Interferometric measurement of the quadrature coherence scale using two replicas of a quantum optical state

C. GRIFFET, MATTHIEU ARNHEM, S. DE BIEVRE, N.J. CERF

LILLE 2023

Multi-copy uncertainty observable inducing a symplectic-invariant uncertainty relation in position and momentum phase space

Anaelle Hertz,[1] Ognyan Oreshkov,[1] and Nicolas J. Cerf[1]

[1] *Centre for Quantum Information and Communication, École polytechnique de Bruxelles, CP 165, Université libre de Bruxelles, 1050 Brussels, Belgium*

We define an *uncertainty observable* acting on several replicas of a continuous-variable state, whose measurement induces phase-space uncertainty relations for a single copy of the state. By exploiting the Schwinger representation of angular momenta in terms of bosonic operators, this observable can be constructed so as to be invariant under symplectic transformations (rotation and squeezing in phase space). We first design a *two-copy* uncertainty observable, which is a discrete-spectrum operator vanishing with certainty if and only if it is applied on (two replicas of) any pure Gaussian state centered at the origin. The non-negativity of its variance translates into the Schrödinger-Robertson uncertainty relation. We then extend our construction to a *three-copy* uncertainty observable, which exhibits additional invariance under displacements (translations in phase space) so that it vanishes on every pure Gaussian state. The resulting invariance under all Gaussian unitaries makes this observable a natural tool to capture the phase-space uncertainty – or the deviation from pure Gaussianity – of continuous-variable bosonic states. In particular, it suggests that the Shannon entropy associated with the measurement of this observable provides a symplectic-invariant entropic measure of uncertainty in position-momentum phase space.

Multicopy observables for the detection of optically nonclassical states

Matthieu Arnhem,[1] Célia Griffet,[1] and Nicolas J. Cerf[1]

[1] *Centre for Quantum Information and Communication, École polytechnique de Bruxelles, CP 165, Université libre de Bruxelles, 1050 Brussels, Belgium*

Distinguishing quantum states that admit a classical counterpart from those that
classicality has long been a central issue in quantum optics. Finding an implem
certifying optical nonclassicality (i.e, the incompatibility with a statistical mixtur
is of major importance as it often is a prerequisite to quantum information
of conditions for detecting whether a quantum state exhibits optical nonc
based on matrices of moments of the optical field [E. V. Shchukin and
043808 (2005)]. Here, we design optical nonclassicality observables
quantum state and whose expectation value coincides with the d
providing witnesses of optical nonclassicality that overcome
multicopy observables are used to construct a family of ph
linear optical operations and photon number detectors
it non-
erion
s)

Measuring polynomial functions of states

Todd A. Brun[1,*]

[1] *Communication Sciences Institute,*

University of Southern California, Los Angeles, CA 90089-2565

(Dated: September 2003)

Abstract

In this paper I show that any mth-degree polyno
matrix ρ can be determined by finding the expectatio
without performing state tomography. Since a circuit ex
observable, in principle one can find a circuit
many runs. I construct some simp

Interferometric measurement of the quadrature coherence scale using two replicas of a quantum optical state

Matthieu Arnhem,[1,2] Stephan De Bièvre,[3] and Nicolas J. Cerf[1,4]

Communication, École polytechnique de Bruxelles,
1050 Brussels, Belgium
77146 Olomouc, Czech Republic
Lille, France
USA

Measuring the quadrature coherence scale on a cloud quantum computer

Aaron Z. Goldberg,[1,2] Guillaume S. Thekkadath,[1] and Khabat Heshami[1,2,3]

[1] *National Research Council of Canada, 100 Sussex Drive, Ottawa, Ontario K1N 5A2, Canada*
[2] *Department of Physics, University of Ottawa, Advanced Research Complex, 25 Templeton Street, Ottawa, Ontario K1N 6N5, Canada*
[3] *Institute for Quantum Science and Technology, Department of Physics and Astronomy, University of Calgary, Alberta T2N 1N4, Canada*

Coherence underlies quantum phenomena, yet it is manifest in classical theories; delineating coherence's role is a fickle business. The quadrature coherence scale (QCS) was invented to remove such ambiguity, quantifying quantum features of any single-mode bosonic system without choosing a preferred orientation of phase space. The QCS is defined for any state, reducing to well-known quantities in appropriate limits including Gaussian and pure states, and, perhaps most importantly for a coherence measure, it is highly sensitive to decoherence. Until recently, it was unknown how to measure the QCS; we here report on an initial measurement of the QCS for squeezed light and thermal states of light. This is performed using Xanadu's machine Borealis, accessed through the
the configurable beam splitters and photon-number-resolving detectors essential
data and theory match well, certifying the usefulness of interferometers

Accessing continuous-variable entanglement witnesses with multimode spin observables

Célia Griffet,[1,*] Tobias Haas,[1,†] and Nicolas J. Cerf[1,‡]

[1] *Centre for Quantum Information and Communication, École polytechnique de Bruxelles, CP 165, Université libre de Bruxelles, 1050 Brussels, Belgium*

We present several measurement schemes for accessing separability criteria for continuous-variable
bipartite quantum systems. Starting from moments of the bosonic mode operators, criteria suitable
to witness entanglement are expressed in terms of multimode spin observables via the J
Schwinger map. These observables are typically defined over a few replicas of the sta
nd can be transformed into simple photon-number measurements by passiv
easurement schemes require only a handful of measurements, ther
ect entanglement without the need for costly state tomo
sically relevant states (Gaussian, mixed Schrödin
al experimental imperfections is shown t

The multicopy method by T. Brun

Measuring polynomial functions of states

Todd A. Brun[1,*]

[1]*Communication Sciences Institute,*

University of Southern California, Los Angeles, CA 90089-2565

(Dated: September 2003)

Abstract

In this paper I show that any mth-degree polynomial function of the elements of the density matrix ρ can be determined by finding the expectation value of an observable on m copies of ρ, without performing state tomography. Since a circuit exists which can approximate the measurement of any observable, in principle one can find a circuit which will estimate any such polynomial function by averaging over many runs. I construct some simple examples and compare these results to existing procedures.

The multicopy technique [1]

Goal : Evaluate the following polynomial function

$$f(\rho) = \sum_{i_1,j_1,\ldots,i_m,j_m} c_{i_1 j_1 \ldots i_m j_m} \rho_{i_1 j_1} \rho_{i_2 j_2} \cdots \rho_{i_m j_m} \qquad \hat{\rho} = \sum_{i,j=0}^{d-1} \rho_{ij} |i\rangle\langle j|$$

Then there always exists a multicopy operator whose mean value on multiple copies of the state evaluate the function :

$$f(\rho) = \langle\langle \hat{A}_f \rangle\rangle_{\hat{\rho}} = Tr\left(\hat{A}_f \hat{\rho}^{\otimes m}\right)$$

where

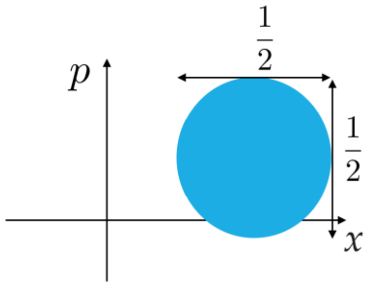
$$\hat{\rho}^{\otimes m} = \sum_{i_1,j_1,\ldots,i_m,j_m} \rho_{i_1 j_1} \cdots \rho_{i_m j_m} |i_1\rangle\langle j_1| \otimes \cdots |i_m\rangle\langle j_m|$$

$$\hat{A}_f = \sum_{i_1,j_1,\ldots,i_m,j_m} c_{i_1 j_1 \ldots i_m j_m} \hat{A}_{i_1 j_1 \ldots i_m j_m}$$

[1] T.A. Brun, Measuring polynomial functions of states, Quant. Inf. Comp. **4**, 401 (2004)

Optical nonclassicality

Glauber-Sudarshan P-function [2] : $\hat{\rho} = \int P(\alpha)|\alpha\rangle\langle\alpha|d^2\alpha$ where $|\alpha\rangle$ coherent state



A state is said to be *classical* if its Glauber-Sudarshan P-function is a probability distribution :

$$P(\alpha) = P_{cl}(\alpha)$$

If the Glauber-Sudarshan P-function of a state $\hat{\rho}$ fails to be interpreted as a probability distribution, then the state is *nonclassical*.

$$P(\alpha) \neq P_{cl}(\alpha)$$

Known witnesses: Mandel parameter, squeezing parameter, **Shchukin *et al.* hierarchy of nonclassicality witnesses**, **quadrature coherence scale**

[2] R. J. Glauber, Phys. Rev. **131**, 2766 (1963) & E. C. G. Sudarshan, Phys. Rev. Lett. **10**, 277 (1963)

Multicopy observables for the detection of optically nonclassical states

Matthieu Arnhem,[1] Célia Griffet,[1] and Nicolas J. Cerf[1]

[1] *Centre for Quantum Information and Communication, École polytechnique de Bruxelles, CP 165, Université libre de Bruxelles, 1050 Brussels, Belgium*

Distinguishing quantum states that admit a classical counterpart from those that exhibit nonclassicality has long been a central issue in quantum optics. Finding an implementable criterion certifying optical nonclassicality (i.e, the incompatibility with a statistical mixture of coherent states) is of major importance as it often is a prerequisite to quantum information processes. A hierarchy of conditions for detecting whether a quantum state exhibits optical nonclassicality can be written based on matrices of moments of the optical field [E. V. Shchukin and W. Vogel, Phys. Rev. A 72, 043808 (2005)]. Here, we design optical nonclassicality observables that act on several replicas of a quantum state and whose expectation value coincides with the determinant of these matrices, hence providing witnesses of optical nonclassicality that overcome the need for state tomography. These multicopy observables are used to construct a family of physically implementable schemes involving linear optical operations and photon number detectors.

Physical Review A, 106: 043705, October 2022.

Nonclassicality and matrix of moments [3]

Shchukin, Richter and Vogel hierarchy of nonclassicality criteria based on normally ordered moments

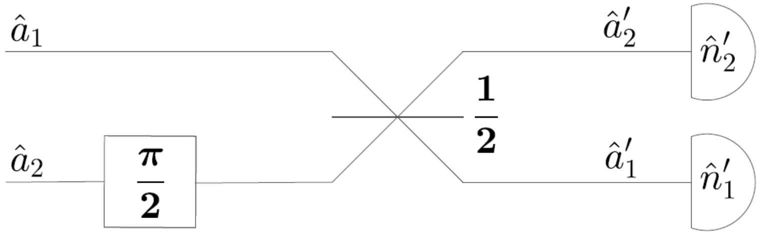
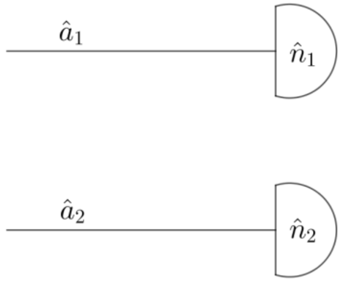
$$d_N = \begin{vmatrix} 1 & \langle \hat{a} \rangle & \langle \hat{a}^\dagger \rangle & \langle \hat{a}^2 \rangle & \langle \hat{a}^\dagger \hat{a} \rangle & \langle \hat{a}^{\dagger 2} \rangle & \dots \\ \langle \hat{a}^\dagger \rangle & \langle \hat{a}^\dagger \hat{a} \rangle & \langle \hat{a}^{\dagger 2} \rangle & \langle \hat{a}^\dagger \hat{a}^2 \rangle & \langle \hat{a}^{\dagger 2} \hat{a} \rangle & \langle \hat{a}^{\dagger 3} \rangle & \dots \\ \langle \hat{a} \rangle & \langle \hat{a}^2 \rangle & \langle \hat{a}^\dagger \hat{a} \rangle & \langle \hat{a}^3 \rangle & \langle \hat{a}^\dagger \hat{a}^2 \rangle & \langle \hat{a}^{\dagger 2} \hat{a} \rangle & \dots \\ \langle \hat{a}^{\dagger 2} \rangle & \langle \hat{a}^{\dagger 2} \hat{a} \rangle & \langle \hat{a}^{\dagger 3} \rangle & \langle \hat{a}^{\dagger 2} \hat{a}^2 \rangle & \langle \hat{a}^{\dagger 3} \hat{a} \rangle & \langle \hat{a}^{\dagger} \rangle & \dots \\ \langle \hat{a}^\dagger \hat{a} \rangle & \langle \hat{a}^\dagger \hat{a}^2 \rangle & \langle \hat{a}^{\dagger 2} \hat{a} \rangle & \langle \hat{a}^\dagger \hat{a}^3 \rangle & \langle \hat{a}^{\dagger 2} \hat{a}^2 \rangle & \langle \hat{a}^{\dagger 3} \hat{a} \rangle & \dots \\ \langle \hat{a}^2 \rangle & \langle \hat{a}^3 \rangle & \langle \hat{a}^\dagger \hat{a}^2 \rangle & \langle \hat{a}^4 \rangle & \langle \hat{a}^\dagger \hat{a}^3 \rangle & \langle \hat{a}^{\dagger 2} \hat{a}^2 \rangle & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{vmatrix}$$

If there exists a N s.t. $d_N < 0$ then the state is optically nonclassical

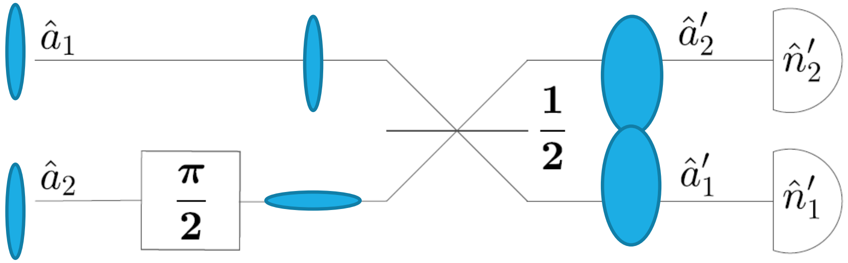
Size	Principal minor	Fock states	Squeezed states	Odd cat states	Even cat states
2	$d_{12} = d_{13}$	n	$\sinh^2(r)$	$\lvert\beta\rvert^2 \frac{N_+}{N_-}$	$\lvert\beta\rvert^2 \frac{N_-}{N_+}$
2	$d_{14} = d_{16}$	$n(n-1)$	$2\sinh^4(r)$	0	0
2	d_{15}	$-n$	$\cosh(2r)\sinh^2(r)$	$\lvert\beta\rvert^4(1 - \frac{N_+^2}{N_-^2})$	$\lvert\beta\rvert^4(1 - \frac{N_-^2}{N_+^2})$
2	d_{23}	n^2	$-\sinh^2(r)$	$\lvert\beta\rvert^4(\frac{N_+^2}{N_-^2} - 1)$	$\lvert\beta\rvert^4(\frac{N_-^2}{N_+^2} - 1)$
2	$d_{24} = d_{26} = d_{34} = d_{36}$	$n^2(n-1)$	$\sinh^4(r)(\cosh^2(r) + 2\sinh^2(r))$	$\lvert\beta\rvert^6 \frac{N_+}{N_-}$	$\lvert\beta\rvert^6 \frac{N_-}{N_+}$
2	$d_{25} = d_{35}$	$n^2(n-1)$	$\sinh^4(r)(\cosh^2(r) + 2\sinh^2(r))$	$\lvert\beta\rvert^6 \frac{N_+}{N_-}$	$\lvert\beta\rvert^6 \frac{N_-}{N_+}$
2	$d_{45} = d_{56}$	$n^2(n-1)^2$	$\frac{1}{2}(5 - 3\cosh(2r))\sinh^4(r)$	$\lvert\beta\rvert^8(1 - \frac{N_+^2}{N_-^2})$	$\lvert\beta\rvert^8(1 - \frac{N_-^2}{N_+^2})$
2	d_{46}	$n^2(n-1)^2$	$-2(1 + 3\cosh(2r))\sinh^4(r)$	0	0
3	d_{123}	n^2	$-\sinh^2(r)$	$\lvert\beta\rvert^4(\frac{N_+^2}{N_-^2} - 1)$	$\lvert\beta\rvert^4(\frac{N_-^2}{N_+^2} - 1)$
3	$d_{124} = d_{126} = d_{134} = d_{136}$	$n^2(n-1)$	$2\sinh^6(r)$	0	0
3	$d_{125} = d_{135}$	$-n^2$	$\sinh^4(r)\cosh(2r)$	$\lvert\beta\rvert^6 \frac{N_+}{N_-}(1 - \frac{N_+^2}{N_-^2})$	$\lvert\beta\rvert^6 \frac{N_-}{N_+}(1 - \frac{N_-^2}{N_+^2})$
3	$d_{145} = d_{156}$	$-n^2(n-1)$	$-2\sinh^6(r)$	0	0
3	d_{146}	$n^2(n-1)^2$	$-4\cosh(2r)\sinh^4(r)$	0	0
3	$d_{234} = d_{236}$	$n^3(n-1)$	$\frac{1}{2}(1 - 3\cosh(2r))\sinh(r)^4$	$\lvert\beta\rvert^8(\frac{N_+^2}{N_-^2} - 1)$	$\lvert\beta\rvert^8(\frac{N_-^2}{N_+^2} - 1)$
3	d_{235}	$n^3(n-1)$	$-\sinh(r)^4(\cosh(r)^2 + 2\sinh(r)^2)$	$\lvert\beta\rvert^8(\frac{N_+^2}{N_-^2} - 1)$	$\lvert\beta\rvert^8(\frac{N_-^2}{N_+^2} - 1)$
3	$d_{245} = d_{256} = d_{345} = d_{356}$	$n^3(n-1)^2$	$\frac{1}{2}(5 - 3\cosh(2r))\sinh^6(r)$	$\lvert\beta\rvert^{10} \frac{N_+}{N_-}(1 - \frac{N_+^2}{N_-^2})$	$\lvert\beta\rvert^{10} \frac{N_-}{N_+}(1 - \frac{N_-^2}{N_+^2})$
3	$d_{246} = d_{346}$	$n^3(n-1)^2$	$-2(1 + 3\cosh(2r))\sinh^6(r)$	0	0
3	d_{456}	$n^3(n-1)^3$	$-8\sinh^6(r)$	0	0
4	d_{1234}	$n^3(n-1)$	$-2\sinh^6(r)$	0	0
4	d_{1235}	$-n^3$	$-\cosh(2r)\sinh^4(r)$	$-\lvert\beta\rvert^8(\frac{N_+^2}{N_-^2} - 1)^2$	$-\lvert\beta\rvert^8(\frac{N_-^2}{N_+^2} - 1)^2$
4	d_{1456}	$-n^3(n-1)^2$	$-4\sinh^6(r)$	0	0
5	d_{12345}	$-n^4(n-1)$	$2\sinh^8(r)$	0	0

[3] E. Shchukin *et al.*, Phys. Rev. A **71**, 011802(R) (2005)

2-copy nonclassicality observables: d_{23} and d_{15}

$d_{23} = \begin{vmatrix} \langle \hat{a}^\dagger \hat{a} \rangle & \langle \hat{a}^{\dagger 2} \rangle \\ \langle \hat{a}^2 \rangle & \langle \hat{a}^\dagger \hat{a} \rangle \end{vmatrix}$	$d_{15} = \begin{vmatrix} 1 & \langle \hat{a}^\dagger \hat{a} \rangle \\ \langle \hat{a}^\dagger \hat{a} \rangle & \langle \hat{a}^{\dagger 2} \hat{a}^2 \rangle \end{vmatrix}$
Squeezed and even cat states	Fock and odd cat states
 $\hat{D}_{23}' = \frac{1}{2}\left((\hat{n}_1' - \hat{n}_2')^2 - (\hat{n}_1' + \hat{n}_2')\right)$	 $\hat{D}_{15} = \frac{1}{2}\left((\hat{n}_1 - \hat{n}_2)^2 - (\hat{n}_1 + \hat{n}_2)\right)$

Interpretation



$$\langle \hat{D}_{23}' \rangle = \frac{1}{2}\Big(\underbrace{\langle(\hat{n}_1' - \hat{n}_2')^2\rangle}_{=0} - \langle(\hat{n}_1' + \hat{n}_2')\rangle\Big) = \frac{-1}{2}\langle(\hat{n}_1 + \hat{n}_2)\rangle = -\sinh^2(r)$$

Interferometric measurement of the quadrature coherence scale using two replicas of a quantum optical state

Célia Griffet,[1] Matthieu Arnhem,[1,2] Stephan De Bièvre,[3] and Nicolas J. Cerf[1,4]

[1] *Centre for Quantum Information and Communication, École polytechnique de Bruxelles, CP 165, Université libre de Bruxelles, 1050 Brussels, Belgium*

[2] *Department of Optics, Palacký University, 17. listopadu 1192/12, 77146 Olomouc, Czech Republic*

[3] *Univ. Lille, CNRS, UMR 8524, INRIA - Laboratoire Paul Painlevé, F-59000 Lille, France*

[4] *James C. Wyant College of Optical Sciences, University of Arizona, Tucson, AZ 85721, USA*

Assessing whether a quantum state $\hat{\rho}$ is nonclassical (*i.e.*, incompatible with a mixture of coherent states) is a ubiquitous question in quantum optics, yet a nontrivial experimental task because many nonclassicality witnesses are nonlinear in $\hat{\rho}$. In particular, if we want to witness or measure the nonclassicality of a state by evaluating its quadrature coherence scale, this *a priori* requires full state tomography. Here, we provide an experimentally friendly procedure for directly accessing this quantity with a simple linear interferometer involving two replicas (independent and identical copies) of the state $\hat{\rho}$ supplemented with photon number measurements. This finding, that we interpret as an extension of the Hong-Ou-Mandel effect, illustrates the wide applicability of the multicopy interferometric technique in order to circumvent state tomography in quantum optics.

Phys. Rev. A **108**, 023730 – Published 31 August 2023

Quadrature coherence scale [4]

Definition for a state $\hat{\rho}$ of n bosonic modes:

$$\mathcal{C}^2(\hat{\rho}) = \frac{1}{2n\,\mathcal{P}(\hat{\rho})}\left(\sum_{j=1}^{2n} \mathrm{Tr}\,[\hat{\rho},\hat{r}_j][\hat{r}_j,\hat{\rho}]\right)$$

where

$$\begin{cases} \hat{\mathbf{r}} = (\hat{x}_1,\hat{p}_1,\cdots,\hat{x}_n,\hat{p}_n) \\ \mathcal{P}(\hat{\rho}) = \mathrm{Tr}(\hat{\rho}^2) \quad \textbf{(purity)} \end{cases}$$

Properties:

- $\mathcal{C}(\hat{\rho}) > 1$ ➡ $\hat{\rho}$ is nonclassical

  Classical states: QCS lower or equal to 1
- Nonclassicality measure: the larger the QCS, the further it is from $\mathcal{C}_{\mathrm{cl}}$
- Average coherence scale of any pair of conjugated quadratures

Measurement of the quadrature coherence scale [4]

QCS for one mode system: $\mathcal{C}^2(\hat\rho) = -\frac{1}{2\,\mathcal{P}(\hat\rho)}\,\mathrm{Tr}\left([\hat\rho,\hat x]^2 + [\hat\rho,\hat p]^2\right)$ where $\mathcal{P}(\hat\rho) = \mathrm{Tr}(\hat\rho^2)$ **(purity)**

$\mathcal{C}(\hat\rho) > 1$ ➡ $\hat\rho$ is nonclassical

How to measure the QCS?

$$\mathcal{C}^2(\hat\rho) = \frac{\mathcal{N}(\hat\rho)}{\mathcal{P}(\hat\rho)}$$

$$\mathcal{N}(\hat\rho) = -\frac{1}{2}\,\mathrm{Tr}\left([\hat\rho,\hat x]^2 + [\hat\rho,\hat p]^2\right)$$

Separated multicopy measurements

$$\mathcal{P}(\hat\rho) = \mathrm{Tr}(\hat\rho^2)$$

◯ Need of 2 copies

[4] S. De Bièvre, D. B. Horoshko, G. Patera & M. I. Kolobov, Phys. Rev. Lett. **122**, 080402 (2019)

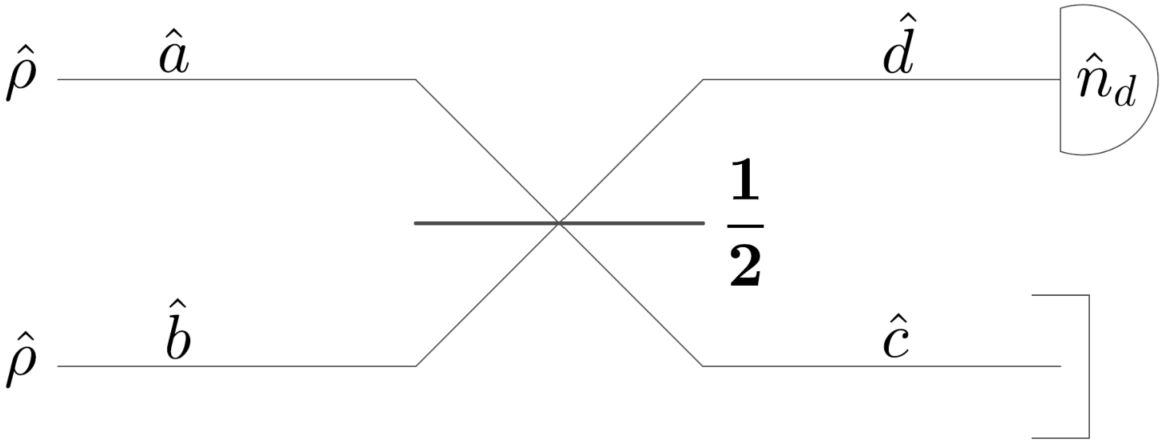
Measurement of the purity $\mathcal{P}(\hat{\rho}) = \mathrm{Tr}(\hat{\rho}^2)$

$$\mathcal{P}(\hat{\rho}) = \mathrm{Tr}\left((\hat{\rho} \otimes \hat{\rho})\,\hat{S}\right)$$

where

$$\hat{S}|\varphi\rangle|\psi\rangle = |\psi\rangle|\varphi\rangle,\ \forall\ |\varphi\rangle, |\psi\rangle$$
$$\hat{S} = e^{i\frac{\pi}{2}(\hat{a}^\dagger - \hat{b}^\dagger)(\hat{a} - \hat{b})}$$

$$\begin{cases} \hat{c} := \hat{U}^\dagger_{BS}\,\hat{a}\,\hat{U}_{BS} = (\hat{a} + \hat{b})/\sqrt{2} \\ \hat{d} := \hat{U}^\dagger_{BS}\,\hat{b}\,\hat{U}_{BS} = (-\hat{a} + \hat{b})/\sqrt{2} \end{cases}$$



➡ $$\hat{S} = e^{i\pi\,\hat{d}^\dagger\hat{d}} = (-1)^{\hat{n}_d}$$

Measurement of the swap operator already implemented in a many-body Bose-Hubbard system [5][6]

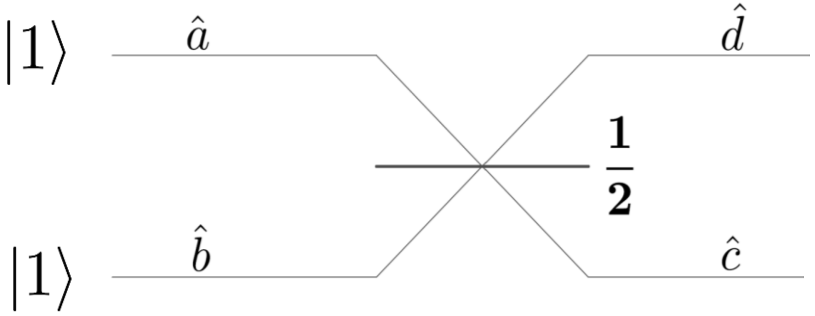
[5] A. J. Daley, H. Pichler, J. Schachenmayer & P. Zoller, Phys. Rev. Lett. **109**, 020505 (2012)
[6] R. Islam, R. Ma, P. M. Preiss, M. Eric Tai, A. Lukin, M. Rispoli & M. Greiner, Nature **528**,77–83 (2015)

Measurement of the purity

$$\mathcal{P}(\hat{\rho}) = \mathrm{Tr}\left((\hat{\rho} \otimes \hat{\rho})\, \hat{S}\right)$$

$$\hat{S} = (-1)^{\hat{n}_d}$$

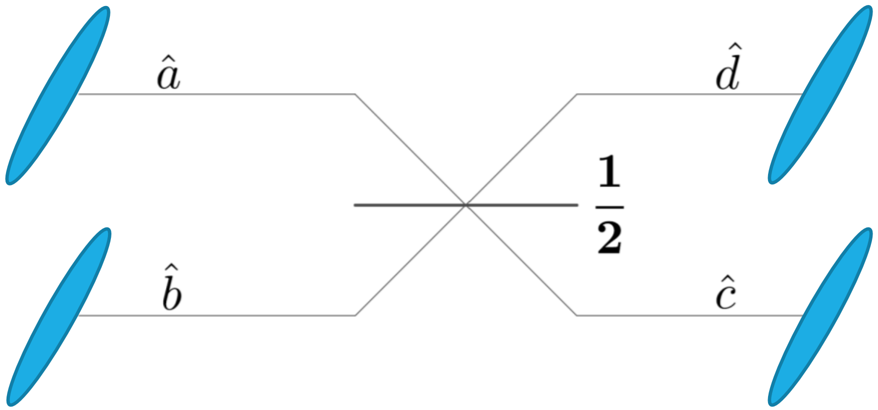
Fock state $|1\rangle$



$\hat{n}_d$

$$\frac{|20\rangle - |02\rangle}{\sqrt{2}} \Rightarrow \mathcal{P}(\hat{\rho}) = 1$$

Squeezed state



$$|S_r\rangle = \frac{1}{\sqrt{\cosh r}} \sum_{k=0}^{\infty} (-e^{i\phi} \tanh r)^k \frac{\sqrt{2k!}}{2^k k!} |2k\rangle \Rightarrow \mathcal{P}(\hat{\rho}) = 1$$

Even

Measurement of the QCS $\mathcal{N}(\hat\rho) = -\frac{1}{2}\operatorname{Tr}\left([\hat\rho,\hat x]^2 + [\hat\rho,\hat p]^2\right)$

Objective $\mathcal{N}(\hat\rho) = \operatorname{Tr}\left((\hat\rho\otimes\hat\rho)\,\hat{N}\right)$

$$\mathcal{N}(\hat\rho) = \frac{1}{2}\left(\int (x-x')^2\,|\langle x|\hat\rho|x'\rangle|^2\,\mathrm{d}x\,\mathrm{d}x' + \int (p-p')^2\,|\langle p|\hat\rho|p'\rangle|^2\,\mathrm{d}p\,\mathrm{d}p'\right)$$

$$= \frac{1}{2}\left(\int (x-x')^2\,\langle x,x'|\hat\rho\otimes\hat\rho|x',x\rangle\,\mathrm{d}x\,\mathrm{d}x' + \int (p-p')^2\,\langle p,p'|\hat\rho\otimes\hat\rho|p',p\rangle\,\mathrm{d}p\,\mathrm{d}p'\right)$$

$$\Rightarrow \quad \hat{N} = \frac{1}{2}\left[(\hat x_a - \hat x_b)^2 + (\hat p_a - \hat p_b)^2\right]\,\hat{S}$$

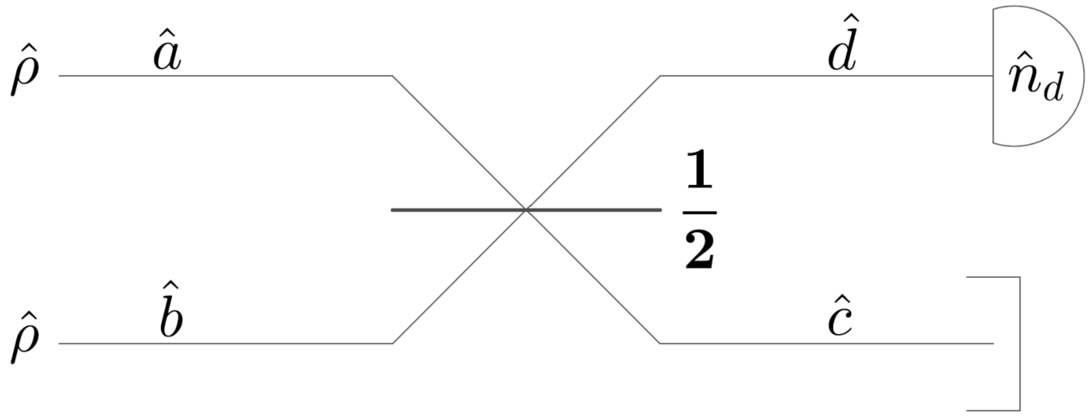
Measurement of the QCS

$$\mathcal{N}(\hat{\rho}) = -\frac{1}{2}\,\mathrm{Tr}\left([\hat{\rho},\hat{x}]^2 + [\hat{\rho},\hat{p}]^2\right)$$

$$\mathcal{N}(\hat{\rho}) = \mathrm{Tr}\left((\hat{\rho}\otimes\hat{\rho})\,\hat{N}\right)$$

where

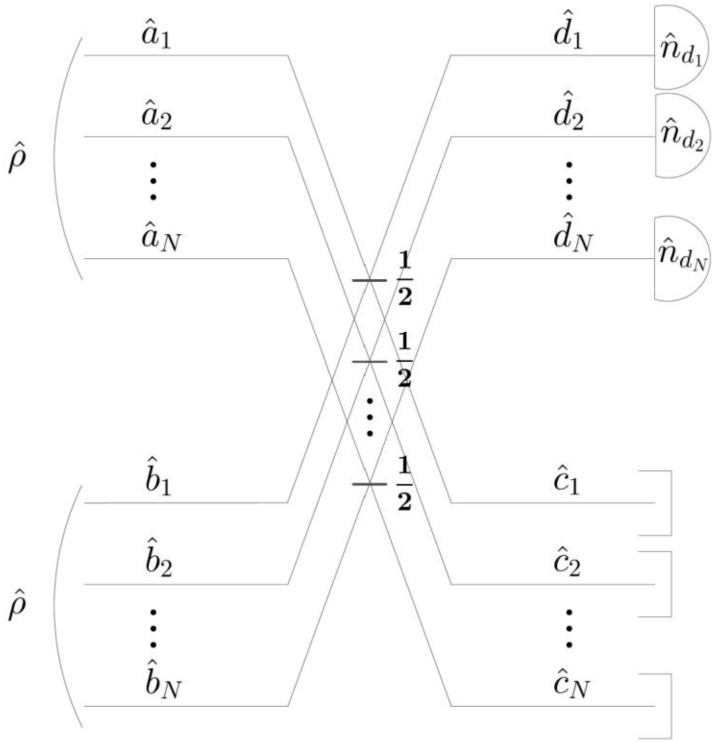
$$\hat{N} = \frac{1}{2}\left[(\hat{x}_a - \hat{x}_b)^2 + (\hat{p}_a - \hat{p}_b)^2\right]\,\hat{S}$$



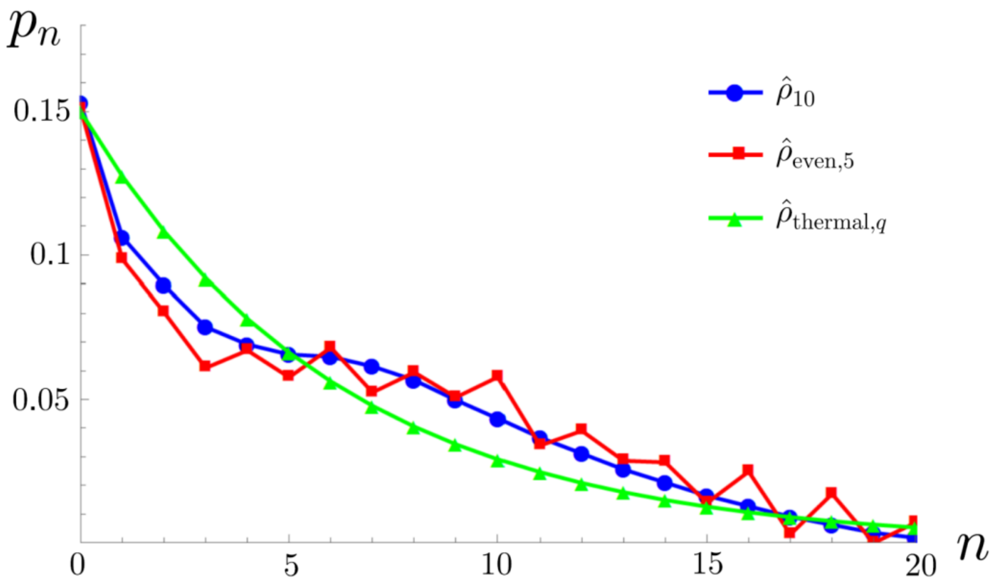
$$\hat{N} = (\hat{x}_d^2 + \hat{p}_d^2)\,\hat{S} = (1 + 2\,\hat{n}_d)\,\hat{S}$$

Other results and comment

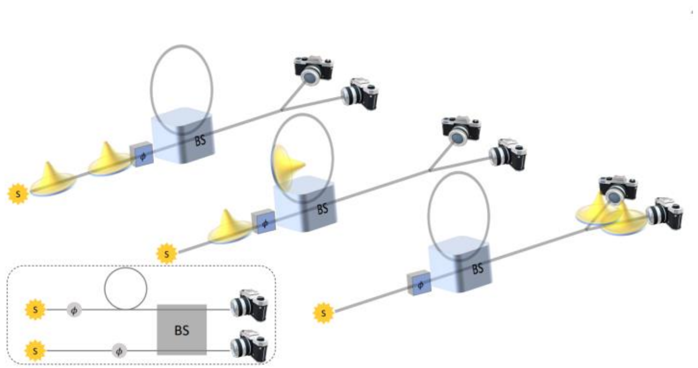
QCS for n modes



Numerical simulations of the result



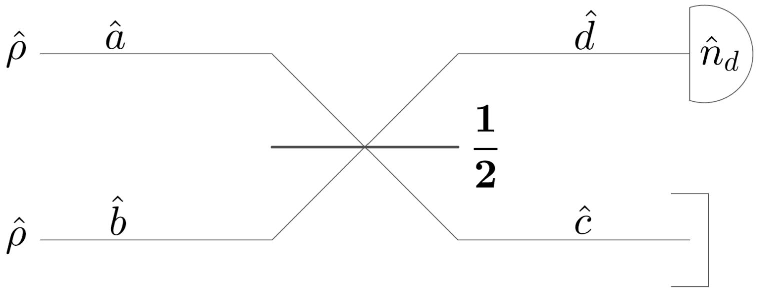
Experimental implementation already done [7]

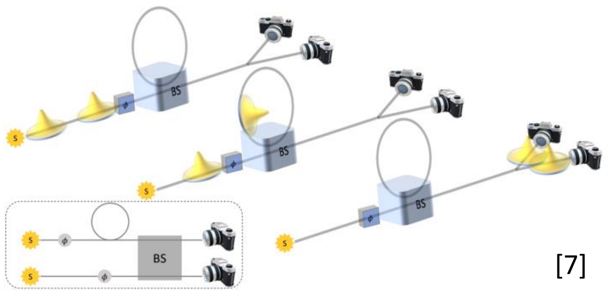


[7] A. Z. Goldberg, G. S. Thekkadath & K. Heshami, Phys. Rev. A **107**, 042610 (2023)

Conclusion and perspectives

- Multicopy technique: efficient way of measuring some expectation values experimentally.





[7]

- Perspectives:
  - Try to improve our results by using homodyne or heterodyne detection and/or active elements (*e.g.* implement d_{1235})
  - Use multicopy technique to implement new criteria
  - Analyze the mathematics behind multicopy (link with algebra, Jordan-Schwinger map)

[7] A. Z. Goldberg, G. S. Thekkadath & K. Heshami, Phys. Rev. A **107,** 042610 (2023)

Thank you for your attention !