Quantum correlations and non-Gaussian operations

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MQO Multimode Quantum Optics

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OQuLus NORDIC veriqub

FRANCE 2030 anr European Innovation Council

Quantum light allows us to...

Perform calculations that no classical computer can do (perhaps one day at least)

Quantum Computers Approach Milestone for Boson Sampling

December 18, 2019 • *Physics* 12, s146

Experiments show that when enough photons travel through a complex optical network, only a quantum computer can efficiently sample the range of possible outcomes.

REPORT

Quantum computational advantage using photons

Han-Sen Zhong[1,2,*], Hui Wang[1,2,*], Yu-Hao Deng[1,2,*], Ming-Cheng Chen[1,2,*], Li-Chao Peng[1,2], Yi-Han Luo[1]...

+ See all authors and affiliations

Science 18 Dec 2020:
Vol. 370, Issue 6523, pp. 1460-1463
DOI: 10.1126/science.abe8770

Article | Open Access | Published: 01 June 2022

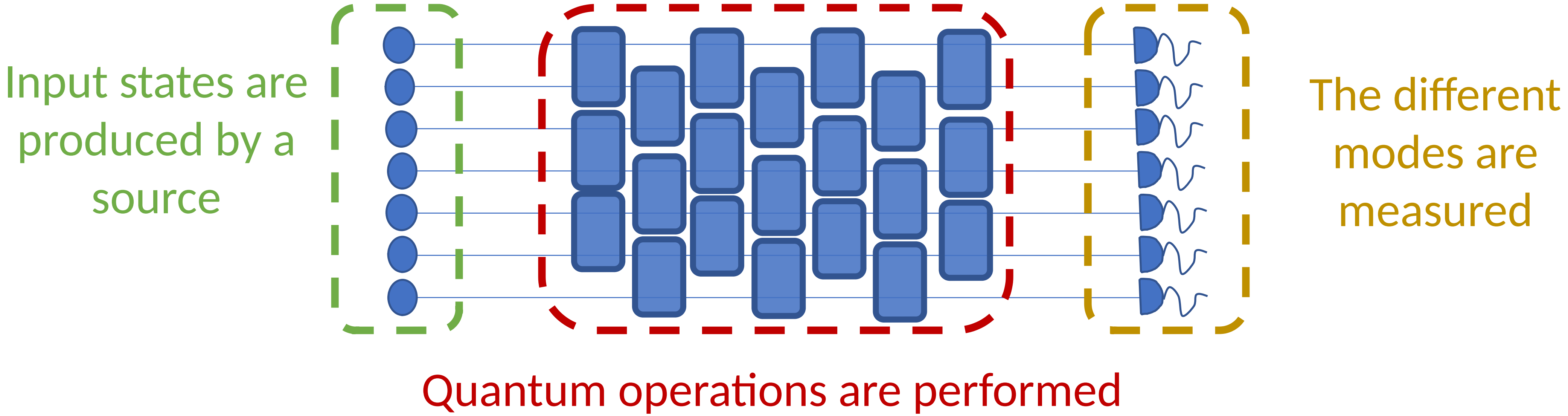
Quantum computational advantage with a programmable photonic processor

Lars S. Madsen, Fabian Laudenbach, Mohsen Falamarzi. Askarani, Fabien Rortais, Trevor Vincent, Jacob F. F. Bulmer, Filippo M. Miatto, Leonhard Neuhaus, Lukas G. Helt, Matthew J. Collins, Adriana E. Lita, Thomas Gerrits, Sae Woo Nam, Varun D. Vaidya, Matteo Menotti, Ish Dhand, Zachary Vernon, Nicolás Quesada & Jonathan Lavoie

Nature **606**, 75–81 (2022) | Cite this article

Sampling problems

A typical photonic sampling setup



We try to find cases where it is hard for a classical computer to simulate these measurements

Quantum computational advantage

VOLUME 88, NUMBER 9 PHYSICAL REVIEW LETTERS 4 MARCH 2002

Efficient Classical Simulation of Continuous Variable Quantum Information Processes

Stephen D. Bartlett and Barry C. Sanders
Department of Physics and Centre for Advanced Computing–Algorithms and Cryptography, Macquarie University, Sydney, New South Wales 2109, Australia

Samuel L. Braunstein and Kae Nemoto

→ **Non-Gaussianity** is needed

PRL **109,** 230503 (2012) PHYSICAL REVIEW LETTERS week ending 7 DECEMBER 2012

Positive Wigner Functions Render Classical Simulation of Quantum Computation Efficient

A. Mari[1,2,3] and J. Eisert[1]

First part of the talk

→ **Wigner negativity** is needed

PHYSICAL REVIEW LETTERS **130,** 090602 (2023)

Resources for Bosonic Quantum Computational Advantage

Ulysse Chabaud[1,2,*] and Mattia Walschaers[3,†]

→ **High stellar rank** and **non-Gaussian entanglement** are needed

second part of the talk

Overview

- Non-Gaussian states from quantum correlations
- Quantum correlations from non-Gaussian operations

Overview

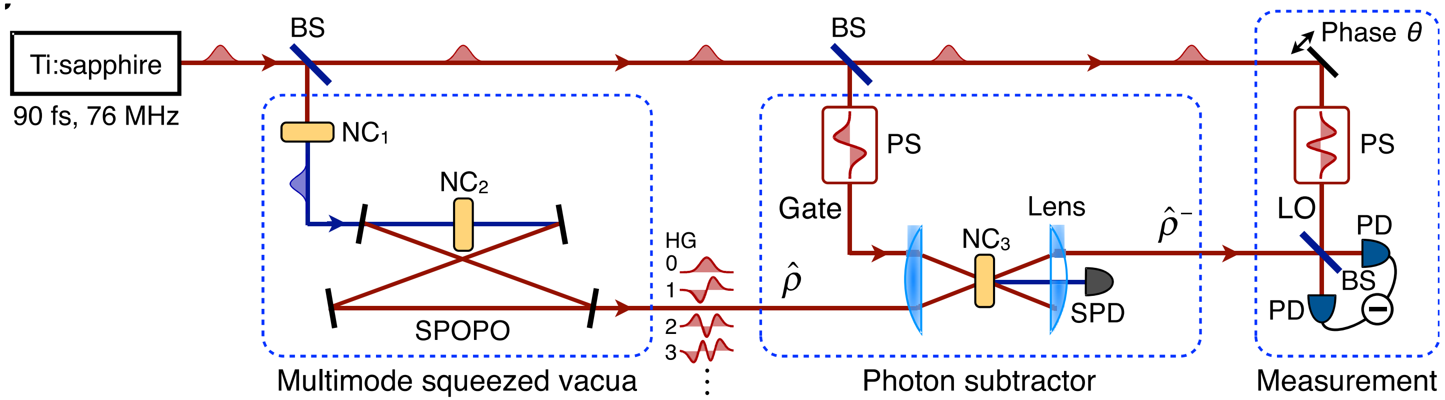
LKB

- Non-Gaussian states from quantum correlations
- Quantum correlations from non-Gaussian operations

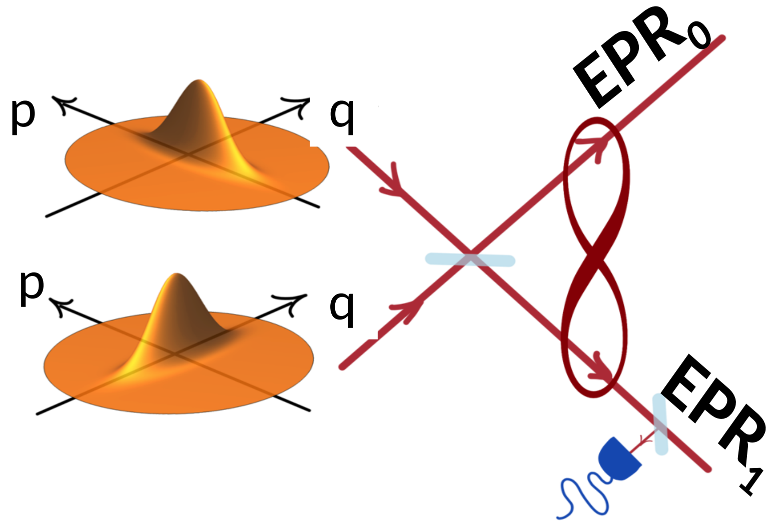
How do we make non-Gaussian states?

Photon-subtraction

How the experimentalists see it

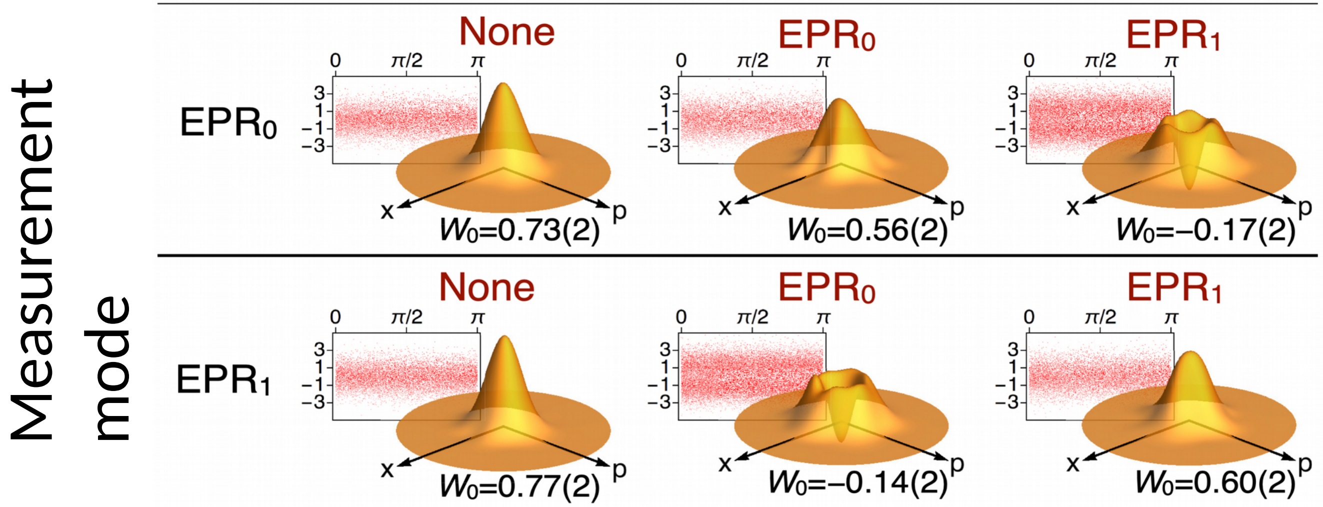


How theorists see it

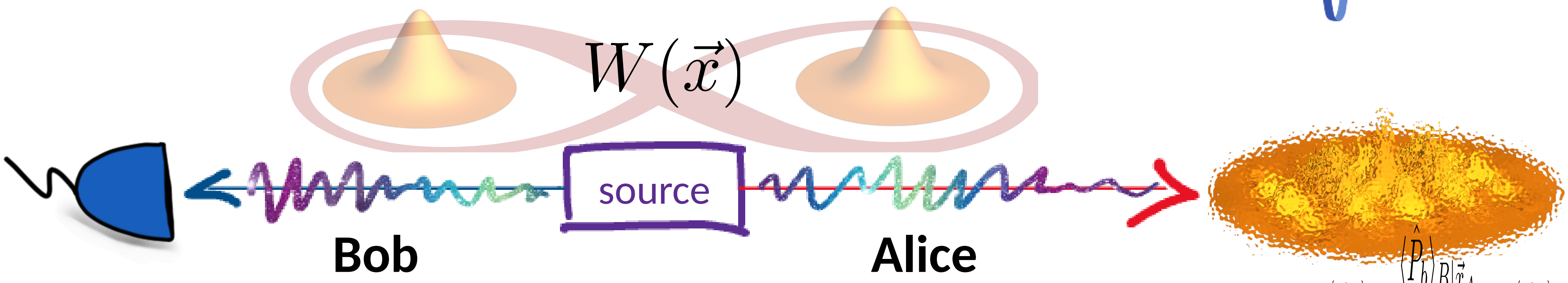


Ra, et al. Nat. Phys. **16**, 144–147 (2020)

Subtraction mode



The making of non-Gaussian states



$$W_{A|\hat{P}_b}(\vec{x}_A) = \frac{\langle \hat{P}_b \rangle_{B|\vec{x}_A}}{\langle \hat{P}_b \rangle} W_A(\vec{x}_A)$$

Bob performs some measurement $\hat{B} = b$

$Prob[\hat{B} = b] = \langle \hat{P}_b \rangle$

We derive a quantum version of Bayes' theorem:

"Quantum conditional probability" → Can be negative!

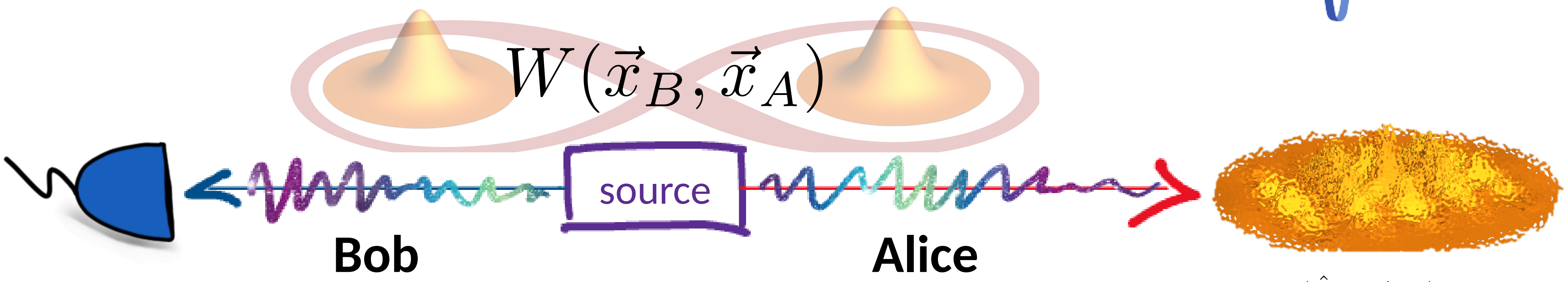
$$W_{A|\hat{P}_b}(\vec{x}_A) = \frac{P(\hat{B} = b|\vec{x}_A)}{\langle \hat{P}_b \rangle} W_A(\vec{x}_A)$$

Probability distribution on phase space

Probability to measure b

Quantum 7, 1038 (2023).

Quantum conditional probability



Bob performs some measurement $\hat{B} = b$

$$W_{A|\hat{P}_b}(\vec{x}_A) = \frac{P(\hat{B} = b|\vec{x}_A)}{\langle \hat{P}_b \rangle} W_A(\vec{x}_A)$$

What is quantum conditional probability?

Wigner function of Bob's measurement

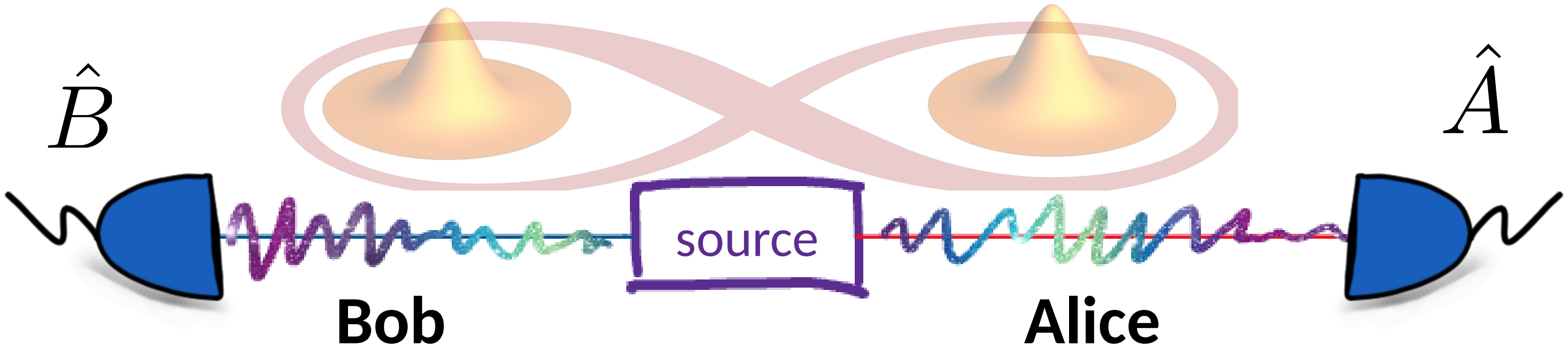
$$P(\hat{B} = b|\vec{x}_A) = (4\pi)^{m_B} \int_{\mathbb{R}^{2m_B}} \mathrm{d}\vec{x}_B W_{\hat{P}_b}(\vec{x}_B) W(\vec{x}_B|\vec{x}_A)$$

Conditional Wigner function

$$= \frac{W(\vec{x}_B, \vec{x}_A)}{W(\vec{x}_A)}$$

When can it be negative?

Joint measurement statistics



We can express the joint probability distribution using Wigner functions

"Quantum conditional probability"

$$P(\hat{A}=a,\hat{B}=b) = \int_{\mathbb{R}^{2m}} (4\pi)^m W_{\hat{A}=a}(\vec{x}_A) P(\hat{B}=b|\vec{x}_A) W_{\hat{\rho}_A}(\vec{x}_A) d\vec{x}_A$$

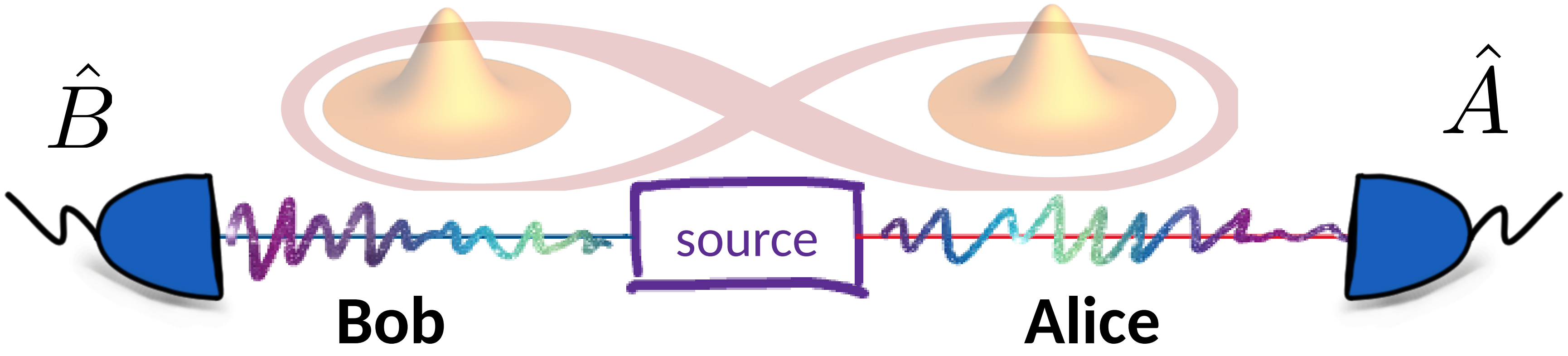
In the case of Wigner-positive measurements

$$P(\hat{A}=a|\vec{x}_A)$$

No Wigner negativity

$$P(\vec{x}_A)$$

Joint measurement statistics



We can express the joint probability distribution using Wigner functions

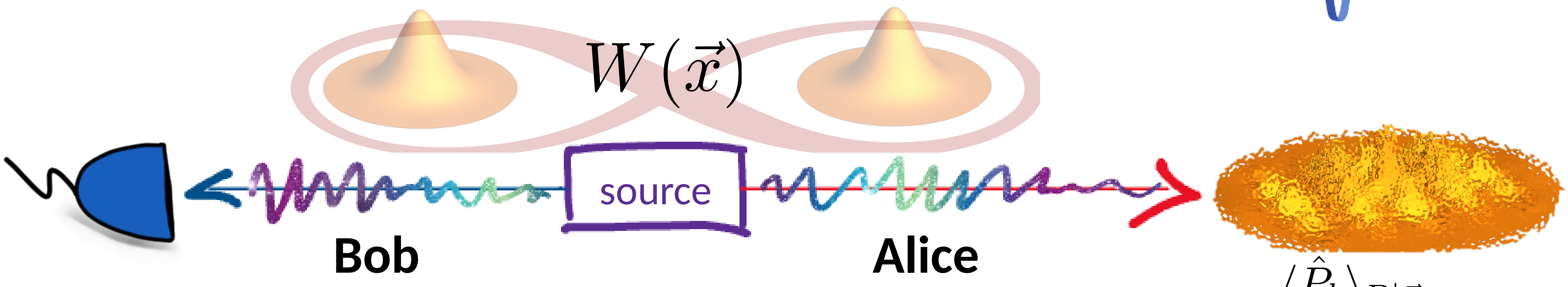
$$P(\hat{A}=a,\hat{B}=b)=\int_{\mathbb{R}^{2m}} P(\hat{A}=a|\vec{x}_A)P(\hat{B}=b|\vec{x}_A)P(\vec{x}_A)d\vec{x}_A$$

If the quantum conditional probability is **postive**, this describes a **local hidden variable model**

$$P(\hat{B}=b|\vec{x}_A)<0$$

If Alice can "steer" Bob with Wigner-positive measurements,
such hidden variable model cannot exist

What did we learn?



Bob performs some measurement $\hat{B} = b$

$$W_{A|\hat{P}_b}(\vec{x}_A) = \frac{\langle \hat{P}_b \rangle_{B|\vec{x}_A}}{\langle \hat{P}_b \rangle} W_A(\vec{x}_A)$$

When can Bob's measurement create Wigner negativity in Alice's subsystem?

Alice and Bob share a Gaussian state → Negativity can be created **if and only if** Alice can steer Bob with Gaussian measurements

PRX Quantum 1, 020305 (2020)

General case → Negativity can be created **if** ~~**and only if**~~ Alice can steer Bob with Wigner-positive measurements

Quantum 7, 1038 (2023).

Overview

LKB

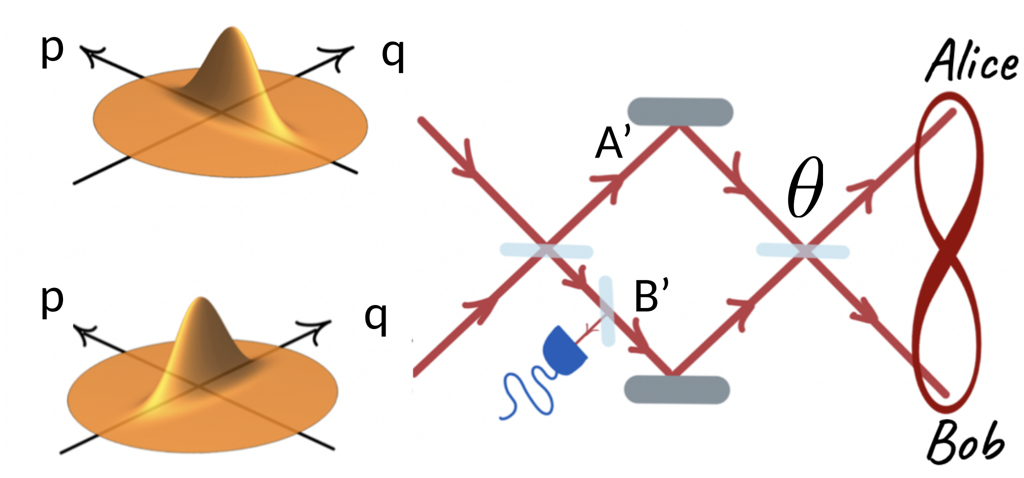
- Non-Gaussian states from quantum correlations

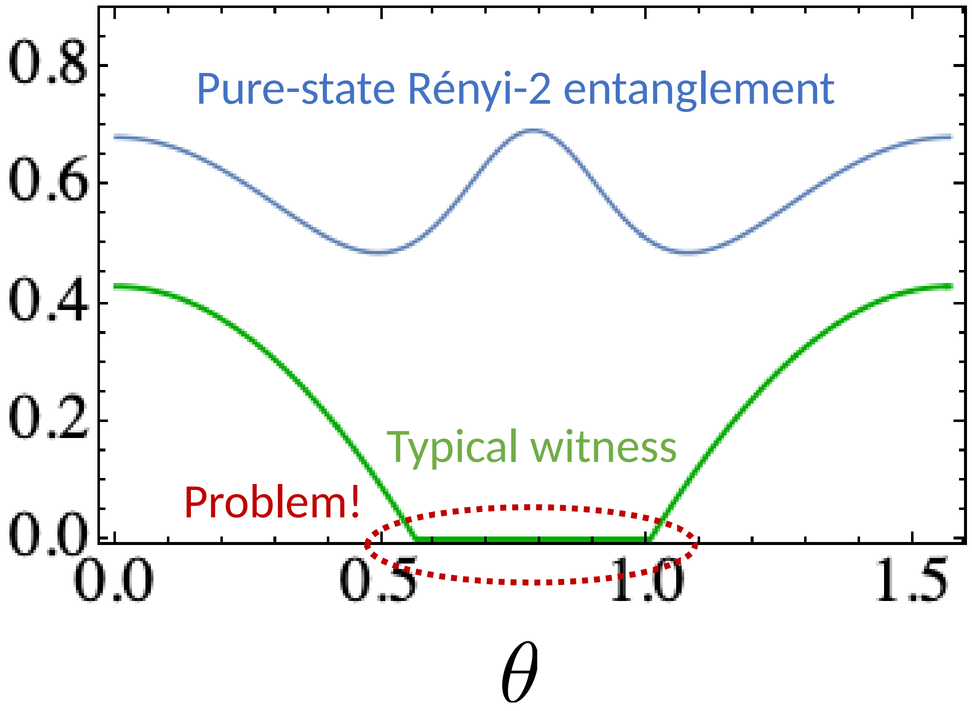
- Quantum correlations from non-Gaussian operations

Entanglement and photon subtraction

Photons subtraction can create states where the entanglement cannot be undone by a beam splitter

K. Zhang, J. Jing, N. Treps, MW, Quantum **6**, 704 (2022).





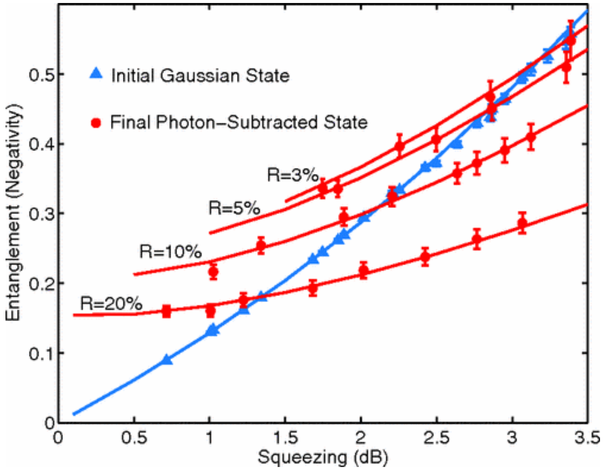
How do we measure non-Gaussian entanglement for **mixed states** in a quantum optics experiment?

MW, C. Fabre, V. Parigi, N. Treps Phys. Rev. Lett. **119**, 183601 (2017)

Non-Gaussian entanglement

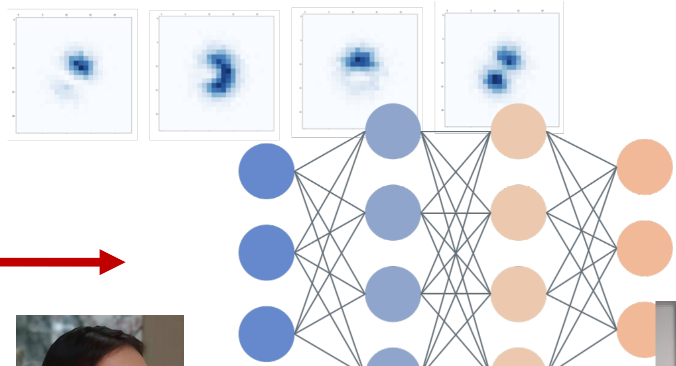
How can we detect non-Gaussian entanglement experimentally?

Quantum-state tomography

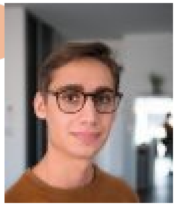


Ourjoumtsev, *et al.*
Phys. Rev. Lett. 98, 030502 (2007)

Machine learning methods?



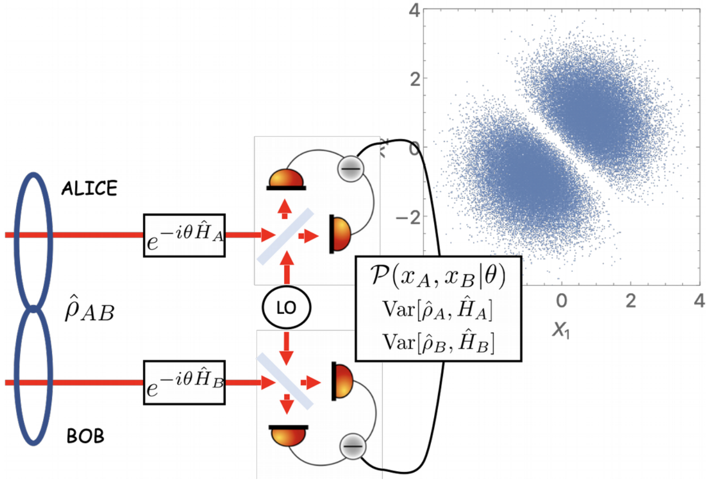
Xiaoting Gao



Mathieu Isoard

See Mathieu's flash talk!

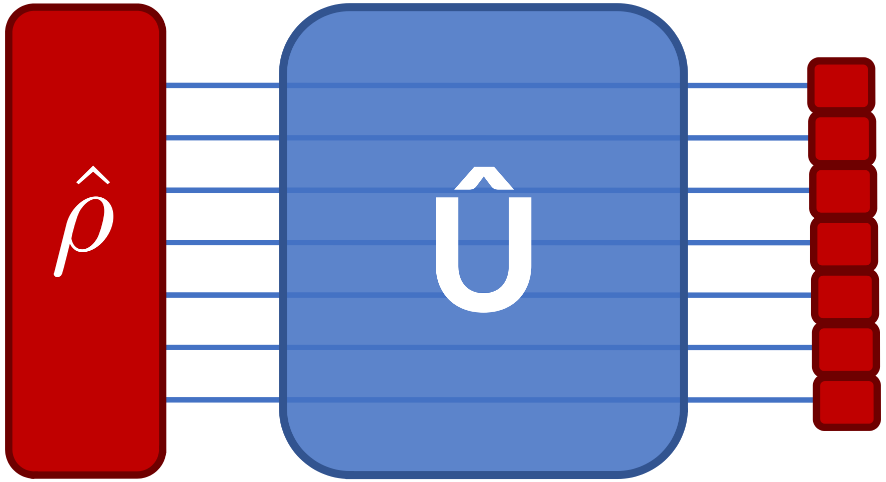
Metrological witnesses



D. Barral, et al.,arXiv 2301.03909v1 (2023)

Passive separability

A state is **passively separable** if there is a passive linear optics circuit that destroys all its entanglement

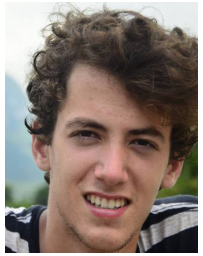


$$\sum_{\lambda} p_\lambda \, \hat{\rho}_1^{(\lambda)} \otimes \cdots \otimes \hat{\rho}_m^{(\lambda)}$$

In some sense we need states that are **not passively separable** to get a quantum computational advantage.

[U. Chabaud and MW, PRL **130**, 090602 (2023)]

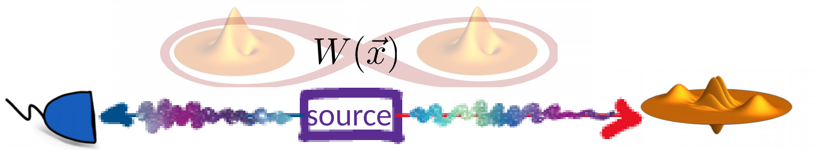
How can we check whether a state is **passively separable** ?



See flash talk by Carlos!

Take home message

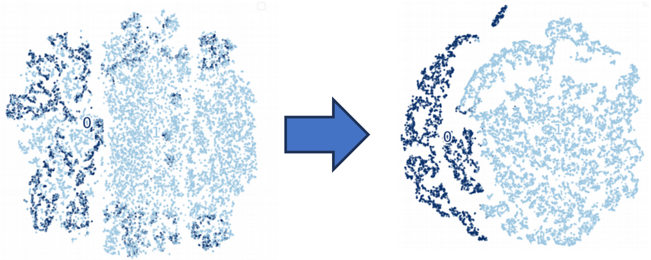
Wigner negativity



Quantum steering is a succent resource for measurement-based creation of Wigner negativity

Quantum **7**, 1038 (2023)

Non-Gaussian entanglement



Homodyne data contain signature of non-Gaussian entanglement. These signatures can amplified and detected by neural networks.

arXiv:2310.20570

Outlook

Outlook

Wigner negativity
Mari and Eisert, Phys. Rev. Lett. 109, 230503 (2012)

How is it all connected?

Non-Gaussian entanglement
U. Chabaud, MW Phys. Rev. Lett. 130, 090602 (2023)

???

Stellar rank
U. Chabaud, MW Phys. Rev. Lett. 130, 090602 (2023)

Quantum steering
MW Quantum 7, 1038 (2023).

Contextuality
Booth, *et al*. Phys. Rev. Lett. 129, 230401 (2022)