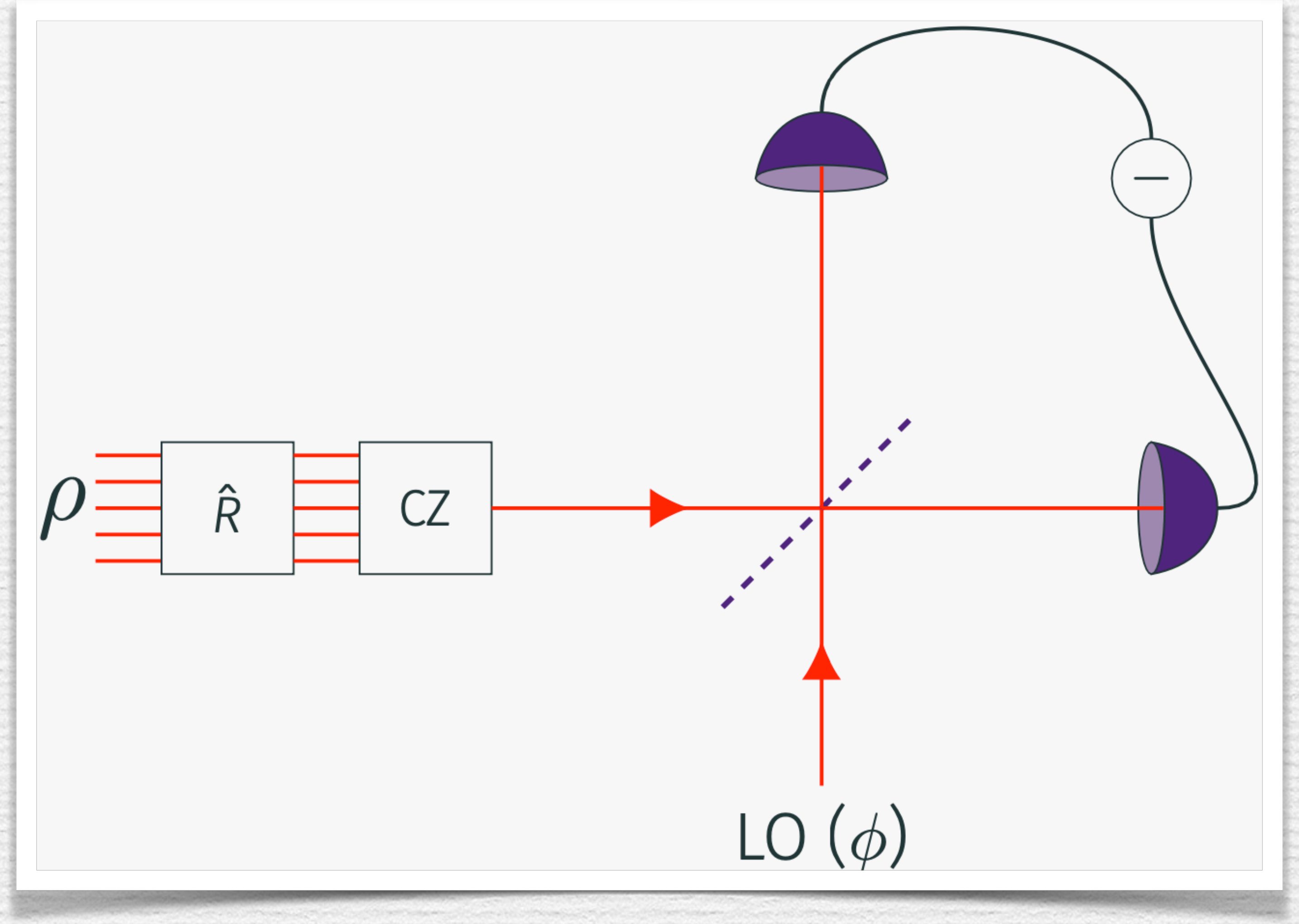


Measurement scenario:

All generalised quadrature measurements



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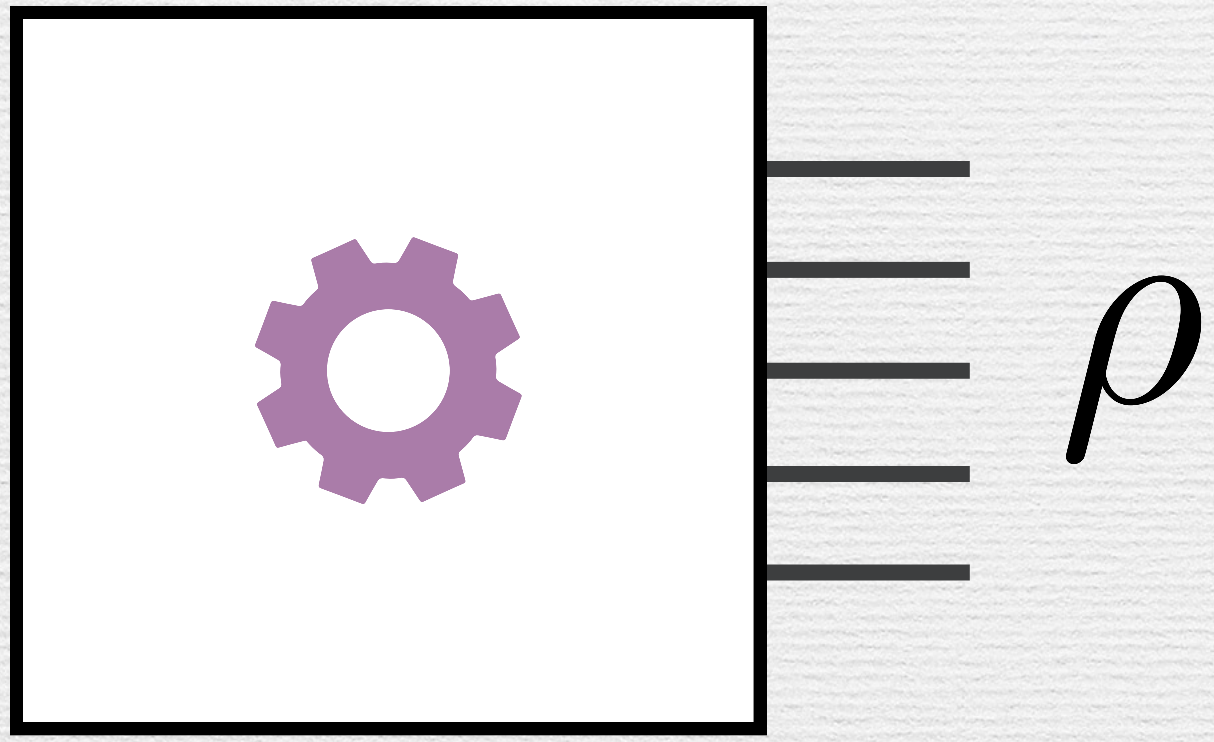

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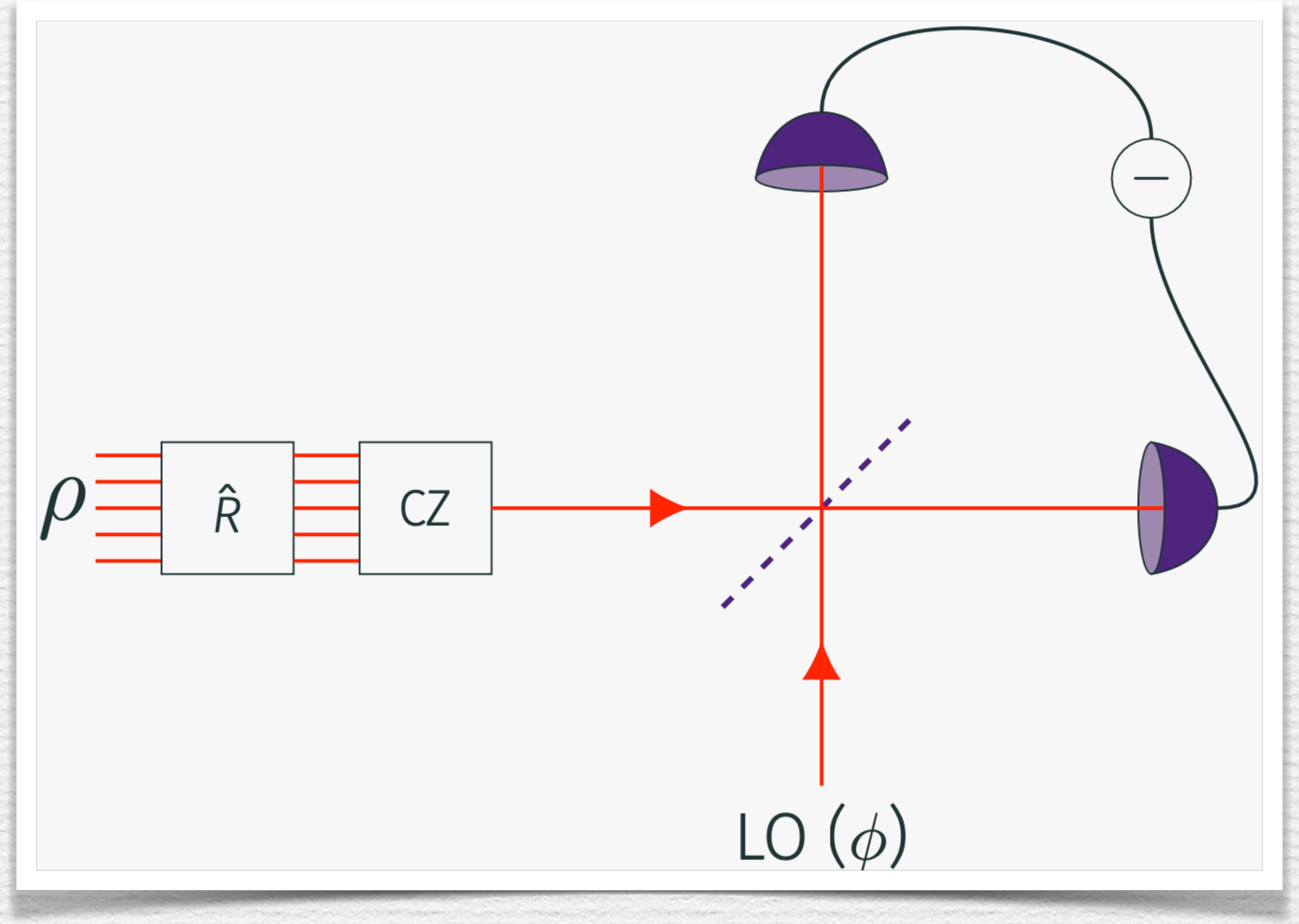

Main result

$W_\rho < 0 \quad \Leftrightarrow \quad$ contextuality

Proof technique

- Measure-theoretic tools
- Wigner function is the unique function which provides the right marginals
- Main technicality: Wigner is a probability distribution over phase space, hidden-variables are distributed over contexts

$\longrightarrow$ Resolved using the linearity of value assignments

The equivalence

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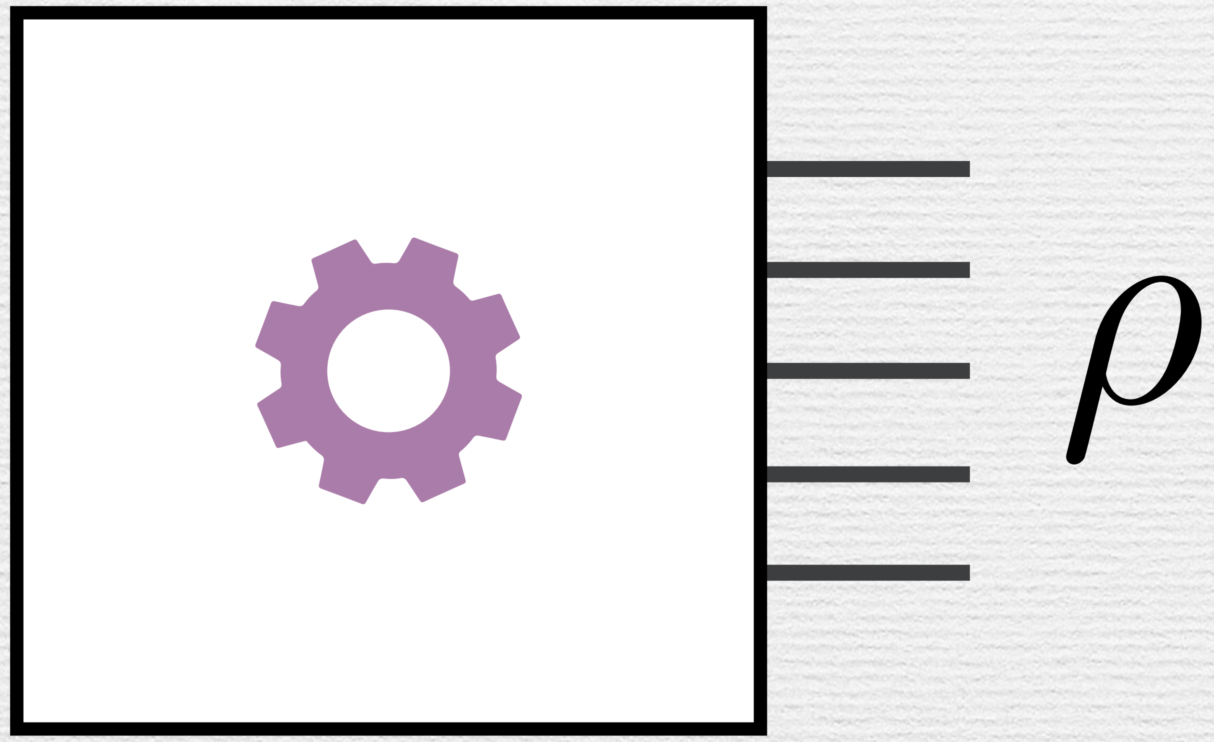

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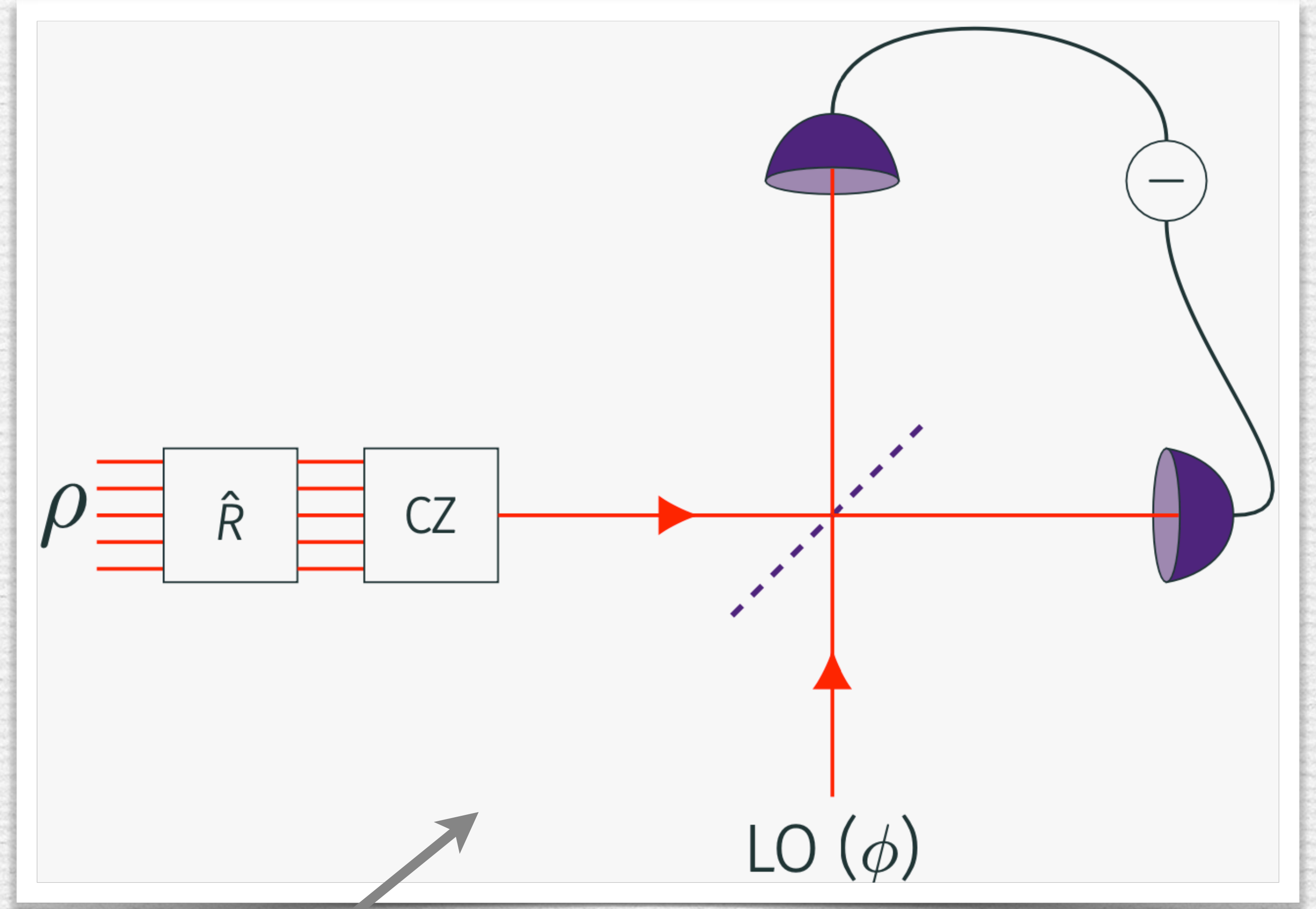

Main result

for this measurement scenario

$W_\rho < 0 \quad \Leftrightarrow \quad$ contextuality

Open questions

- Quantitative version of the theorem: Contextuality Fraction *vs* Negativity Volume

- Minimal number of quadratures such that the result holds?

- Genuine CV Bell inequalities and CV contextuality inequalities: finding the simplest measurement scenarios using SDPs

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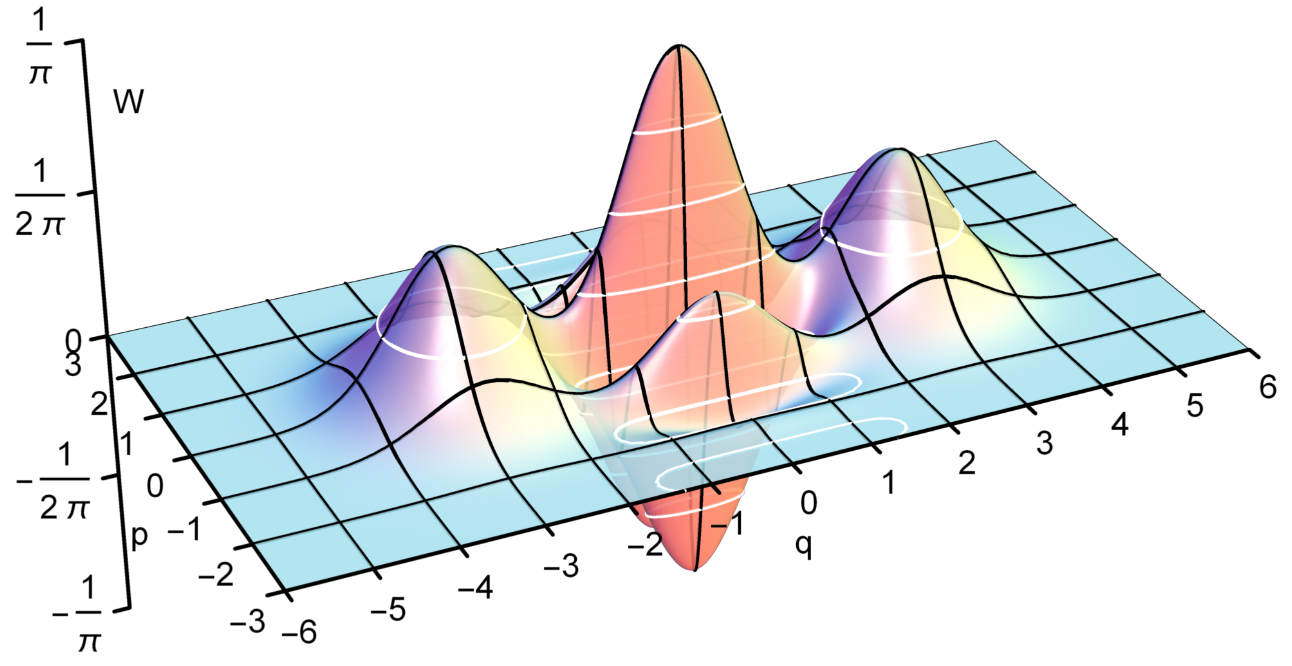

QuiDiQua

Thank you!

ENS | PSL

Inria

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Equivalence results in CV

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