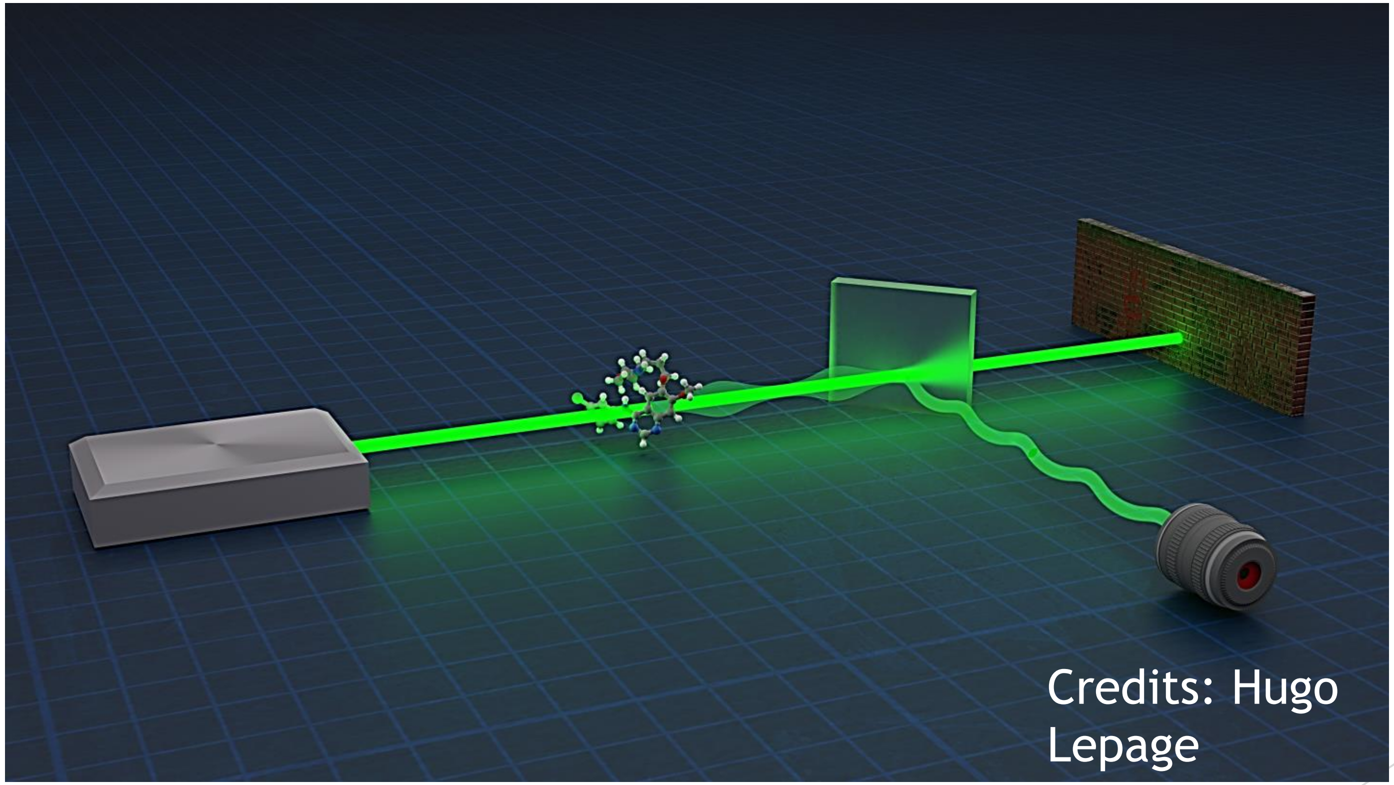
Enhancing Phase Estimation by Harnessing Negative Quasiprobabilities



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Negative Quasiprobabilities Enhance Phase Estimation in Quantum-Optics Experiment

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Operator noncommutation, a hallmark of quantum theory, limits measurement precision, according to uncertainty principles. Wielded correctly, though, noncommutation can boost precision. A recent foundational result relates a metrological advantage with negative quasiprobabilities—quantum extensions of probabilities—engendered by noncommuting operators. We crystallize the relationship in an equation that we prove theoretically and observe experimentally. Our proof-of-principle optical experiment features a filtering technique that we term partially postselected amplification (PPA). Using PPA, we measure a wave plate's birefringent phase. PPA amplifies, by over two orders of magnitude, the information obtained about the phase per detected photon. In principle, PPA can boost the information obtained from the average filtered photon by an arbitrarily large factor. The filter's amplification of systematic errors, we find, bounds the theoretically unlimited advantage in practice. PPA can facilitate any phase measurement and mitigates challenges that scale with trial number, such as proportional noise and detector saturation. By quantifying PPA's metrological advantage with quasiprobabilities, we reveal deep connections between quantum foundations and precision measurement.

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Outline

❑ Metrology

- Quantum Fisher Information
- Postselected Metrology

❑ Foundations

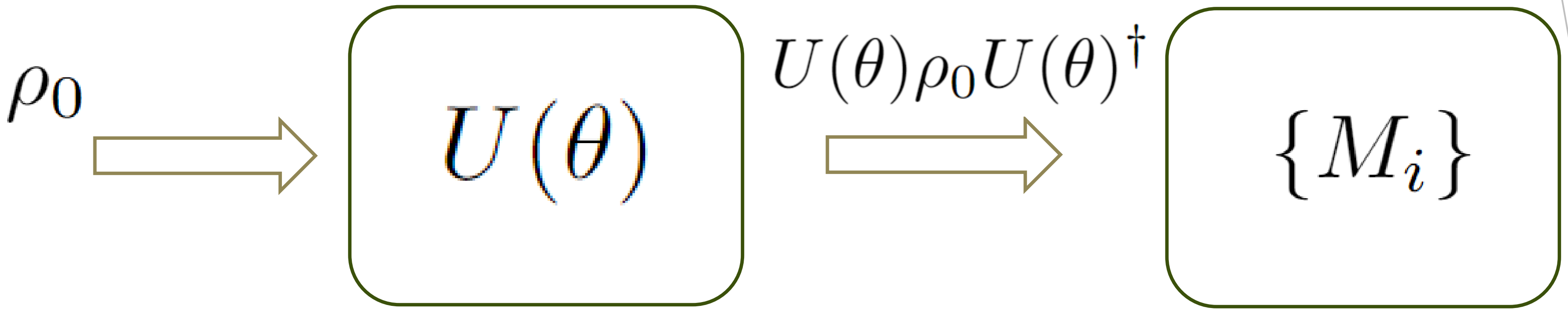
- Quasiprobabilities and Negativity
- Kirkwood-Dirac (KD) Distribution

❑ KD negativity and distilling Fisher Information

❑ Experiment

Parameter Estimation

Parameter estimation:



$$U(\theta) = e^{iA\theta/2}$$

Estimating an unknown parameter Θ encoded in a unitary

Parameter Estimation

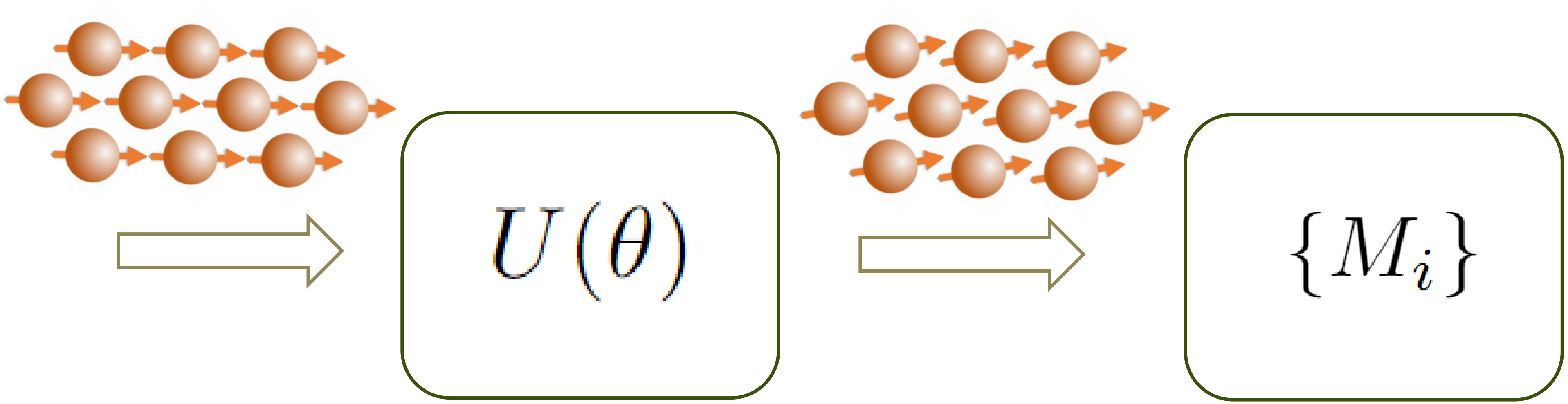
We want to measure the state to estimate Θ. The variance is limited by Quantum Cramer-Rao Bound:

$$Var(\theta) \geq \frac{1}{\mathcal{I}(\theta)}$$

$\mathcal{I}(\theta)$ is the quantum Fisher information(QFI).

Parameter Estimation

Repeating the experiment multiple times with independent trial improves precision



$$N_{in} = N_{det} \longrightarrow \mathcal{I}_T(\theta) = N_{in}\mathcal{I}(\theta) = N_{det}\mathcal{I}(\theta)$$

Per-photon information $\mathcal{I}(\theta)$

Postselected Metrology

For pure states, the QFI can be written as:

$$\hat{\rho}_\theta = |\Psi_\theta\rangle\langle\Psi_\theta| \qquad \mathcal{I}_{\mathrm{Q}}(\theta|\hat{\rho}_\theta) = 4\langle\dot{\Psi}_\theta|\dot{\Psi}_\theta\rangle - 4|\langle\dot{\Psi}_\theta|\Psi_\theta\rangle|^2$$

QFI is bounded by the contrast of the eigenvalues of the generator A of the unitary $U(\theta) = e^{iA\theta/2}$

$$\max_{\hat{\rho}_0}\{\mathcal{I}_{\mathrm{Q}}(\theta|\hat{\rho}_\theta)\} = 4\max_{\hat{\rho}_0}\left\{\mathrm{Var}\,(\hat{A})_{\hat{\rho}_0}\right\} = (\Delta a)^2$$

Postselected Metrology

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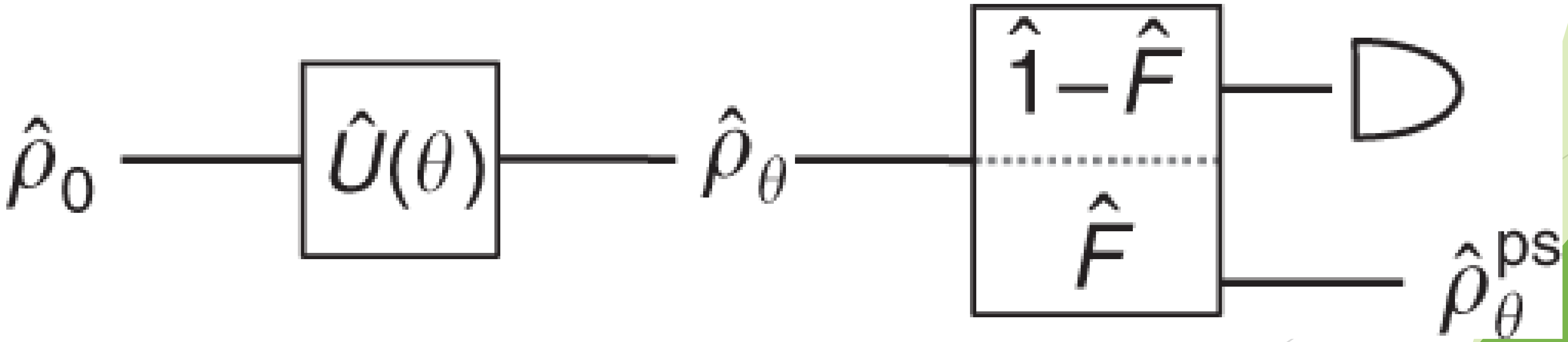
QFI is bounded by the contrast of the eigenvalues of the generator A of the unitary $U(\theta) = e^{iA\theta/2}$

$$\max_{\hat{\rho}_0}\{\mathcal{I}_Q(\theta|\hat{\rho}_\theta)\} = 4\max_{\hat{\rho}_0}\left\{\mathrm{Var}\,(\hat{A})_{\hat{\rho}_0}\right\} = (\Delta a)^2$$

What if the state goes through a post-selection?



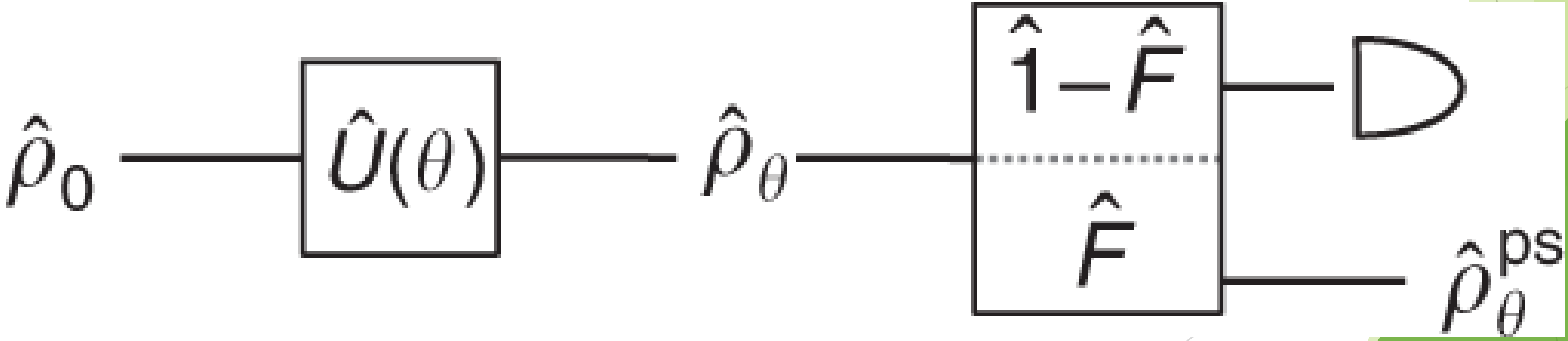
Arvidsson-Shukur et. al. (2020), «Quantum advantage in postselected metrology» *Nature communications*, *11*(1), 3775.

Postselected Metrology

Per-state QFI can exceed the previous bound for some postselection.

$$|\Psi_\theta^{\mathrm{ps}}\rangle \equiv |\psi_\theta^{\mathrm{ps}}\rangle / \sqrt{p_\theta^{\mathrm{ps}}}$$

$$\mathcal{I}_{\mathrm{Q}}(\theta|\Psi_\theta^{\mathrm{ps}}) = 4\langle\dot{\psi}_\theta^{\mathrm{ps}}|\dot{\psi}_\theta^{\mathrm{ps}}\rangle\frac{1}{p_\theta^{\mathrm{ps}}} - 4|\langle\dot{\psi}_\theta^{\mathrm{ps}}|\psi_\theta^{\mathrm{ps}}\rangle|^2\frac{1}{(p_\theta^{\mathrm{ps}})^2} \nleq (\Delta a)^2$$



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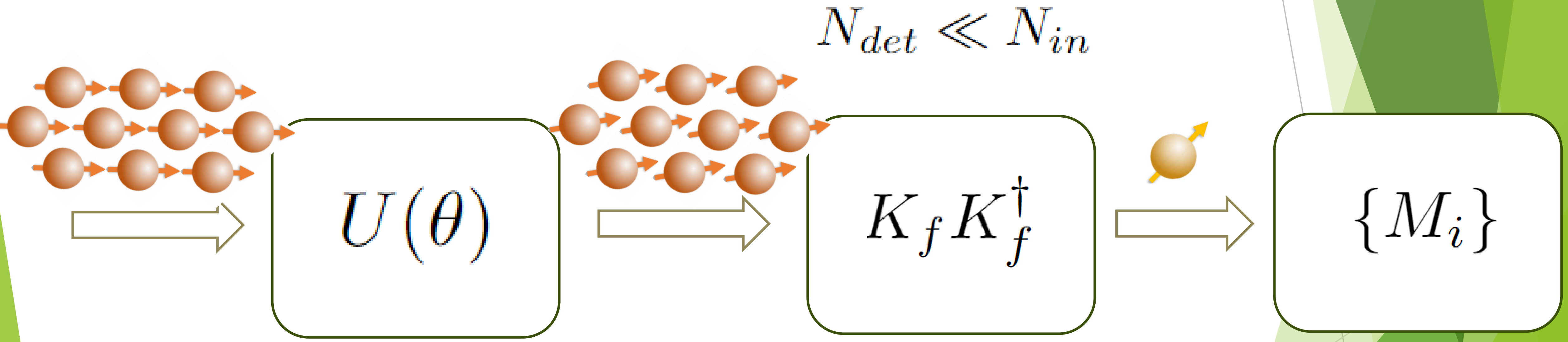
Postselected Metrology

Distilling information with a filter

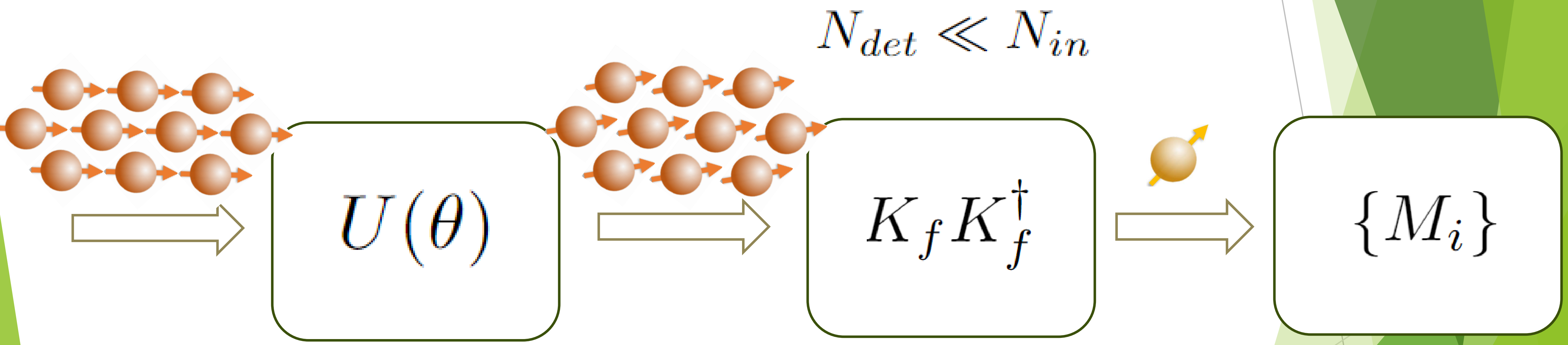


$$\mathcal{I}_T(\theta) = N_{in}\mathcal{I}(\theta)$$

Per-input-photon information ≤ $(\Delta a)^2$

Postselected Metrology

Distilling information with a filter



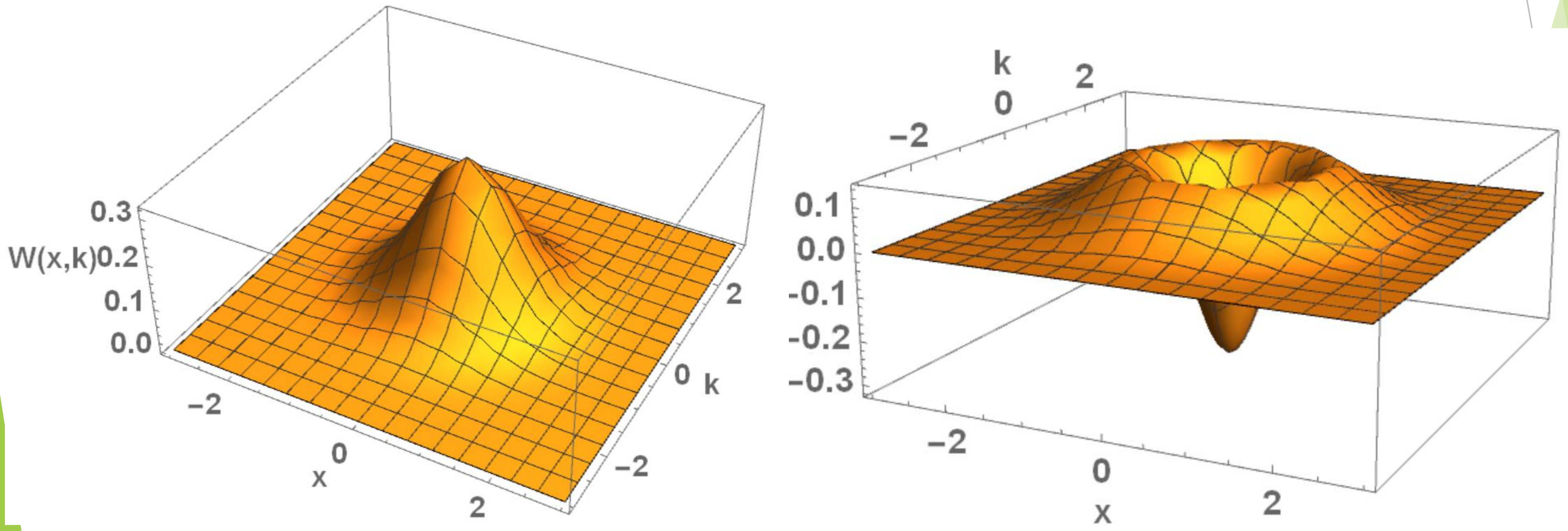
$$\mathcal{I}_T(\theta) = N_{in}\mathcal{I}(\theta) \gg N_{det}\mathcal{I}(\theta)$$

Information per-detected-photon >> $(\Delta a)^2$

Uncertainty and Noncommutation

Uncertainty principle: $\sigma_x \sigma_p \geq \hbar/2$

Position and momentum can't be precisely known simultaneously.
A joint probability distribution P(x,p) for a quantum system may have negativities.



Negativities

A negative probability?

Does negativity have a meaning?

Negativities

A negative probability?

Does negativity have a meaning?

- Sperling and Vogel (2009): Negativity as a measure of entanglement

Negativities

A negative probability?

Does negativity have a meaning?

- Sperling and Vogel (2009): Negativity as a measure of entanglement
- Veitch et. al. (2012): Negativity as a resource for quantum computation

Kirkwood-Dirac Distribution

For observables A,F with eigenbases $\{|a_i\rangle\}, \{|f_i\rangle\}$, KD distribution for a state $\rho(\theta)$ is defined as:

$$\tilde{p}_{\rho(\theta)}(a_i, f_i) = Tr[|f_i\rangle \langle f_i| |a_i\rangle \langle a_i| \rho(\theta)]$$

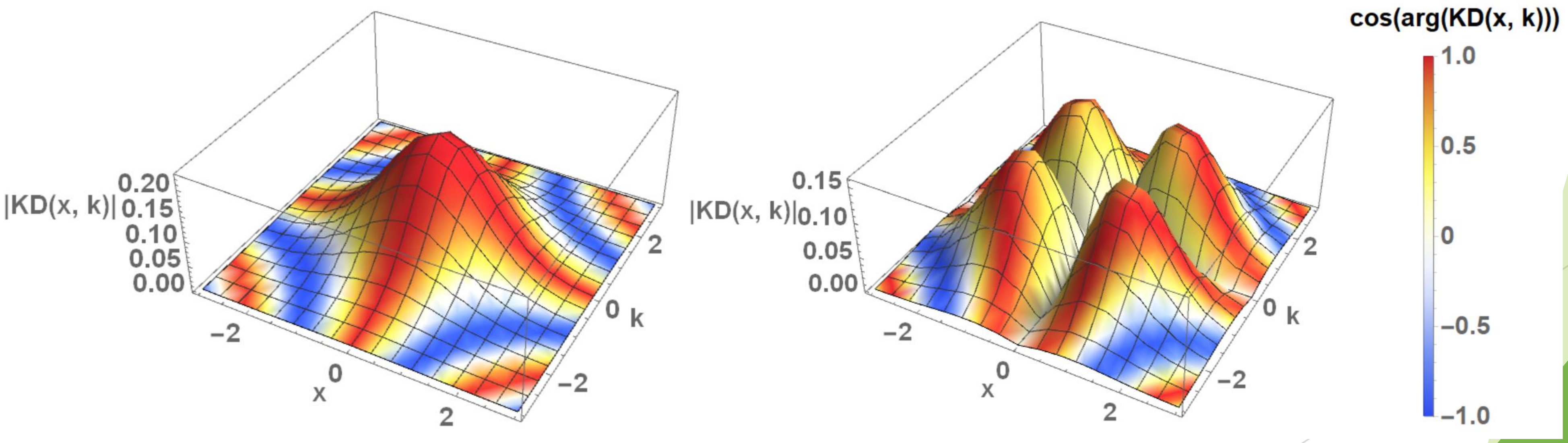
- Can have nonreal values.

- Applications to weak values, quantum thermodynamics, quantum chaos

Kirkwood-Dirac Distribution

- Negative or non-real values in KD distribution quantifies non-classical phenomena in quantum chaos and quantum thermodynamics.

- Negativity in Kirkwood-Dirac distribution can lead to a metrological advantage.



Kirkwood-Dirac Distribution

The anomalous values of QFI can be obtained when the KD distribution has negativities.

$$\tilde{p}_{\rho(\theta)}(a, f, a') := Tr[(\rho(\theta) |a\rangle \langle a| K_f^\dagger K_f |a'\rangle \langle a'|)^\dagger]$$

Preparation-Transformation

Projections

Filtering/Postselection

To have negativity, the filter shouldn't commute with the unitary's generator.

$$[A, K_f^\dagger K_f] \neq 0$$

Kirkwood-Dirac Distribution

The anomalous values of QFI can be obtained when the KD distribution has negativities.

$$\tilde{p}_{\rho(\theta)}(a, f, a') := Tr[(\rho(\theta)\,|a\rangle\langle a|\,K_f^\dagger K_f\,|a'\rangle\langle a'|)^\dagger]$$

Preparation-Transformation **Projections** **Filtering/Postselection**

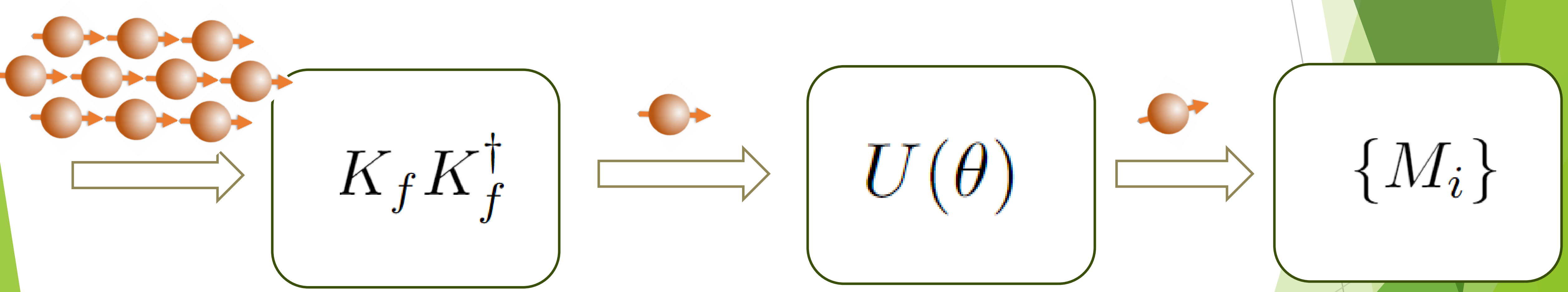
Conditioned on f being successful:

0<x<1

p	a'=a+	a'=a-
a=a+	1-x	0
a=a-	0	x

Kirkwood-Dirac Distribution

If they commute, it is no different than sending fewer input photons.



Kirkwood-Dirac Distribution

The anomalous values of QFI can be obtained when the KD distribution has negativities.

$$\tilde{p}_{\rho(\theta)}(a, f, a') := Tr[(\rho(\theta) |a\rangle \langle a| K_f^\dagger K_f |a'\rangle \langle a'|)^\dagger]$$

Preparation-Transformation

Projections

Filtering/Postselection

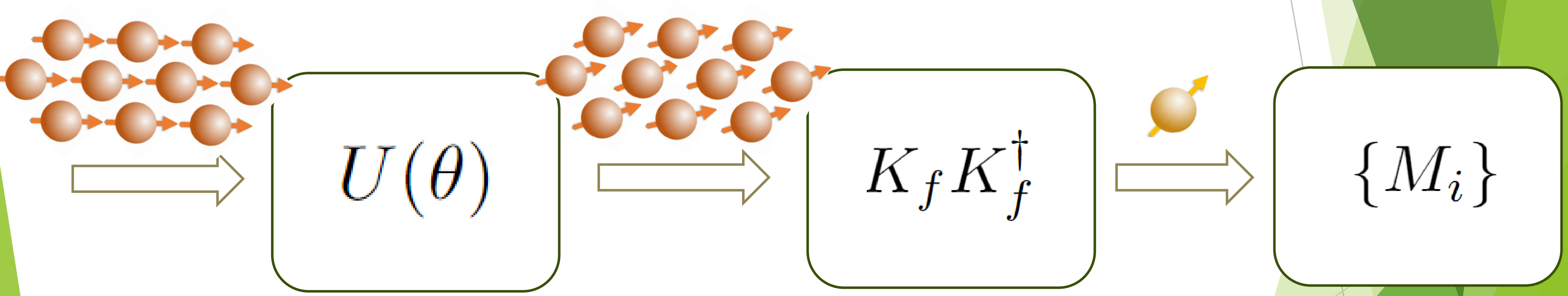
Conditioned on f being successful:

0<x<1
y>0

p	a’=a+	a’=a-
a=a+	1-x+y	-y
a=a-	-y	x+y

Kirkwood-Dirac Distribution

The anomalous QFI is observed when they don't commute.



KD Negativity and Distillation

We use a particular witness of negativity for the conditional KD distribution we call non-classicality gap. This negativity arises from non-commutation between the transformation and the postselection.

$$\left[\max_{a,a'}\left\{|\tilde{p}_{\rho(\theta)}(a,a'|+)|^2\right\} - \min_{a,a'}\left\{|\tilde{p}_{\rho(\theta)}(a,a'|+)|^2\right\}\right]$$

0.5	0
0	0.5

NC gap: 0.5^2 = 0.25 < 1

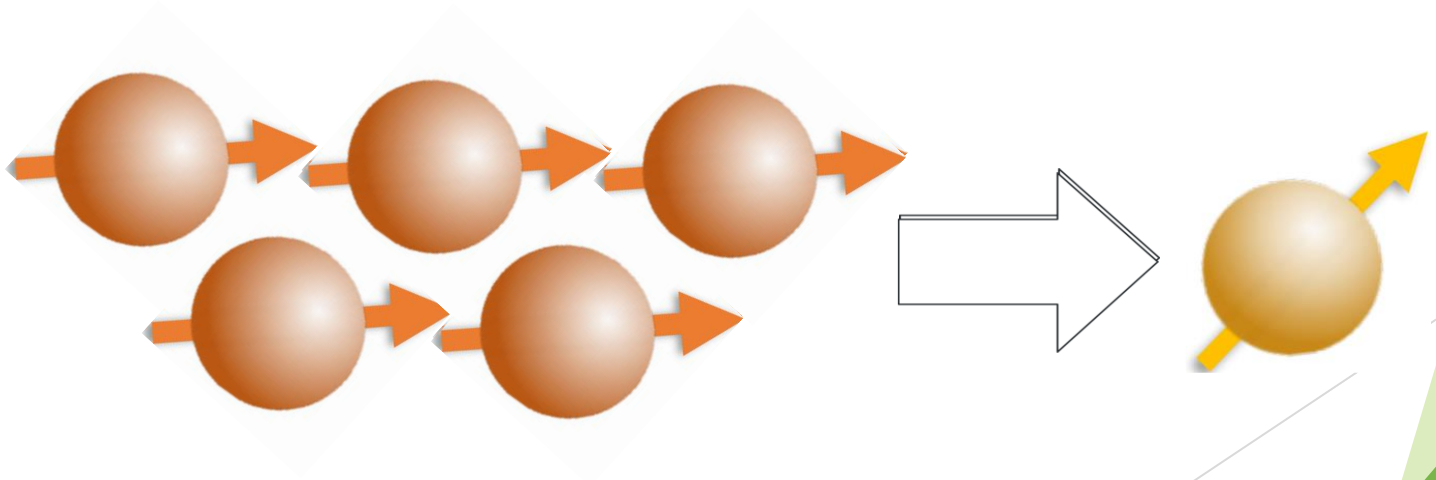
1.5	-1
-1	1.5

NC gap: 1.5^2 - (-1)^2 = 1.25 > 1

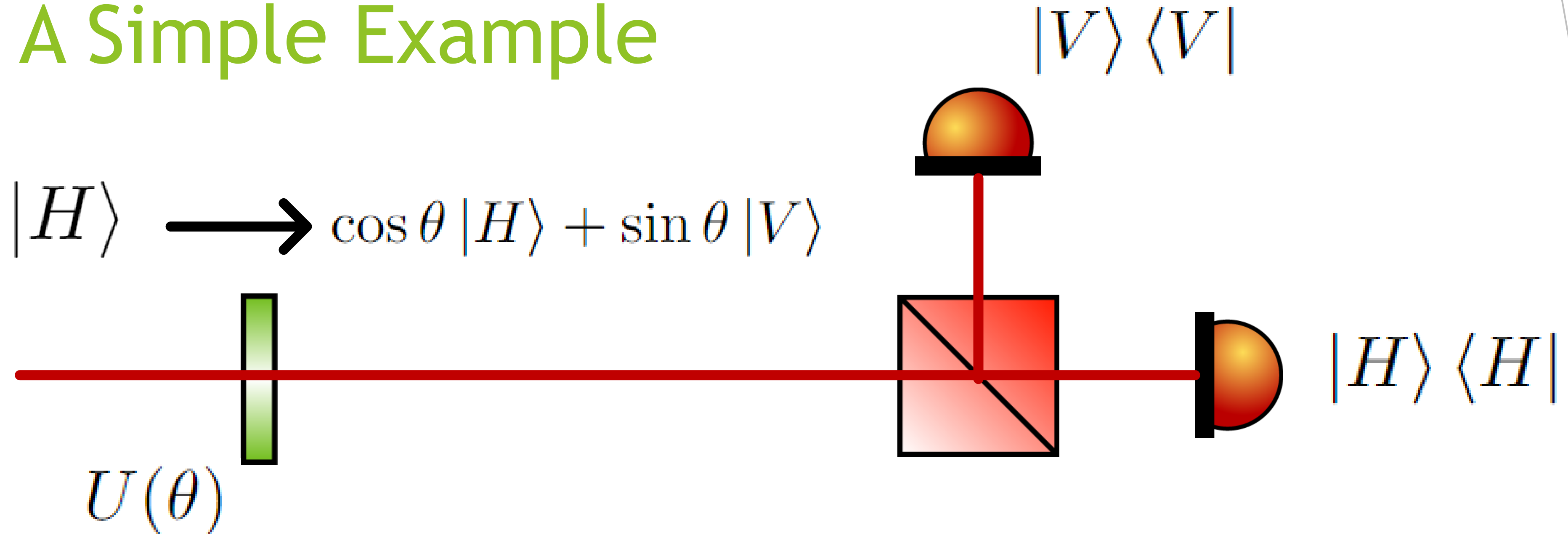
KD Negativity and Distillation

$$\mathcal{I}(\theta) = 4(\Delta a)^2 \times \left[\max_{a,a'} \left\{ |\tilde{p}_{\rho(\theta)}(a,a'|+)|^2 \right\} - \min_{a,a'} \left\{ |\tilde{p}_{\rho(\theta)}(a,a'|+)|^2 \right\} \right]$$

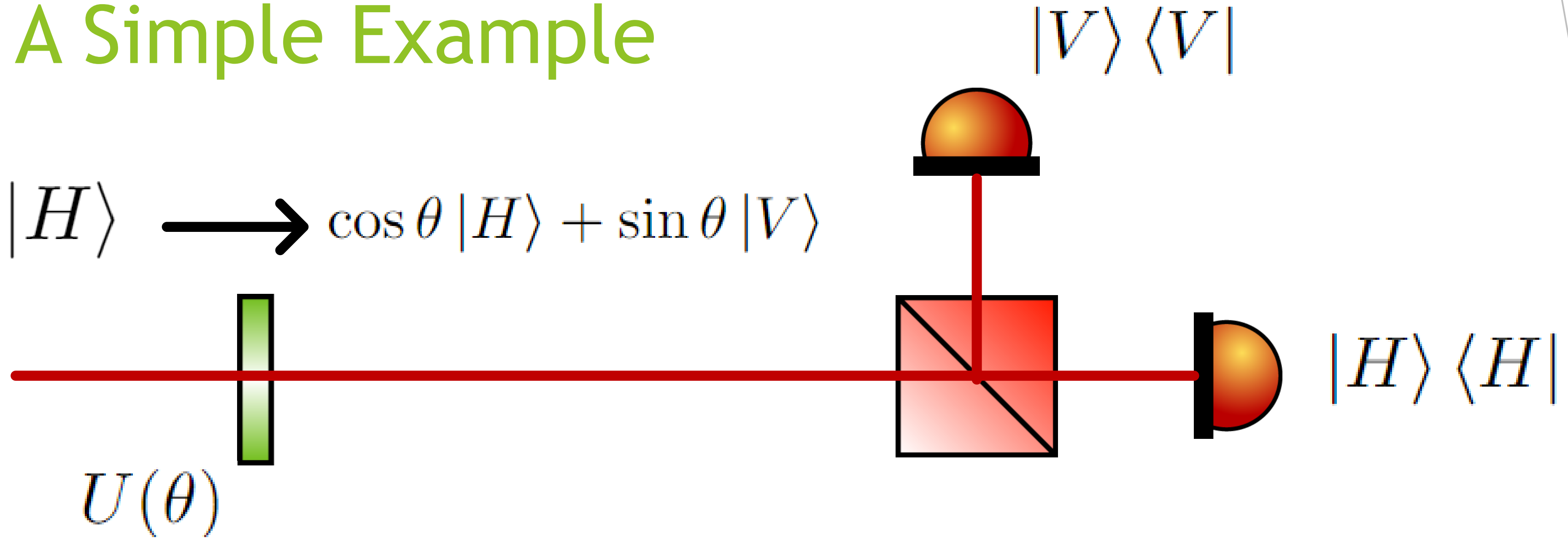
Negativity in the KD distribution is a measure of how much we can «distill» the information by filtering the states in the correct way.



A Simple Example



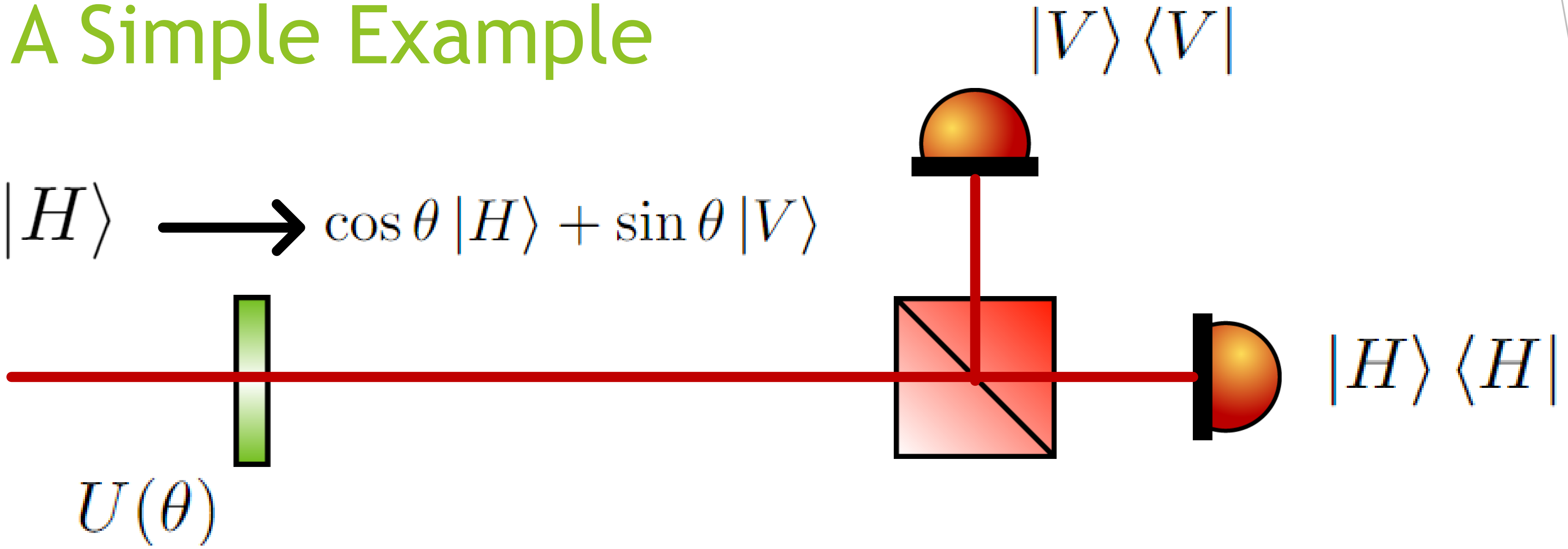
A Simple Example



If it is a small rotation, most of the times we will measure H.

M={H,H,H,…,V,H,H,H,H,H}

A Simple Example



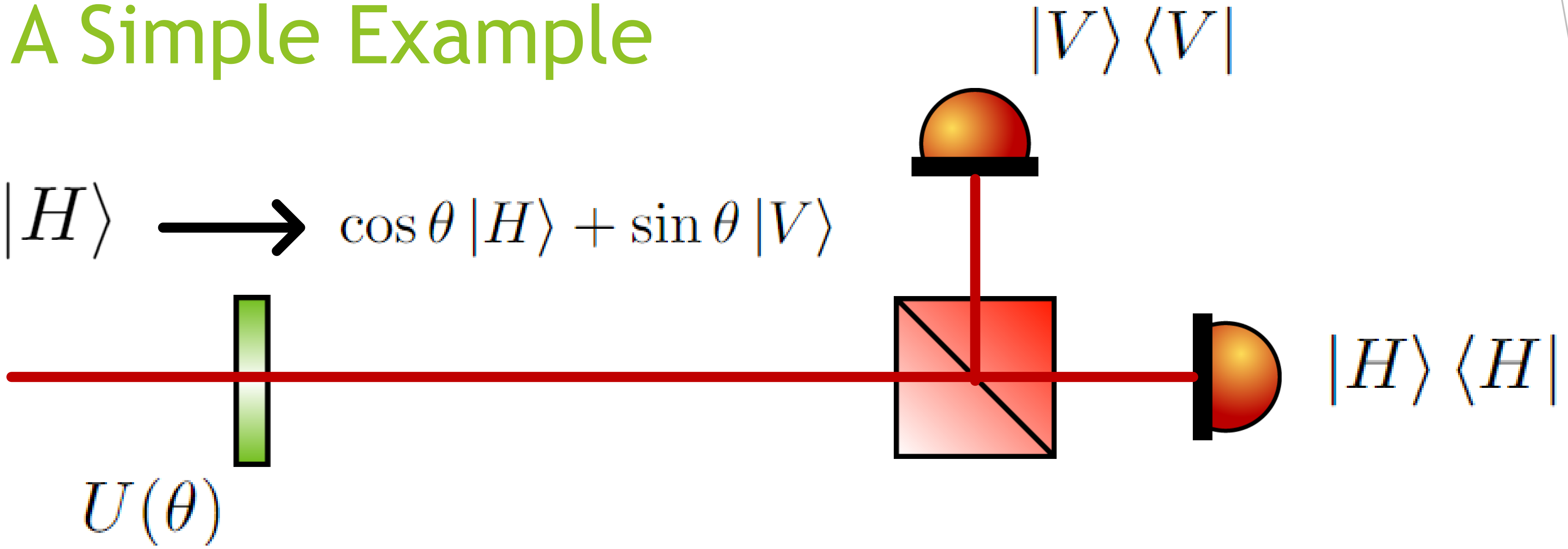
M={H,H,H,…,V,H,H,H,H,H}

$$\hat{\theta} = \arcsin\left(\sqrt{\frac{N_V}{N_H + N_V}}\right)$$

N_V=1, N_H = 999
Θ=31.63 mrad

A Simple Example

$$|H\rangle \longrightarrow \cos\theta\,|H\rangle + \sin\theta\,|V\rangle$$



M={H,H,H,…,V,H,H,H,H,H,H}

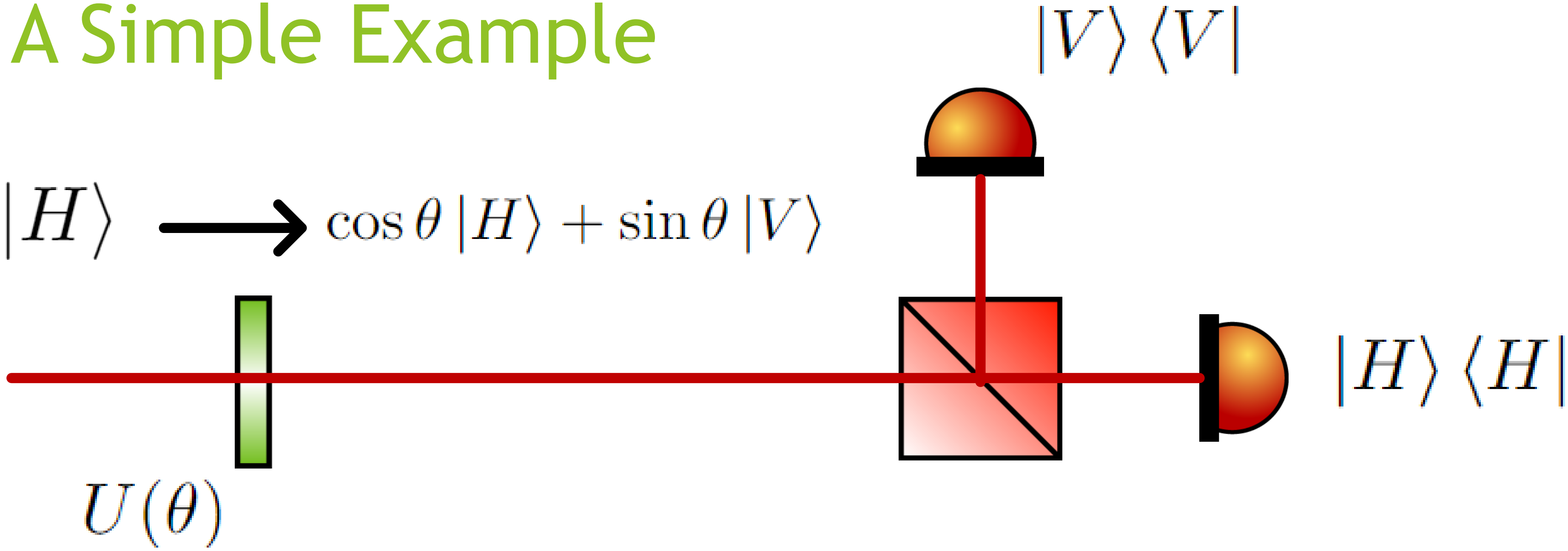
$$\hat{\theta} = \arcsin\left(\sqrt{\frac{N_V}{N_H + N_V}}\right)$$

Nv=1, NH = 1000

Θ= 31.61 mrad

0.06% change

A Simple Example

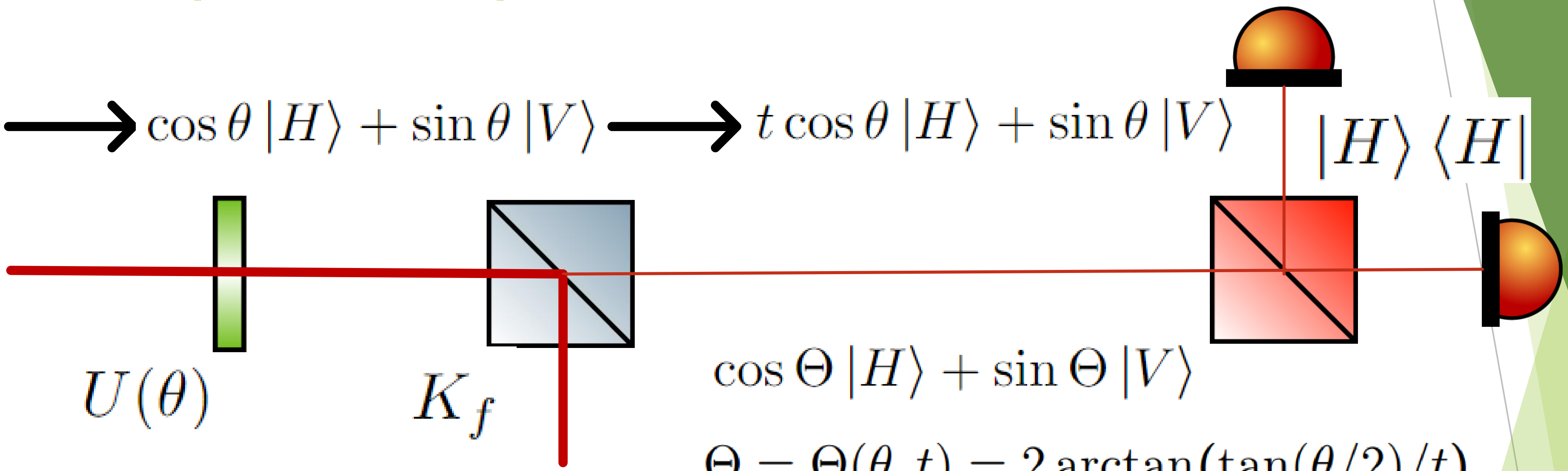


M={H,H,H,…,V,H,H,H,H,H,H,V}

$$\hat{\theta} = \arcsin\left(\sqrt{\frac{N_V}{N_H + N_V}}\right)$$

N_V=2, N_H = 1000
Θ=44.71 mrad
41% change!

A Simple Example



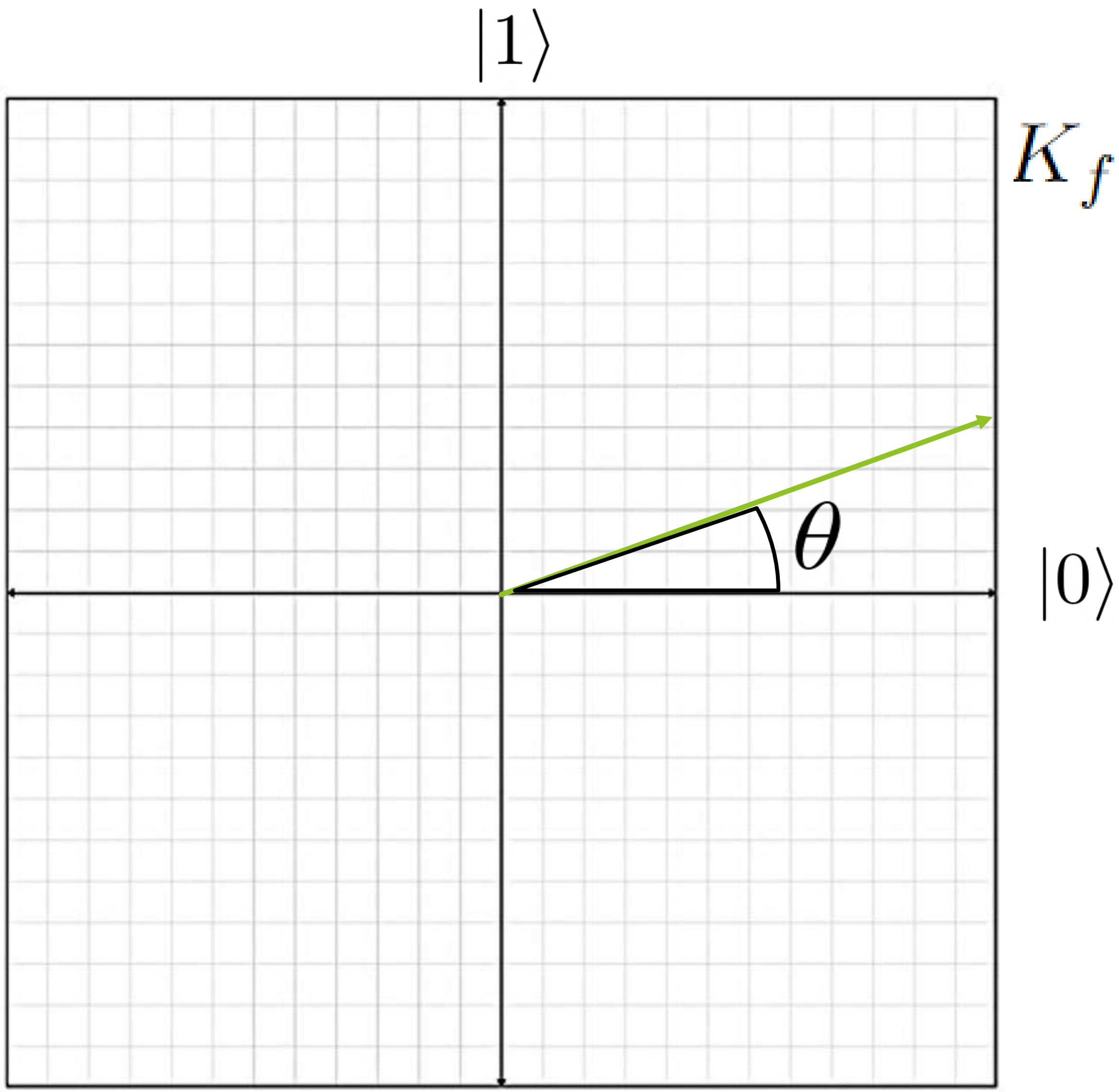
$$\Theta = \Theta(\theta, t) = 2\arctan(\tan(\theta/2)/t)$$

M={H,H,...,V,H,H,V}

$$\hat{\theta} = \arcsin\left(\sqrt{\frac{N_V}{N_H/t^2 + N_V}}\right)$$

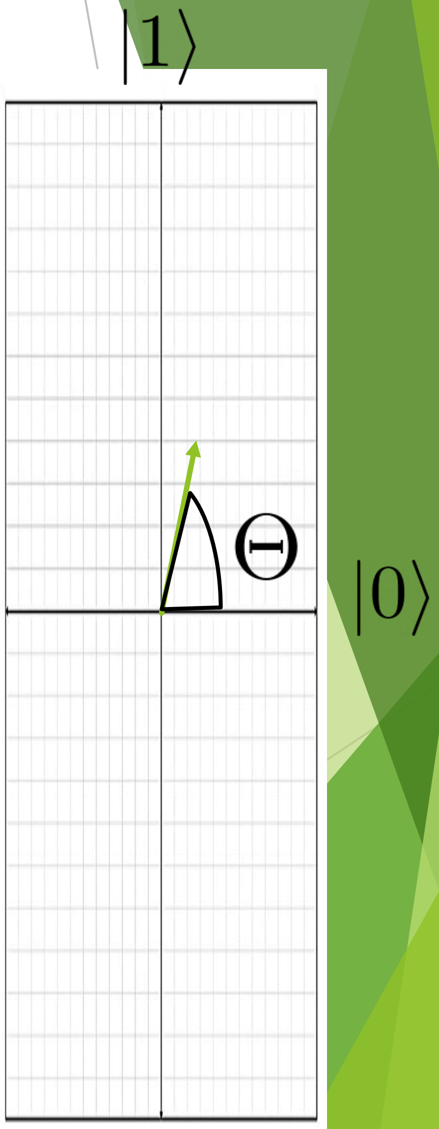
Nv=2, NH = 100
Θ=44.71 mrad

The Experiment



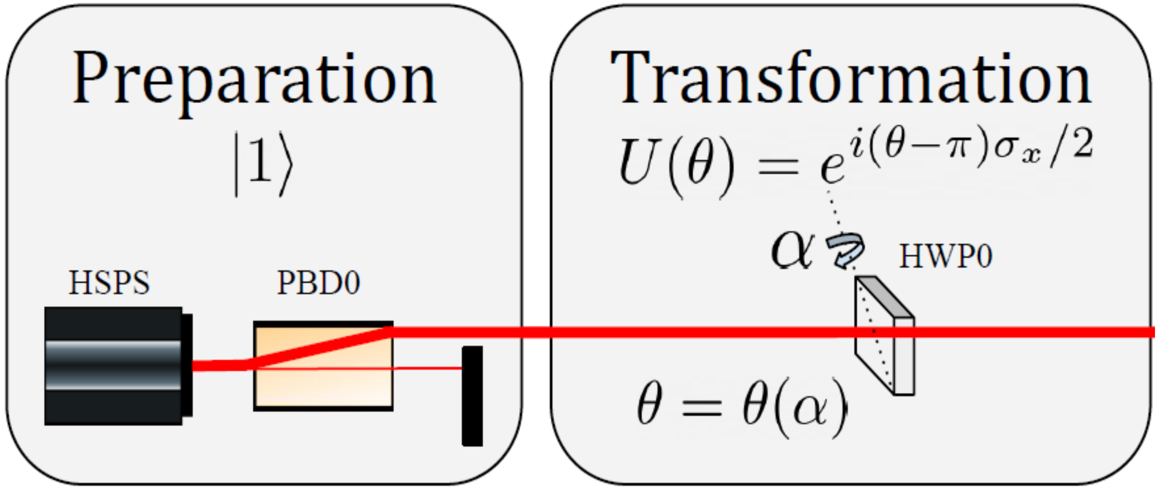
$$K_f = t\,|0\rangle\langle 0| + |1\rangle\langle 1|$$

$$|0\rangle \rightarrow t|0\rangle$$



The Experiment

PPA: Partially Postselected Amplification

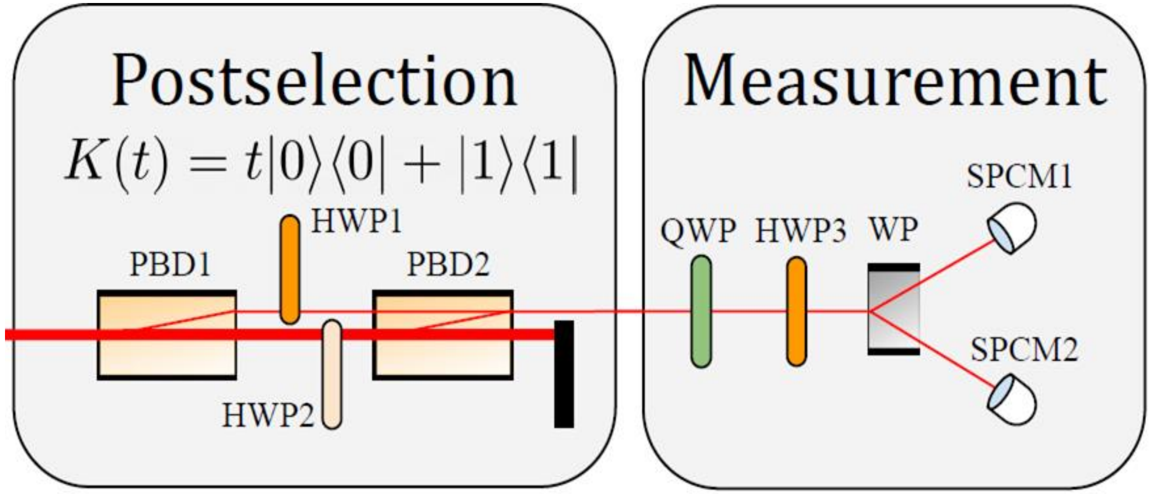


Lupu-Gladstein et. al. (2022) «Negative quasiprobabilities enhance phase estimation in quantum optics experiment» *Physical Review Letters*, *128*(22), 220504.

- Vertically polarized photons are prepared.
- Photons go through a HWP with variable retardance.

The Experiment

PPA: Partially Postselected Amplification

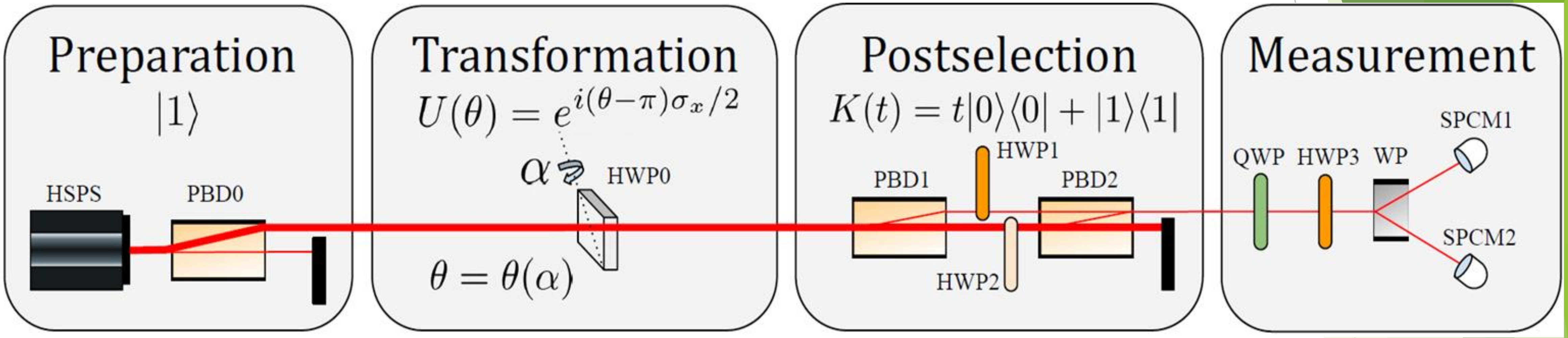


Lupu-Gladstein et. al. (2022) «Negative quasiprobabilities enhance phase estimation in quantum optics experiment» *Physical Review Letters*, *128*(22), 220504.

- Postselection is done on a polarization interferometer.
- Final state is measured.

The Experiment

PPA: Partially Postselected Amplification



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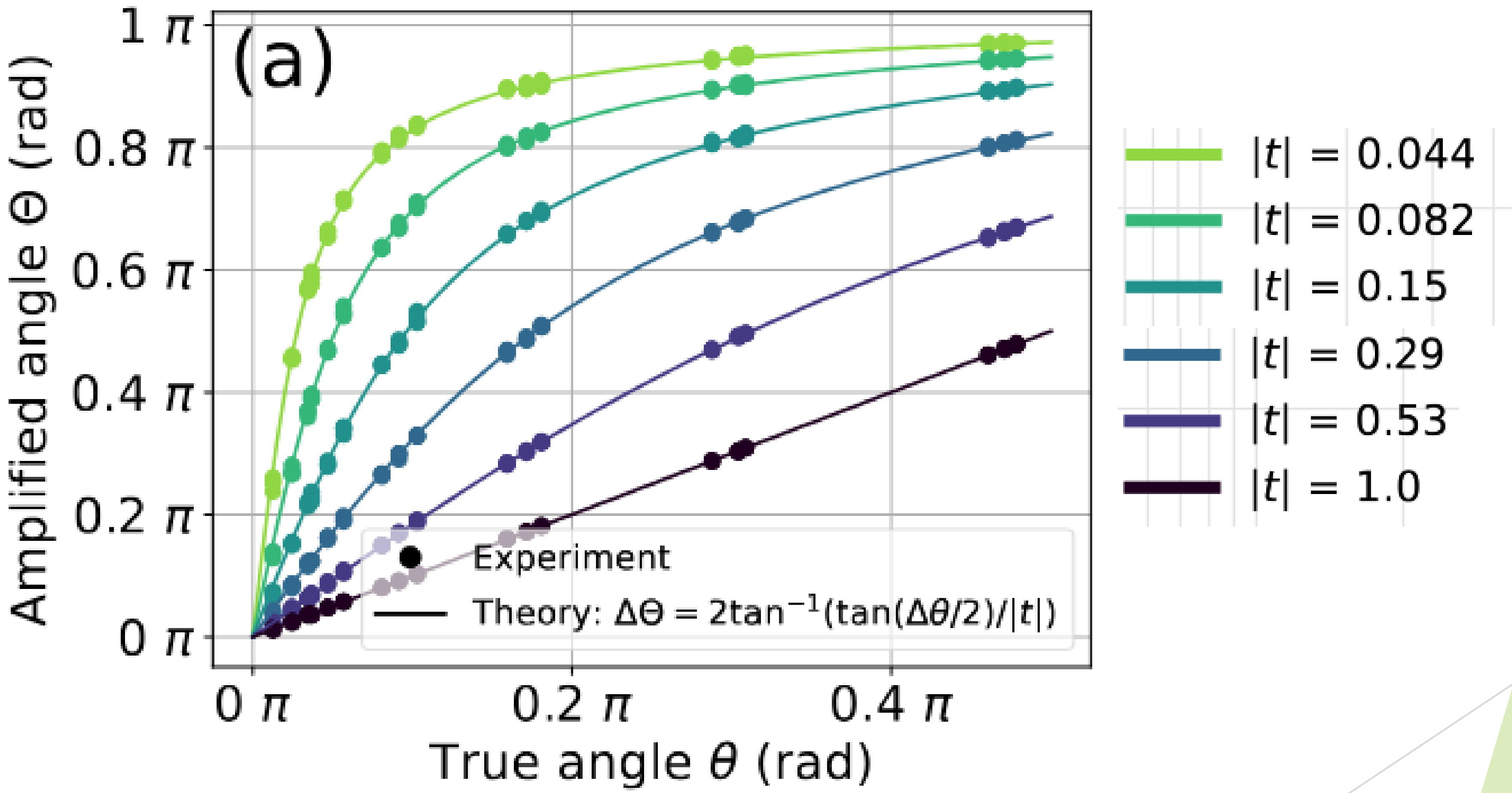
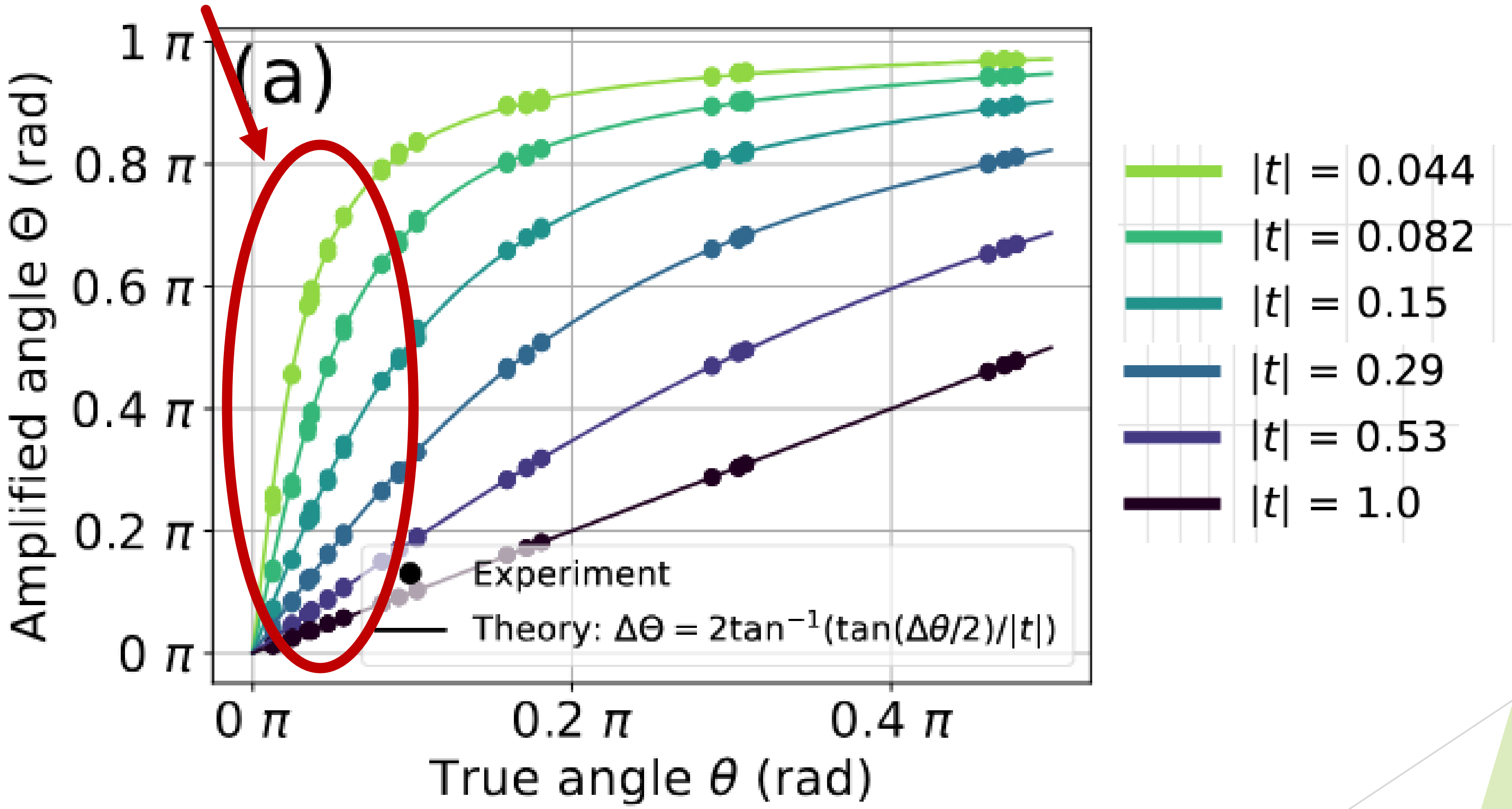
The Experiment



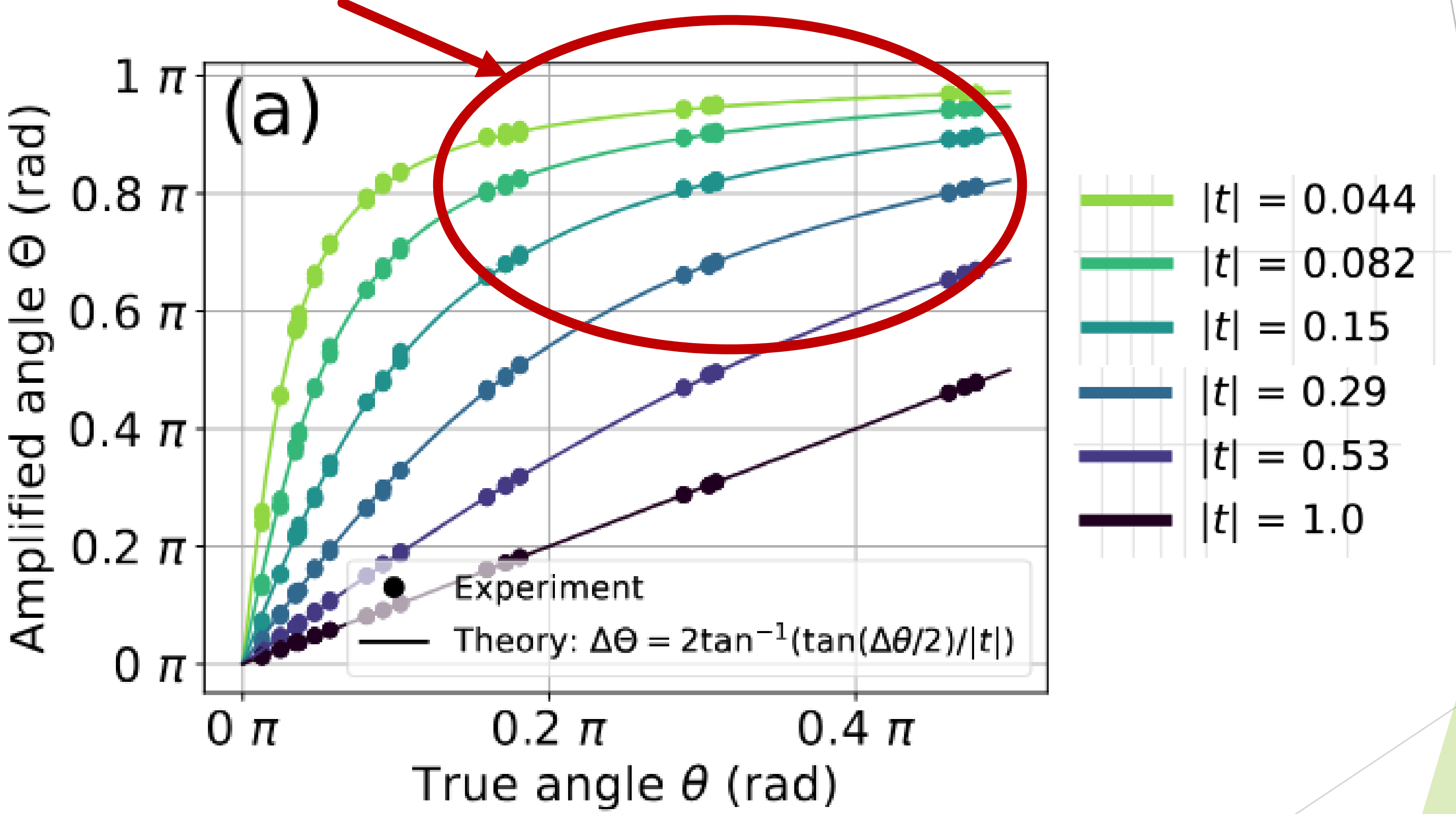
The Experiment

Linear region: Slope increases with increasing filtering



The Experiment

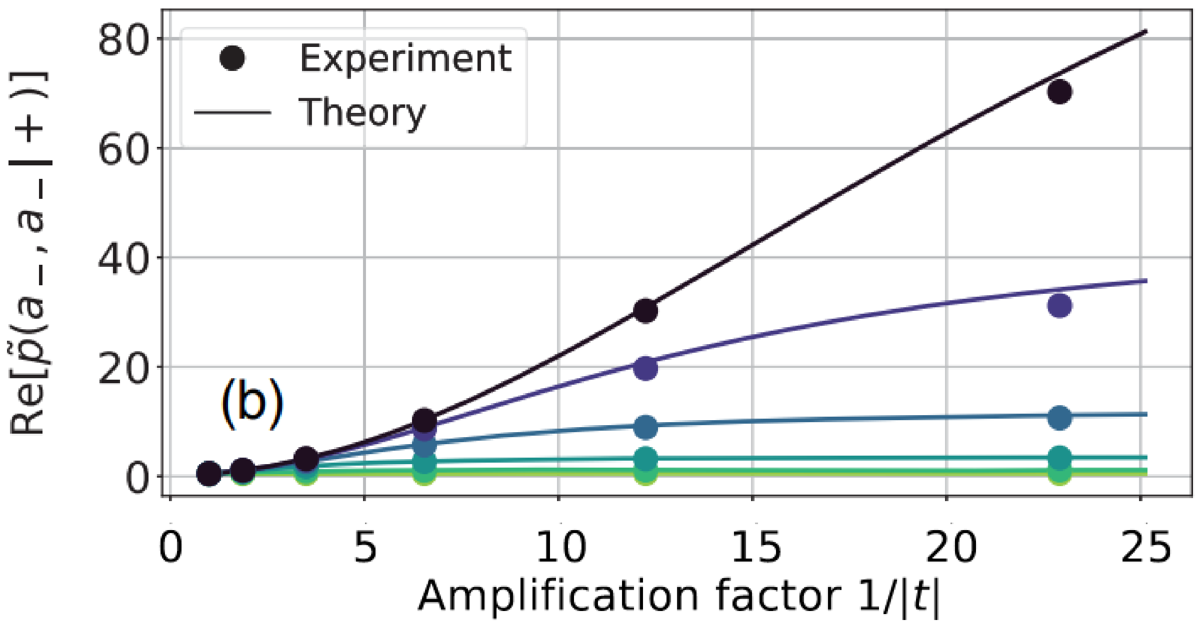
Saturation region: Sensitivity gets worse.



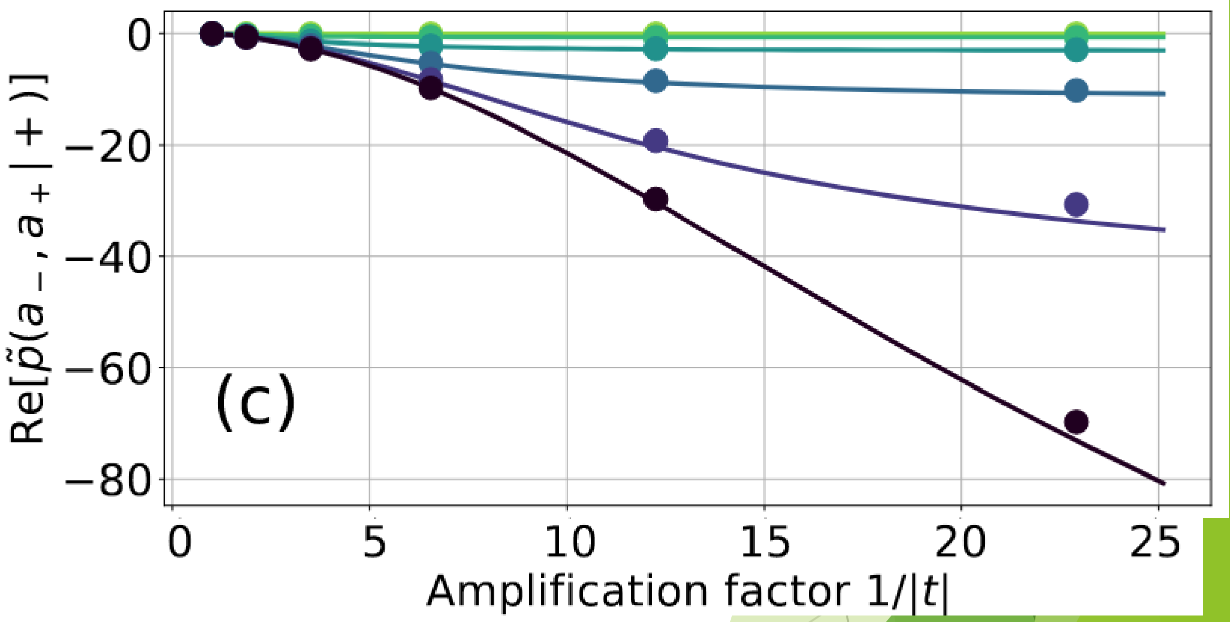
Results

Reconstruction of the KD distribution:

Diagonal element of the KD distribution



Off-diagonal element of the KD distribution:



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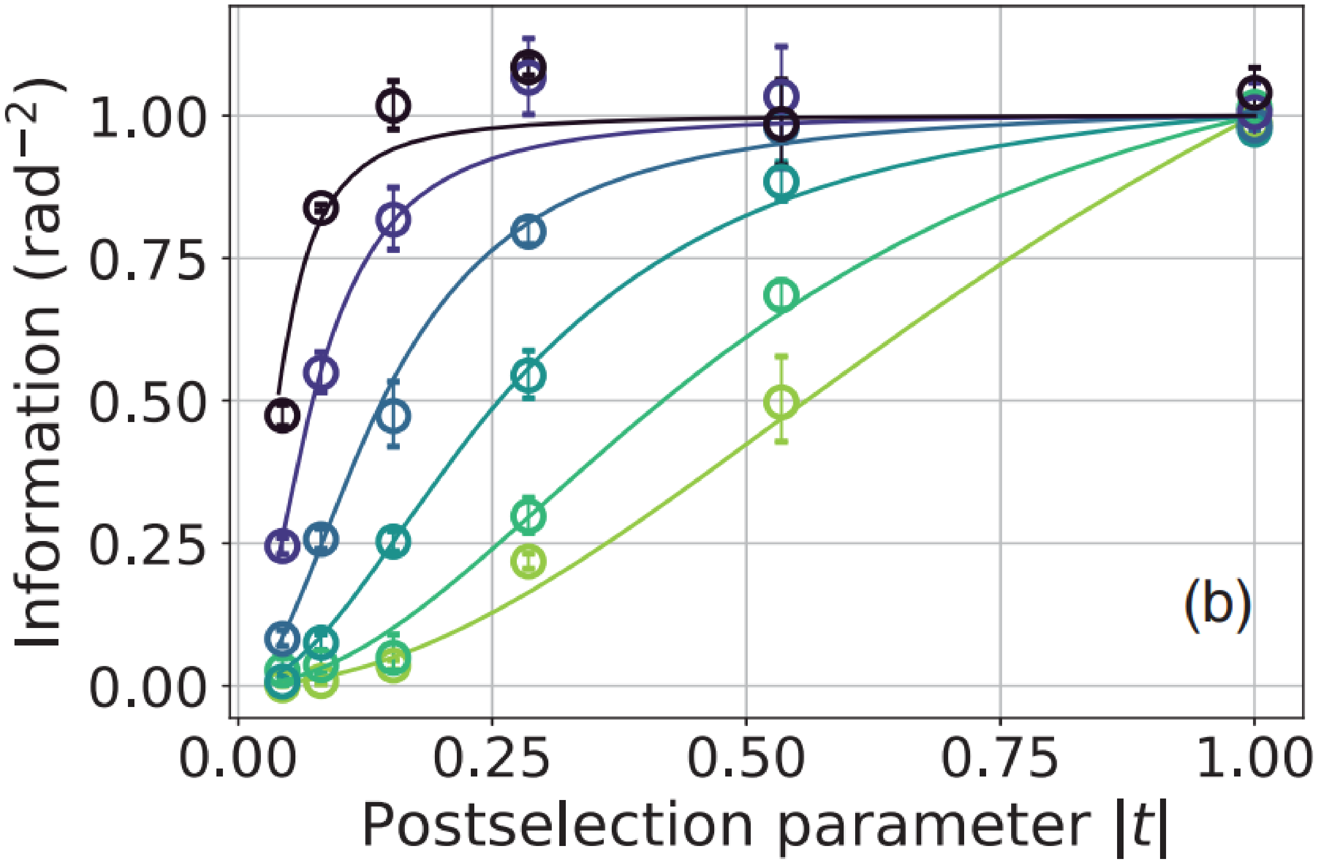
Results

Information per-input photon



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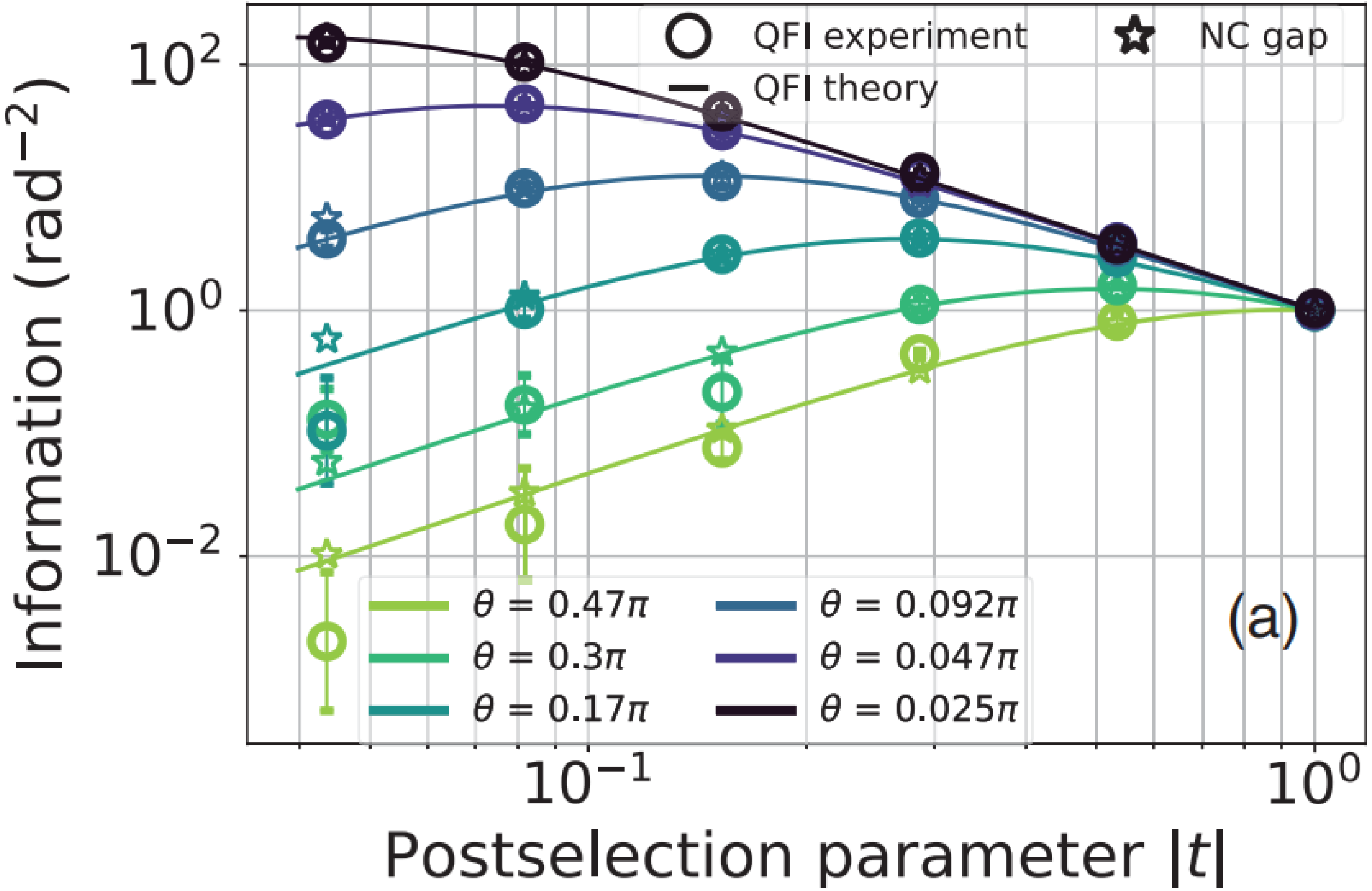
Results

Information per-detected photon and non-classicality gap:



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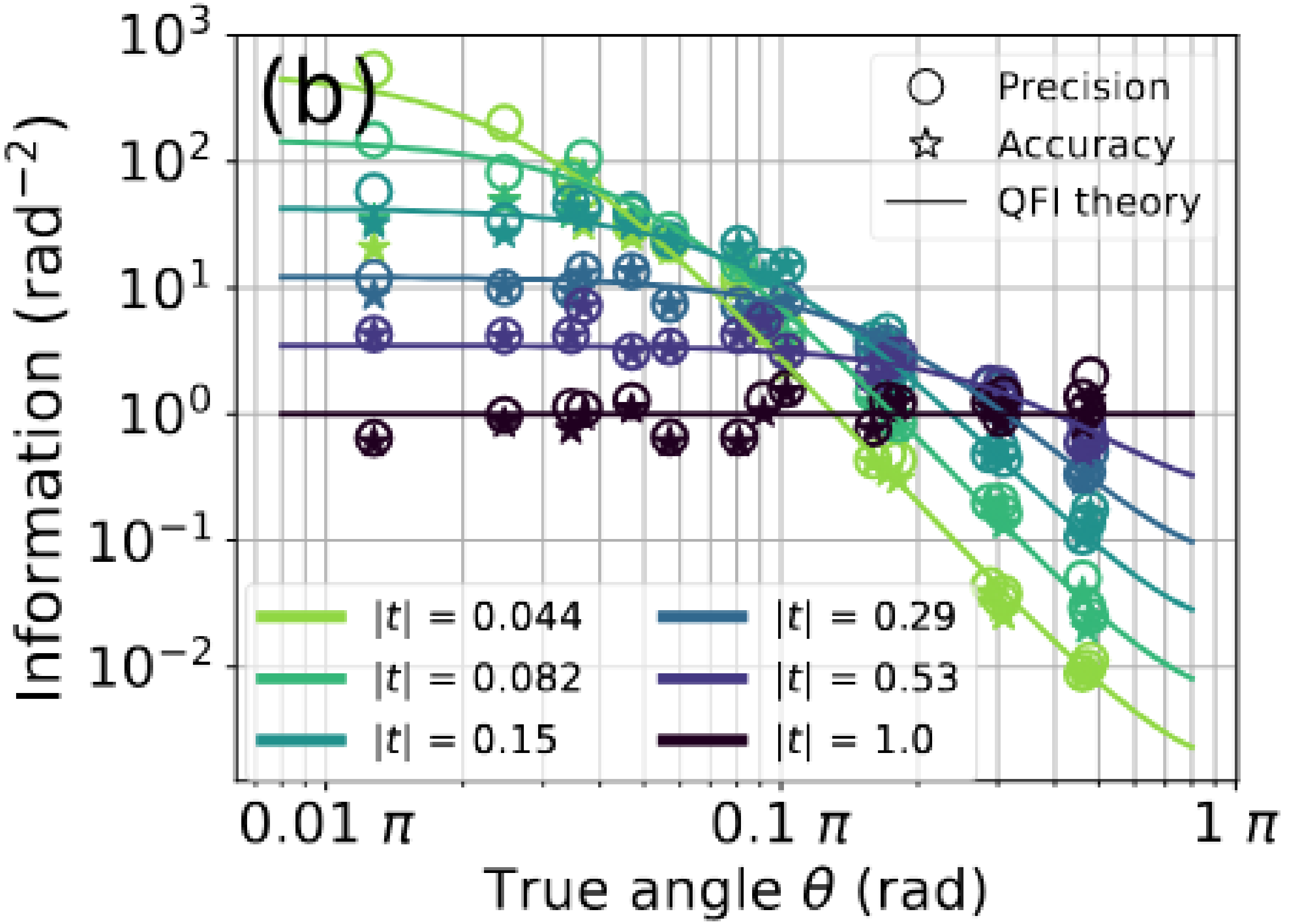
Results

Precision = 1/Variance

Accuracy=1/MSE = 1/(Variance+Bias^2)

Both quantities are calculated for per-detected photon.



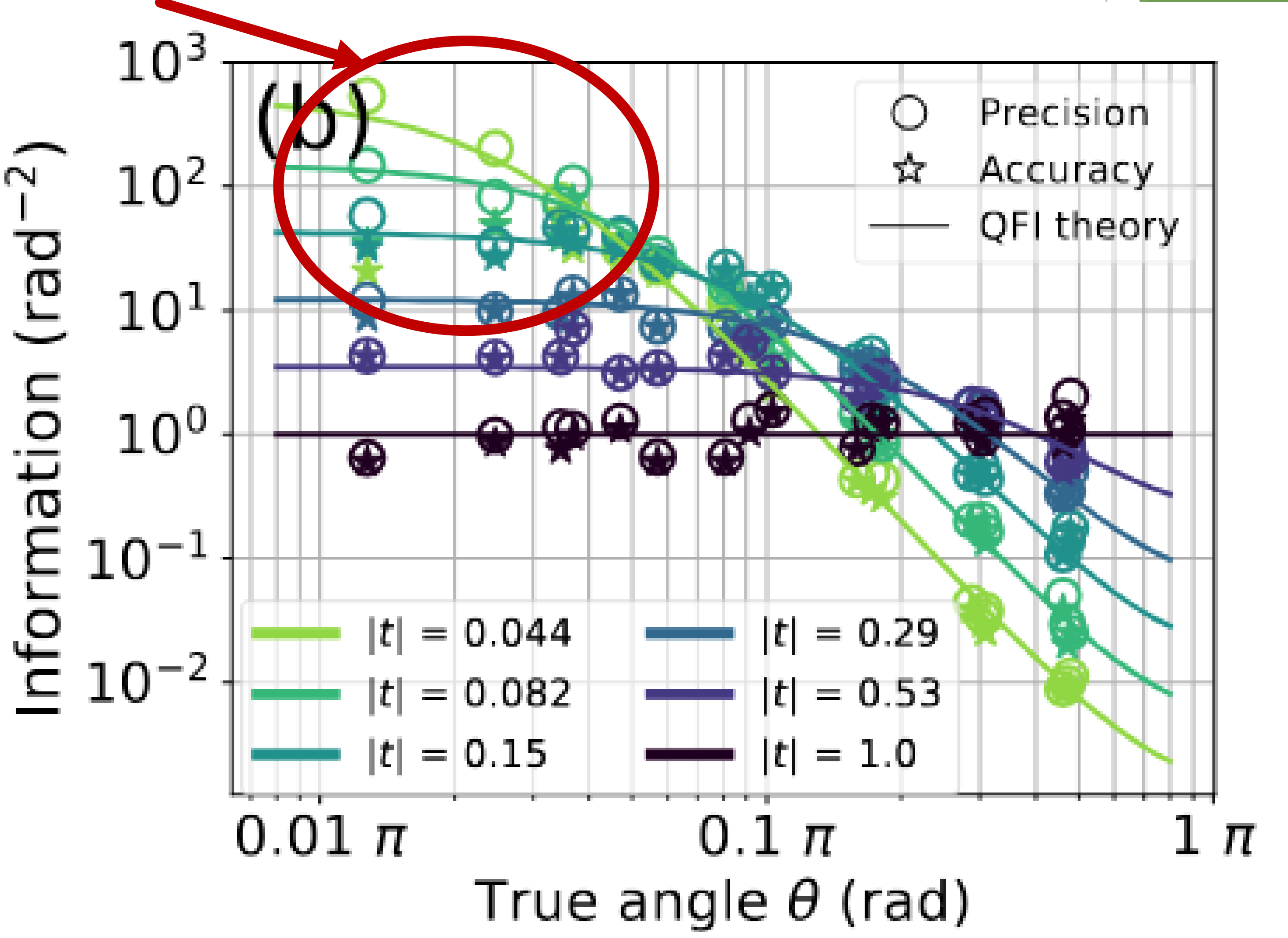
Results

Mismatch between accuracy
And precision due to systematic errors.

Precision = 1/Variance

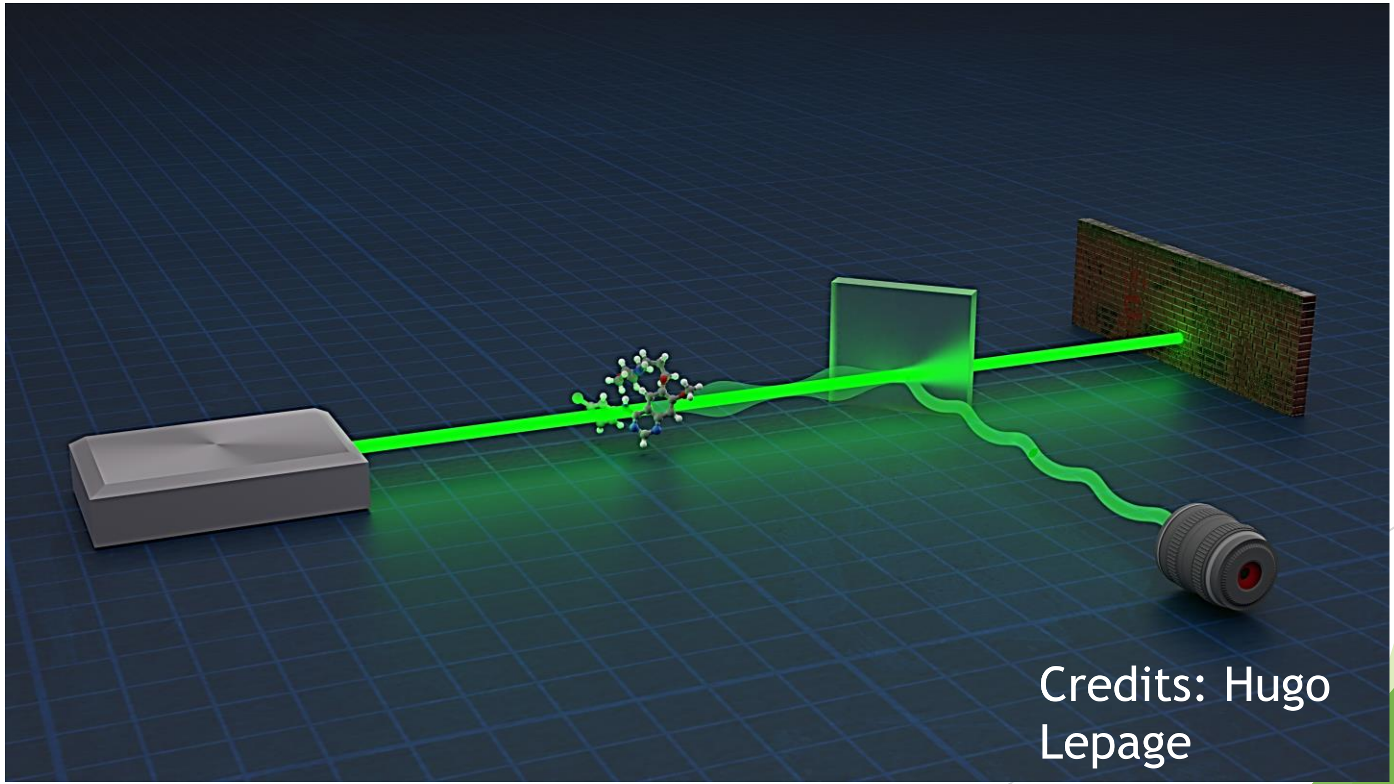
Accuracy=1/MSE =
1/(Variance+Bias^2)

Both quantities
are calculated for
per-detected
photon.



Conclusion

Non-commutation between the postselection and the transformation stages leads to negativities in the KD distribution. This negativity is directly related to the how much the information is distilled. This idea can be useful when there is more signal than the detectors can process.



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CQIQC

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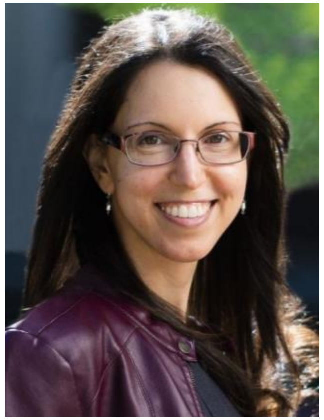
IonQ/ Entangled Networks

Collaborators:



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