Max Planck - University of Ottawa Centre
for Extreme and Quantum Photonics

Lundeen Lab
Quantum Photonics

# Direct Measurement: from the wavefunction to the Kirkwood-Dirac distribution

**Jeff Lundeen**

**Lundeen Lab**
**www.photonicquantum.info**
Department of Physics
Centre for Research in Photonics, Joint NRC-uOttawa Centre for Extreme Photonics

QuiDiQua 2023 (Lille)

uOttawa

# Direct Measurement Group Members

Thomas Bailey

Aldo Becerril

Jeff Lundeen

# Direct Measurement Alumni

Charles Bamber

Corey Stewart

Aabid Patel

Brandon Sutherland

Raphael Abrahao

Jash Banker

Yamn Chalich

Matthew Horton

Rebecca Saaltink

Davor Curic

Felix Hufnagel

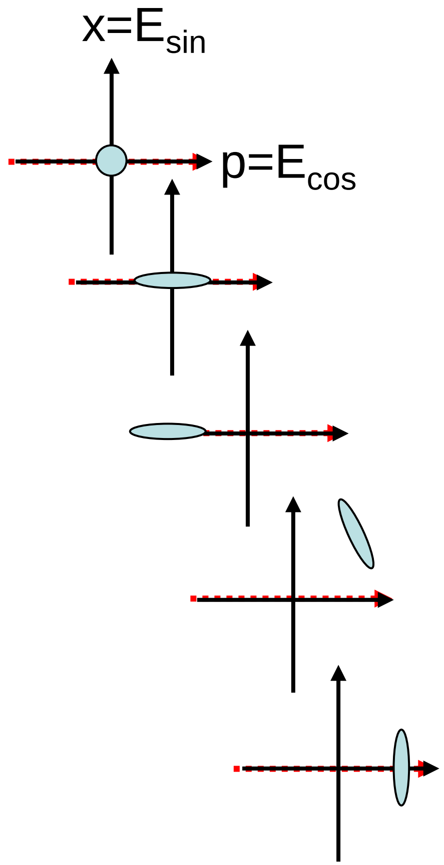
Guillaume Thekkadath

Abdulkarim Hariri

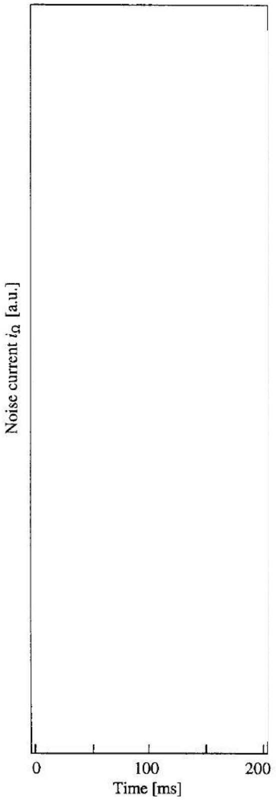
Lambert Giner

# Quantum State Tomography

**Wavefunction of an Electric Field**

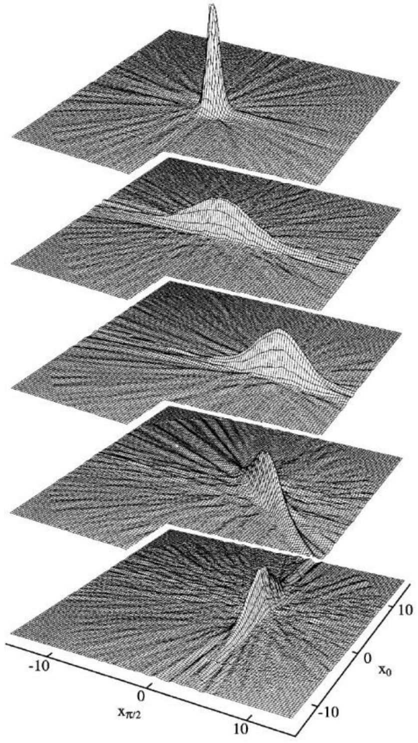



Homodyne Detection

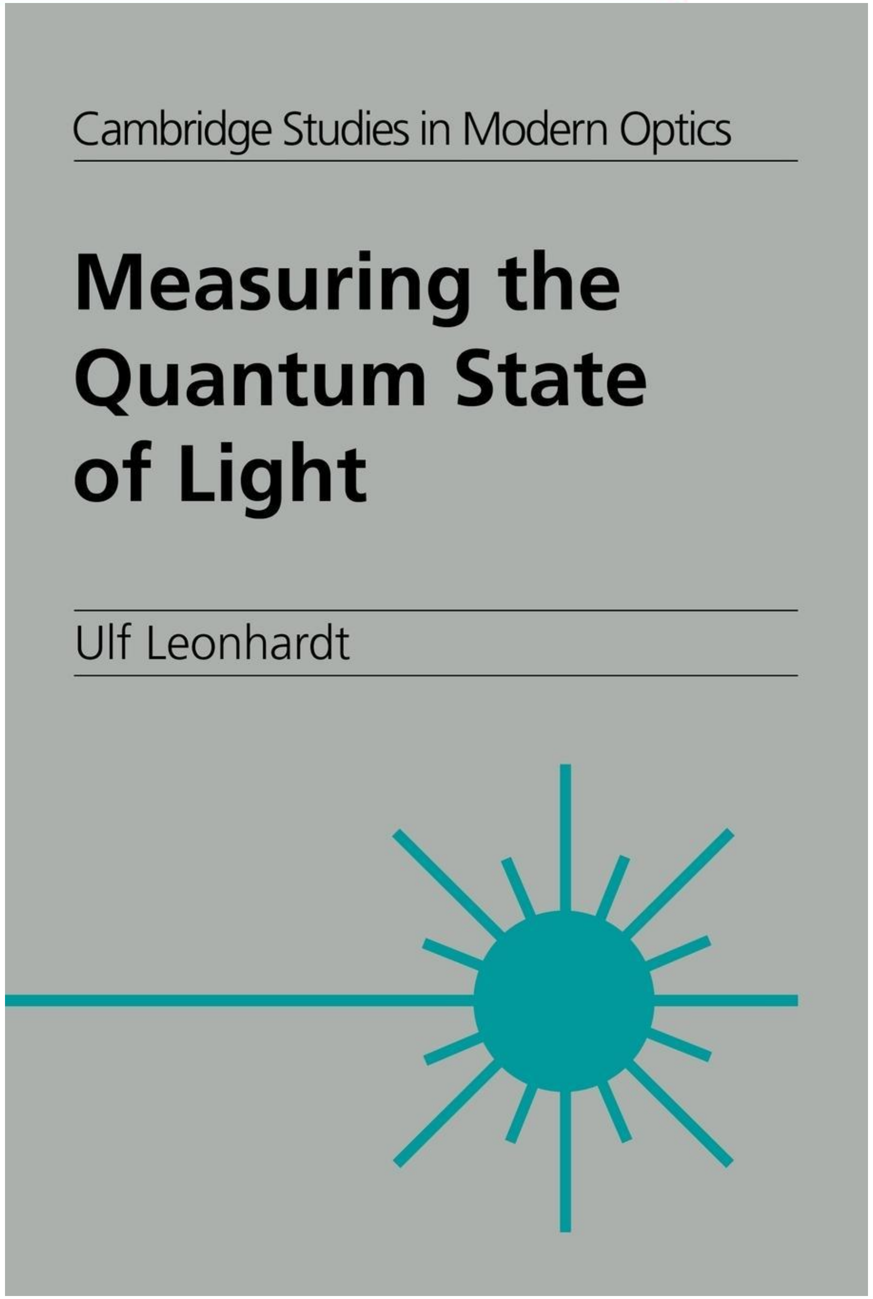



Gerd Breitenbach, Science (2000)

Find the Wavefunction (or Wigner Function) most compatible with those measurements.




- Proposed by K. Vogel. and H. Risken, Phys. Rev. A 40: 2487 (1989).
- Demonstrated by Mike Raymer: Phys. Rev. Lett. 70, 1244 (1993).
- Reconstruction is effective and well developed but indirect.




p 8 - “we cannot measure position and momentum simultaneously and precisely… What we do see are only the different aspects of a quantum object, the "quantum shadows" in the sense of Plato's famous parable”

p 98. - “Consequently, **we can not see quantum states directly**...”

# Joint measurements of $x$ and $p$

- A classical particle's state is given by its position $x$ and momentum $p$


XKCD

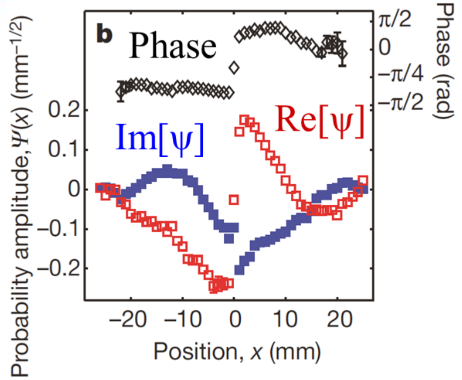
- Heisenberg's measurement-disturbance relation:

$$\Delta x\, \Delta p \geq \hbar/2$$

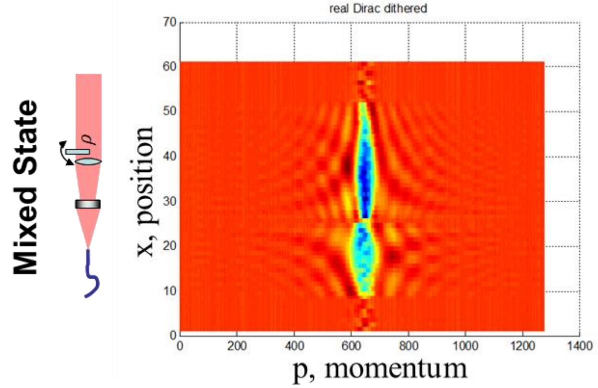
Cannot directly observe a quantum particle's state

Are there strategies to get around this?

# Outline



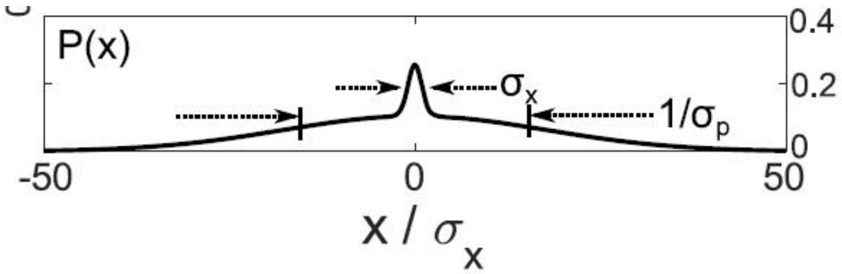

1. Direct Measurement of the wavefunction using weak measurement



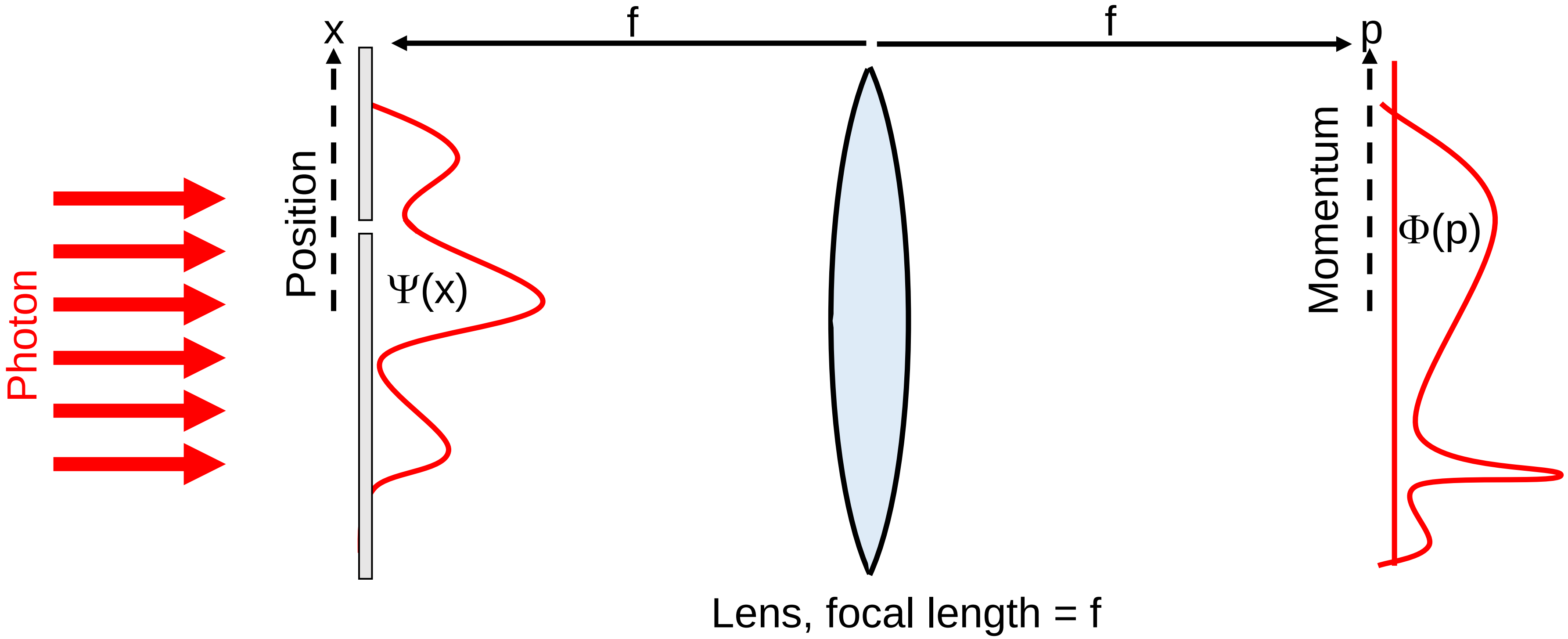

2. Generalizations: Kirkwood-Dirac, Entangled States,…

3. Other direct measurement methods: Cloning

4. Understanding what is happening in direct measurement



# Example of the Heisenberg measurement-disturbance relation




- Measure $x$ precisely and we cause $\Delta p \rightarrow \infty$
  - ➢ Can not know $x$ and $p$ perfectly at the same time

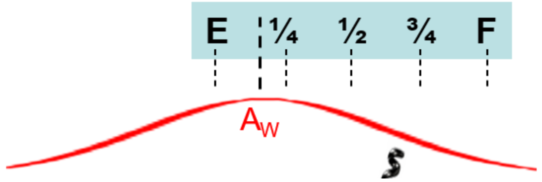
What if we decrease our certainty of $x$ ?

# Gently measure X so that you don't disturb P

- What if we do a weak measurement of $\boldsymbol{X}$, and then make a strong measurement of $\boldsymbol{P}$?

i.e. **A** = $|x\rangle\langle x| = \pi$, Initial state= $|\psi\rangle$, Strong measurement result $P=p$

Average shift of the pointer: $A_W = \dfrac{\langle b|A|\psi\rangle}{\langle b|\psi\rangle}$




$$\pi_w = \frac{\langle p|x\rangle\langle x|\psi\rangle}{\langle p|\psi\rangle}$$

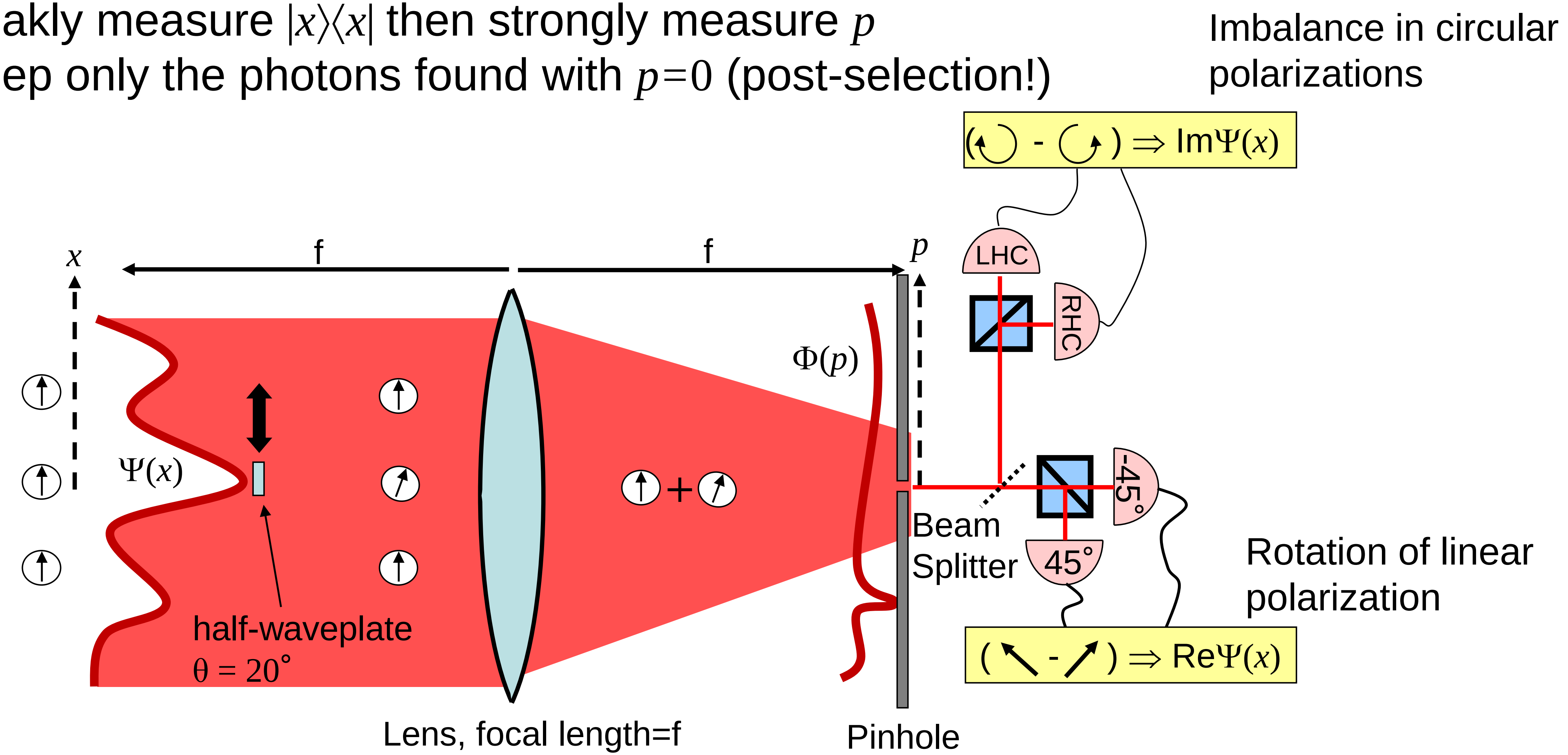
And if $p=0$, $$\pi_w = \frac{1/\sqrt{2\pi}\cdot\langle x|\psi\rangle}{\sqrt{\mathrm{Prob}(p=0)}} = \boxed{k\cdot\psi(x)}$$

- **The average shift of the pointer (i.e. rotation of the polarization) is proportional to the wavefunction**

Lundeen Nature, 474, 188 (2011)

# Direct Measurement of the Wavefunction

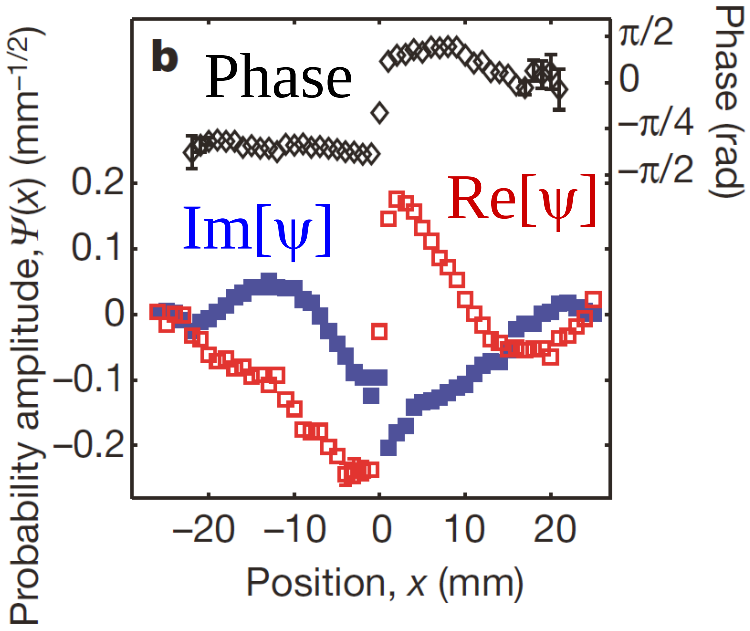
- Weakly measure $|x\rangle\langle x|$ then strongly measure $p$
- Keep only the photons found with $p=0$ (post-selection!)

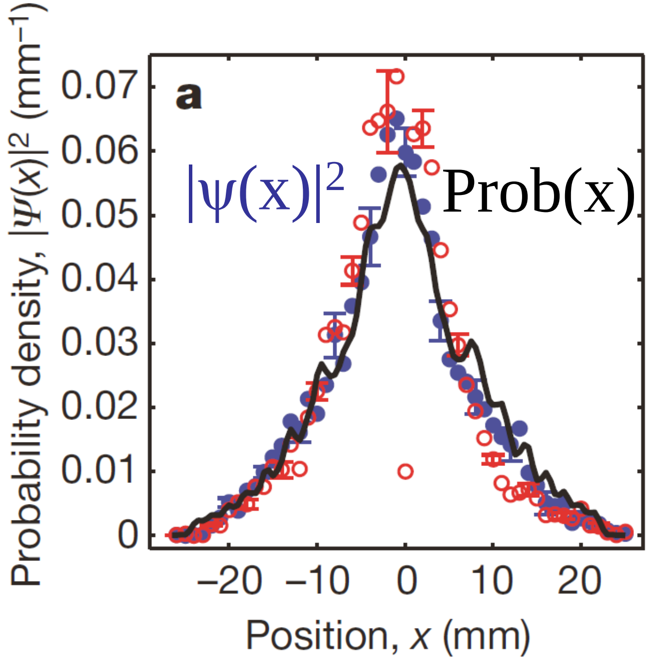



- The average result of the weak measurement is the real and imaginary components of the wavefunction

# Direct Measurement of the Wavefunction

- Demonstrate method with $\Psi(x)$ of photons exiting a single-mode fibre

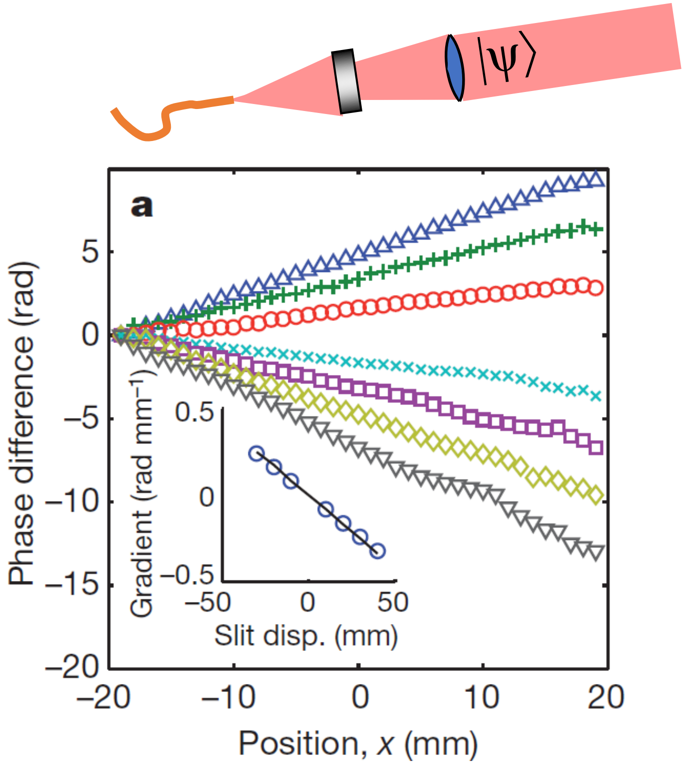




Lundeen Nature, 474, 188 (2011)

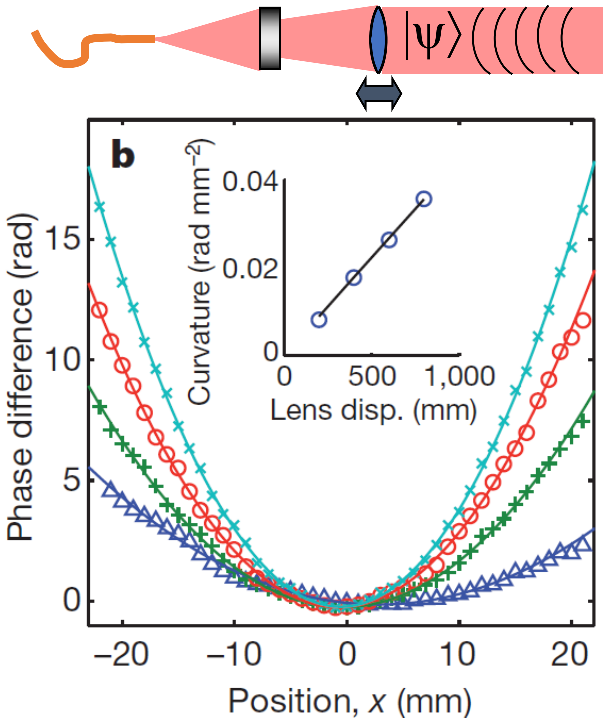
- The two signals directly give Im[ψ] and Re[ψ].
- Direct measurement accurately shows phase and magnitude of ψ(x)

# Testing other wavefunctions phase profiles

Phase Gradient



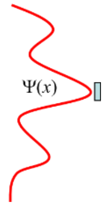

Phase Curvature



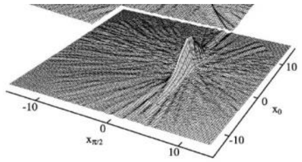



Lundeen Nature, 474, 188 (2011)

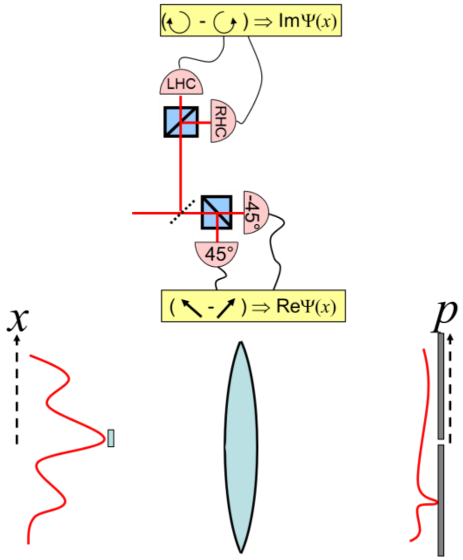
# Why it is Direct




1. It is local - measures $\psi(x)$ at $x$



2. No complicated mathematical reconstruction




3. The value of $\psi(x)$ appears right on our measurement apparatus

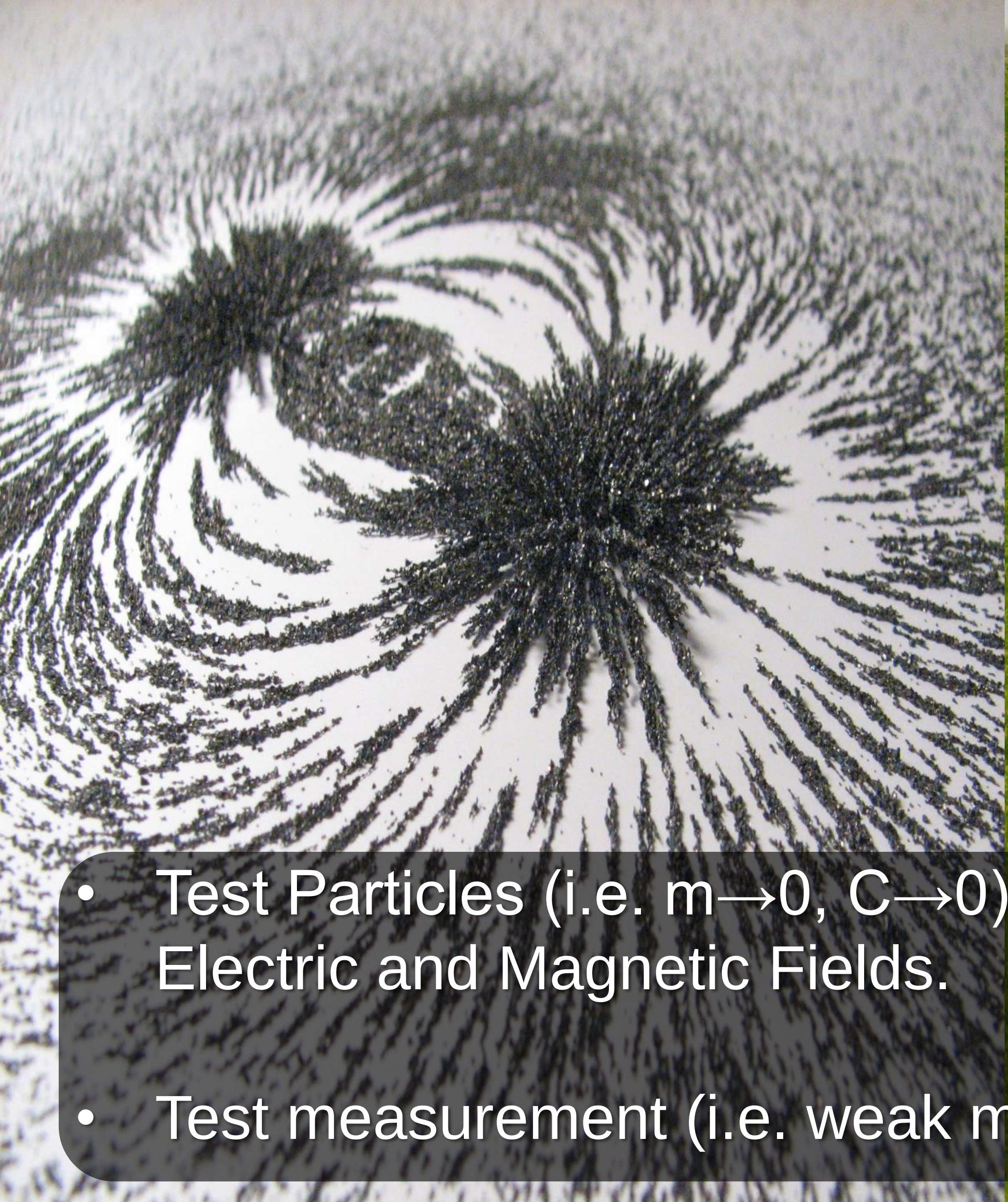
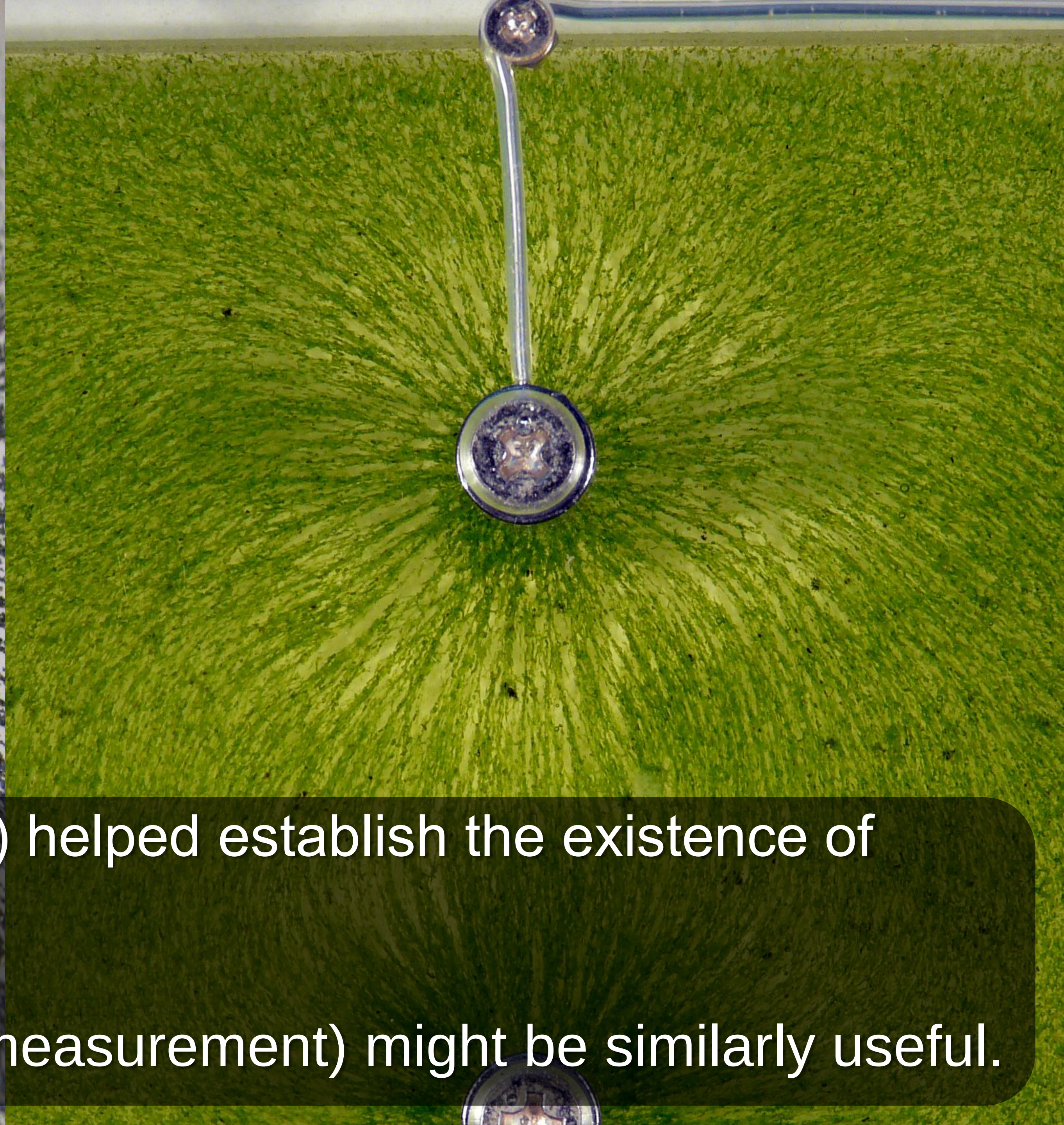
4. The procedure is simple and general - measure $x$ and then $p$

# An operational definition of the wavefunction

- Currently there is no definition of the wavefunction.
- Clarity can come from "Operational" definitions of physical concepts.
  - i.e. the set of operations used in the lab to observe something.

Bridgman, P. The Logic of Modern Physics (1927).

*"The wavefunction is the average result of a weak measurement of a variable followed by a strong measurement of the complementary variable"*

- Test Particles (i.e. m→0, C→0) helped establish the existence of Electric and Magnetic Fields.

- Test measurement (i.e. weak measurement) might be similarly useful.

- Standard tomography: Measurement bases scale with dimension
- Direct Measurement: Measurements in only two bases always

nature photonics ARTICLES
PUBLISHED ONLINE: 3 MARCH 2013 | DOI: 10.1038/NPHOTON.2013.24

**Full characterization of polarization states of light via direct measurement**

Jeff Z. Salvail[1]*, Megan Agnew[1], Allan S. Johnson[1], Eliot Bolduc[1], Jonathan Leach[1] and Robert W. Boyd[1,2]

Research Article Vol. 2, No. 4 / April 2015 / Optica 388
optica

**Scan-free direct measurement of an extremely high-dimensional photonic state**

Zhimin Shi,[1,*] Mohammad Mirhosseini,[2] Jessica Margiewicz,[1] Mehul Malik,[2,3] Freida Rivera,[1] Ziyi Zhu,[1] and Robert W. Boyd[2,4]

nature COMMUNICATIONS
ARTICLE
Received 22 Oct 2013 | Accepted 16 Dec 2013 | Published 20 Jan 2014 DOI: 10.1038/ncomms4115

**Direct measurement of a 27-dimensional orbital-angular-momentum state vector**

Mehul Malik[1,2], Mohammad Mirhosseini[1], Martin P.J. Lavery[3], Jonathan Leach[4,5], Miles J. Padgett[3] & Robert W. Boyd[1,5]

PRL 113, 090402 (2014) PHYSICAL REVIEW LETTERS 29 AUGUST 2014

**Compressive Direct Measurement of the Quantum Wave Function**

Mohammad Mirhosseini,[1,*] Omar S. Magaña-Loaiza,[1] Seyed Mohammad Hashemi Rafsanjani,[2] and Robert W. Boyd[1,3]

# Directly Measuring Entangled States

- Weakly measure where the particle pair is in H… …adox

measure e.g. $\pi_{IO} = |I…$

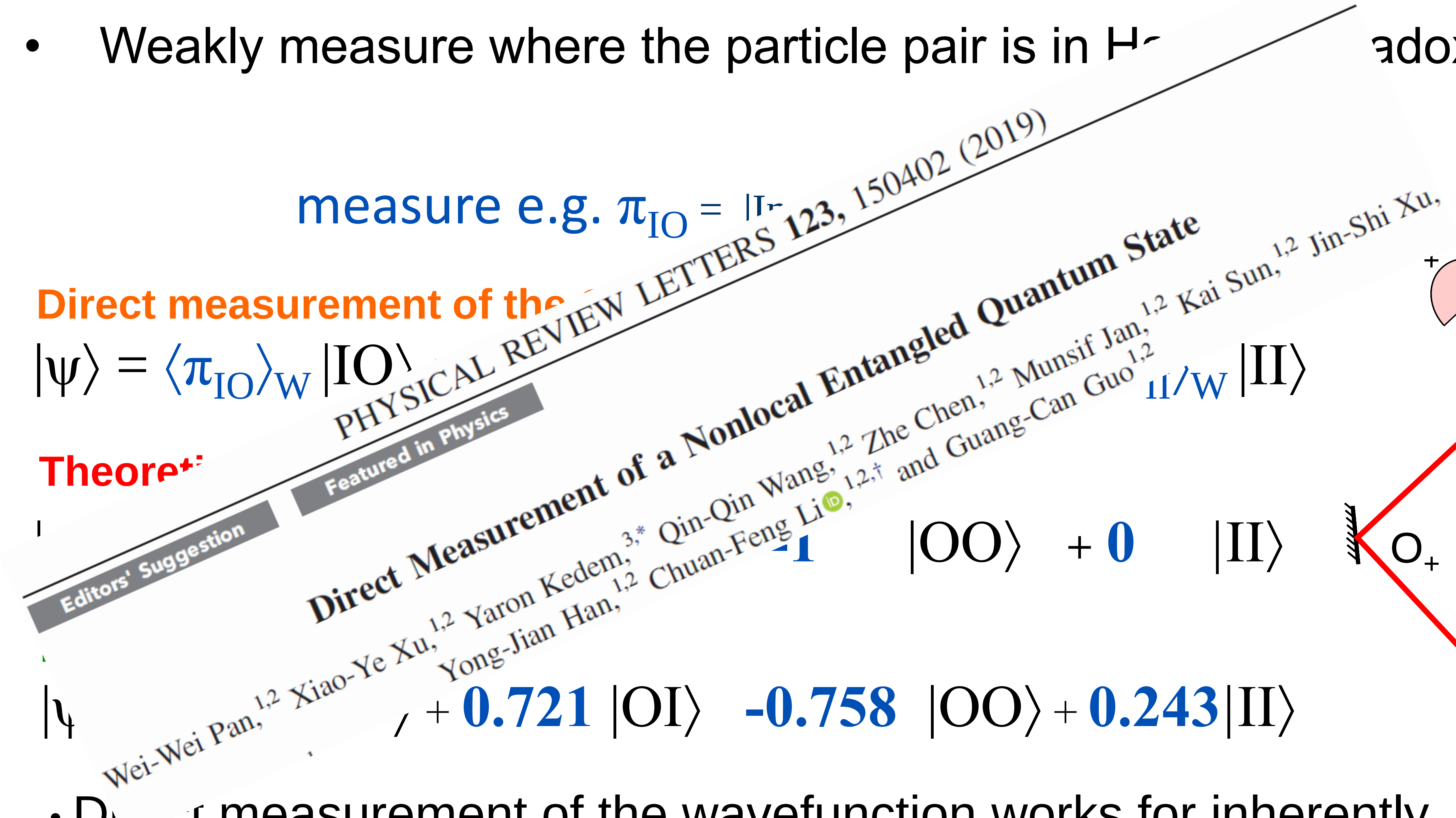
**Direct measurement of the …**

$|\psi\rangle = \langle\pi_{IO}\rangle_W |IO\rangle$ … $\ldots\rangle_W |II\rangle$

**Theoret…**

… $|OO\rangle + 0\ |II\rangle$

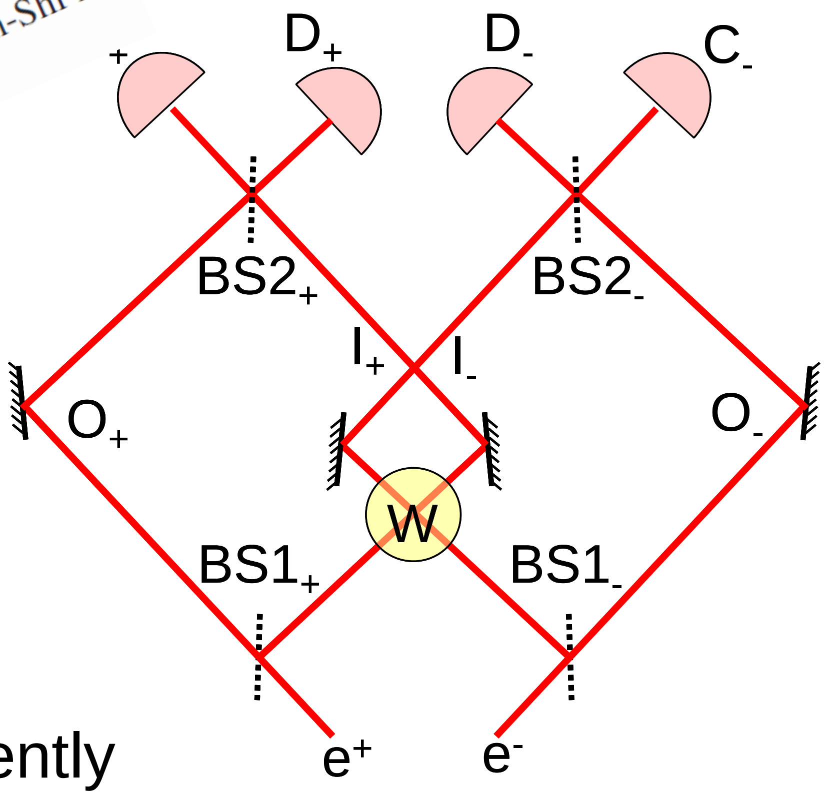
$|\psi$ … $+ \mathbf{0.721}\ |OI\rangle\ \ \mathbf{-0.758}\ |OO\rangle + \mathbf{0.243}|II\rangle$


PHYSICAL REVIEW LETTERS 123, 150402 (2019)

Editors' Suggestion | Featured in Physics

**Direct Measurement of a Nonlocal Entangled Quantum State**

Wei-Wei Pan,[1,2] Xiao-Ye Xu,[1,2] Yaron Kedem,[3,*] Qin-Qin Wang,[1,2] Zhe Chen,[1,2] Munsif Jan,[1,2] Kai Sun,[1,2] Jin-Shi Xu,[1,2] Yong-Jian Han,[1,2] Chuan-Feng Li,[1,2,†] and Guang-Can Guo[1,2]



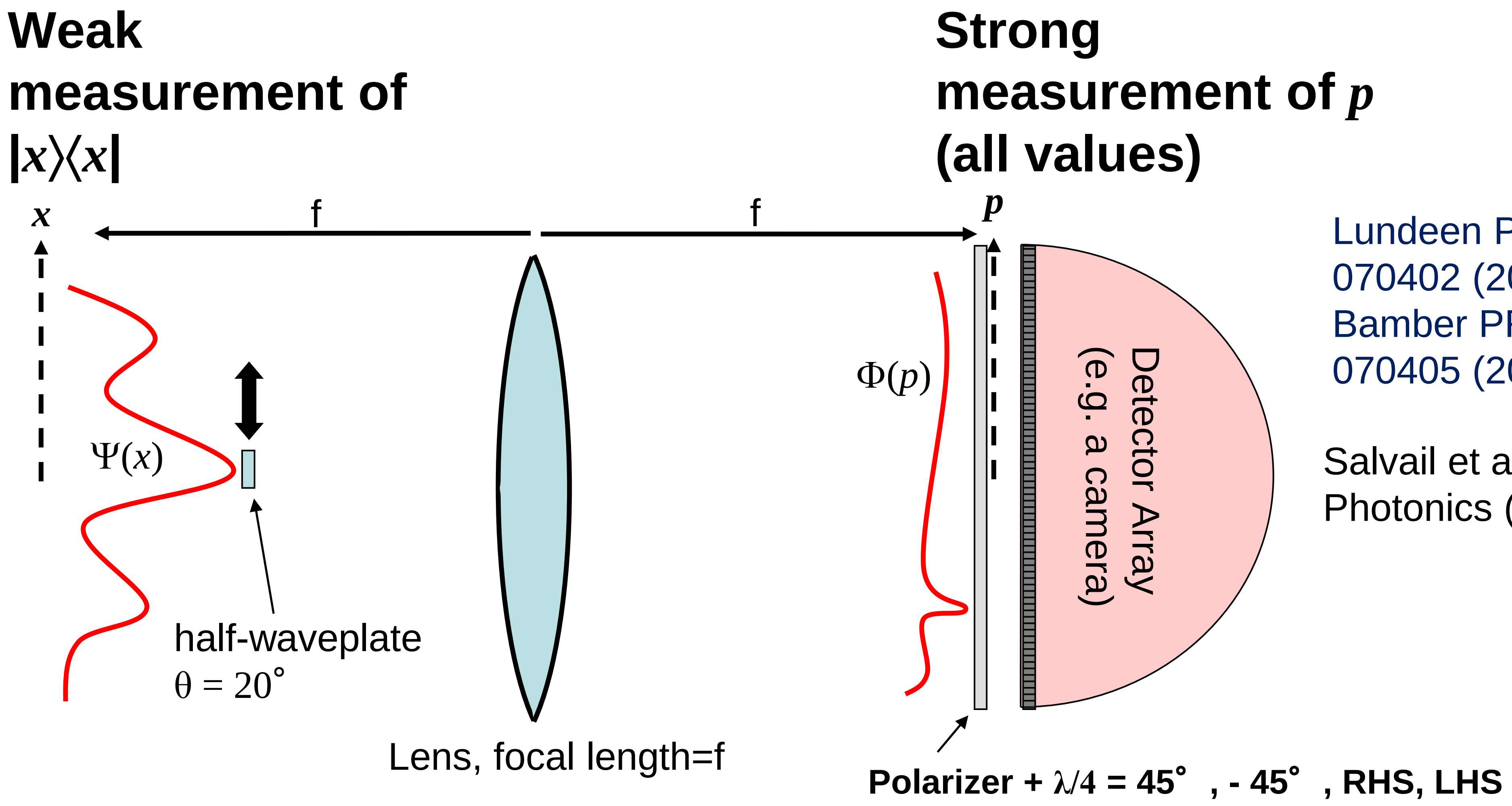

- Di…ct measurement of the wavefunction works for inherently quantum systems (i.e. entangled particles).

PRL 102, 020404 (2009) PHYSICAL REVIEW LETTERS week ending 16 JANUARY 2009

**Experimental Joint Weak Measurement on a Photon Pair as a Probe of Hardy's Paradox**

J. S. Lundeen and A. M. Steinberg

# Joint measurement of X and every P




Lundeen PRL 108, 070402 (2012), Bamber PRL 112, 070405 (2014)

Salvail et al. Nature Photonics (2013)

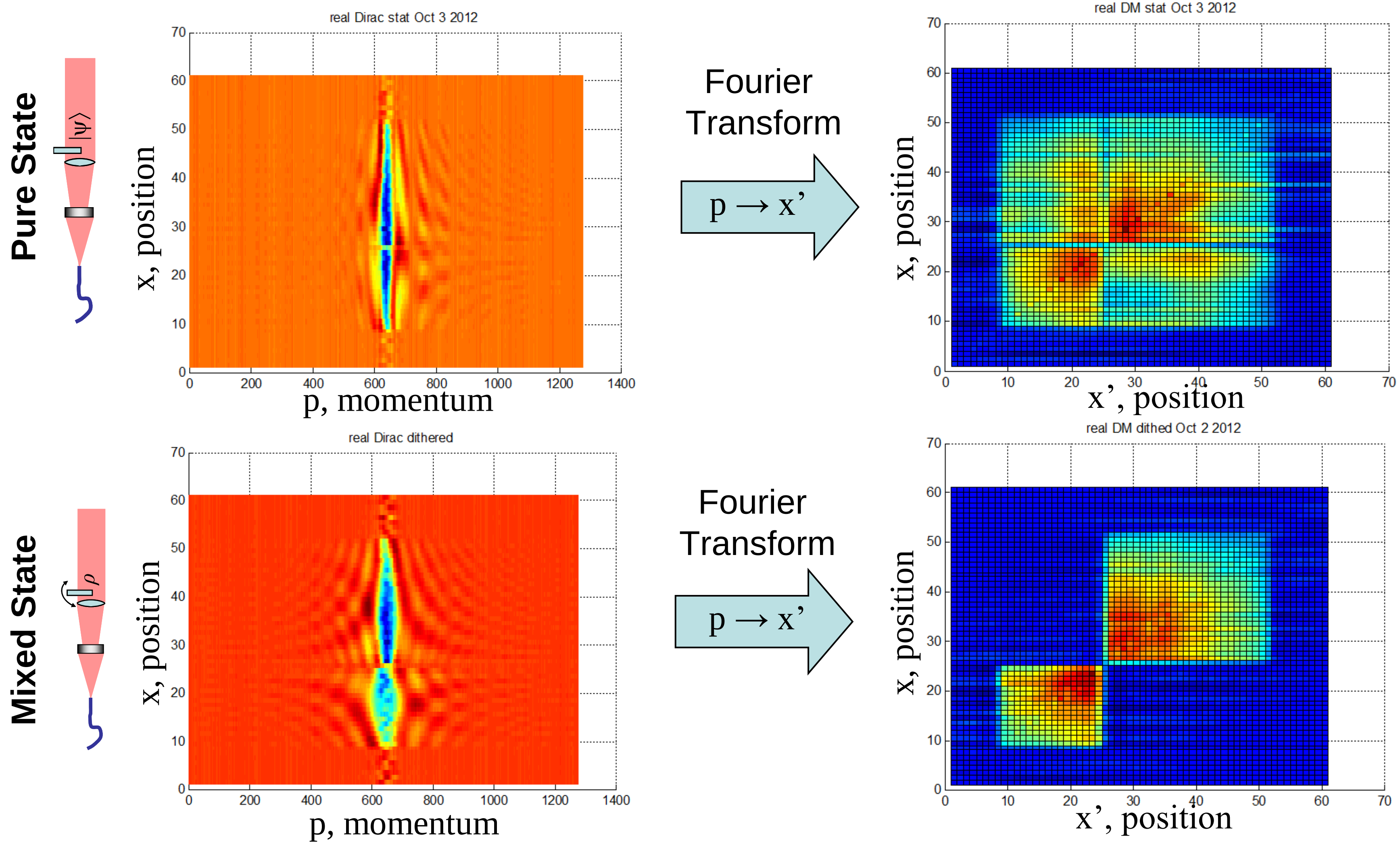
•Joint measurement of $\pi_x = |x\rangle\langle x|$ and $\pi_p = |p\rangle\langle p|$ gives the Kirkwood-Dirac Distribution:

$$D(x,p) = \langle \pi_x \pi_p \rangle = \mathrm{Tr}[\pi_x \pi_p \rho]$$

# Direct Measurement of Mixed States: The Dirac Distribution

Dirac Distribution, $D(x,p) = \mathrm{Tr}[\pi_x \pi_p \rho]$

Density Matrix $\rho(x,x')$




Lundeen PRL 108, 070402 (2012), Bamber PRL 112, 070405 (2014),

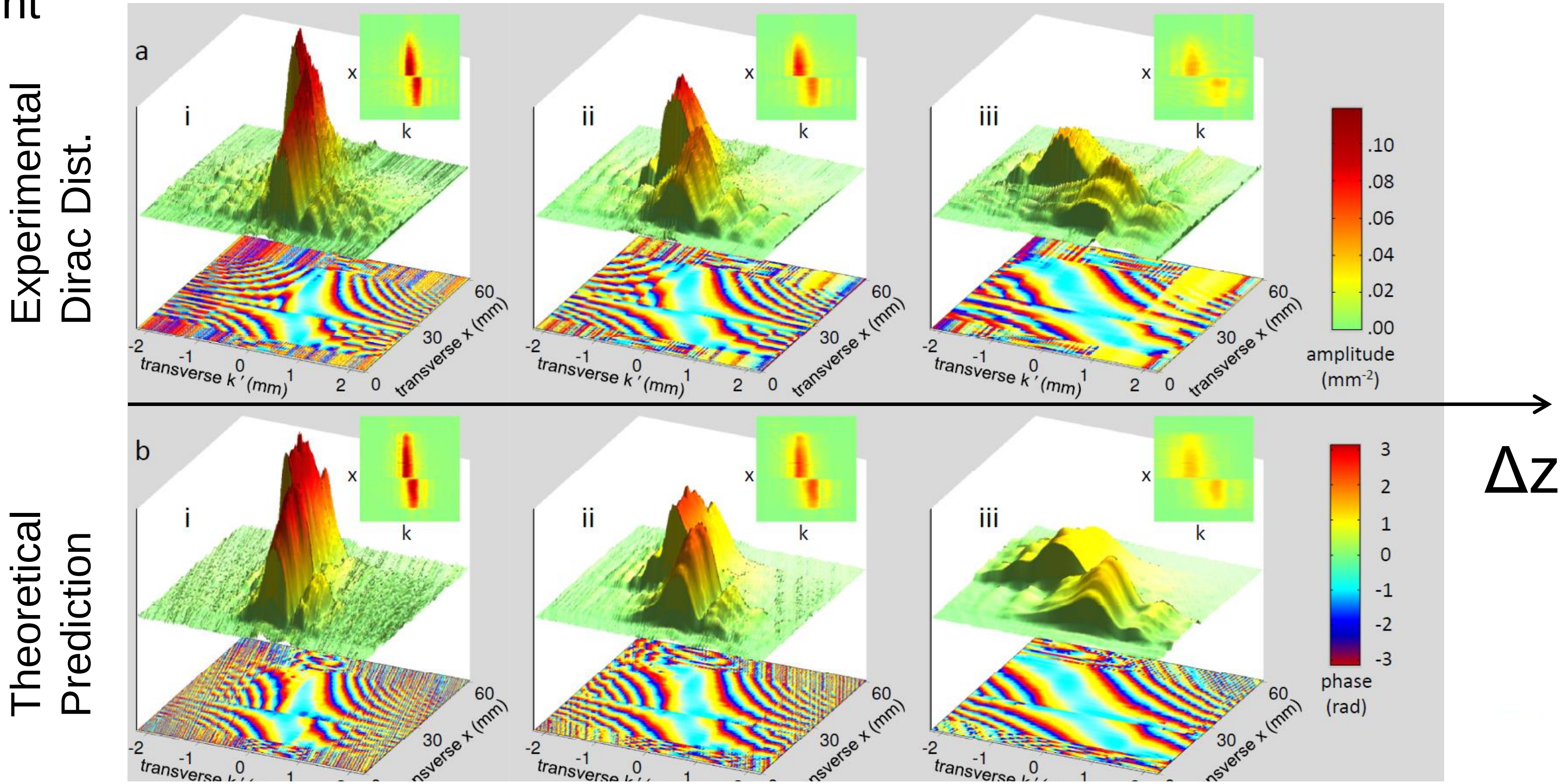
- Simple generalization allows us to completely measure mixed states

# Bayesian Propagation of the Dirac Distribution

H. F. Hofmann, New Journal of Physics, 14, 043031 (2012):

Use Baye's law to propagate the Dirac Distribution

Move camera by Δz to allow the Dirac Distribution to evolve under free propagation before the strong measurement




- The experiment confirms that the Dirac Distribution evolves in much the same way that a classical probability distribution evolves

# Bayes' Law and Weak Measurement

A. M. Steinberg, Phys. Rev. A, 52, 32 (1995):
Weakly measured probabilities (e.g. Dirac Dist.) satisfy Bayes' Law.

H. F. Hofmann, New Journal of Physics, 14, 043031 (2012):
Use Baye's law to propagate the Dirac Distribution (like in classical physics!)

1. Generalize Dirac Distribution (no longer anti-standard ordered):

$$\mathrm{Pq}_D(x,q,k,p) = \langle \delta(\mathbf{P}-p)\delta(\mathbf{K}-k)\delta(\mathbf{Q}-q)\delta(\mathbf{X}-x)\rangle$$

2. Use Baye's Law to propagate the Dirac Dist:

$$\begin{aligned}\mathrm{Pq}_{AS}(x,k) &= \sum_{x,p} \mathrm{Pq}_{\mathrm{D}}(x,q,k,p) \\ &= \sum_{x,p} \mathrm{Pq}_{\mathrm{D}}(q,k|x,p)\cdot \mathrm{Pq}_{AS}(x,p)\end{aligned}$$

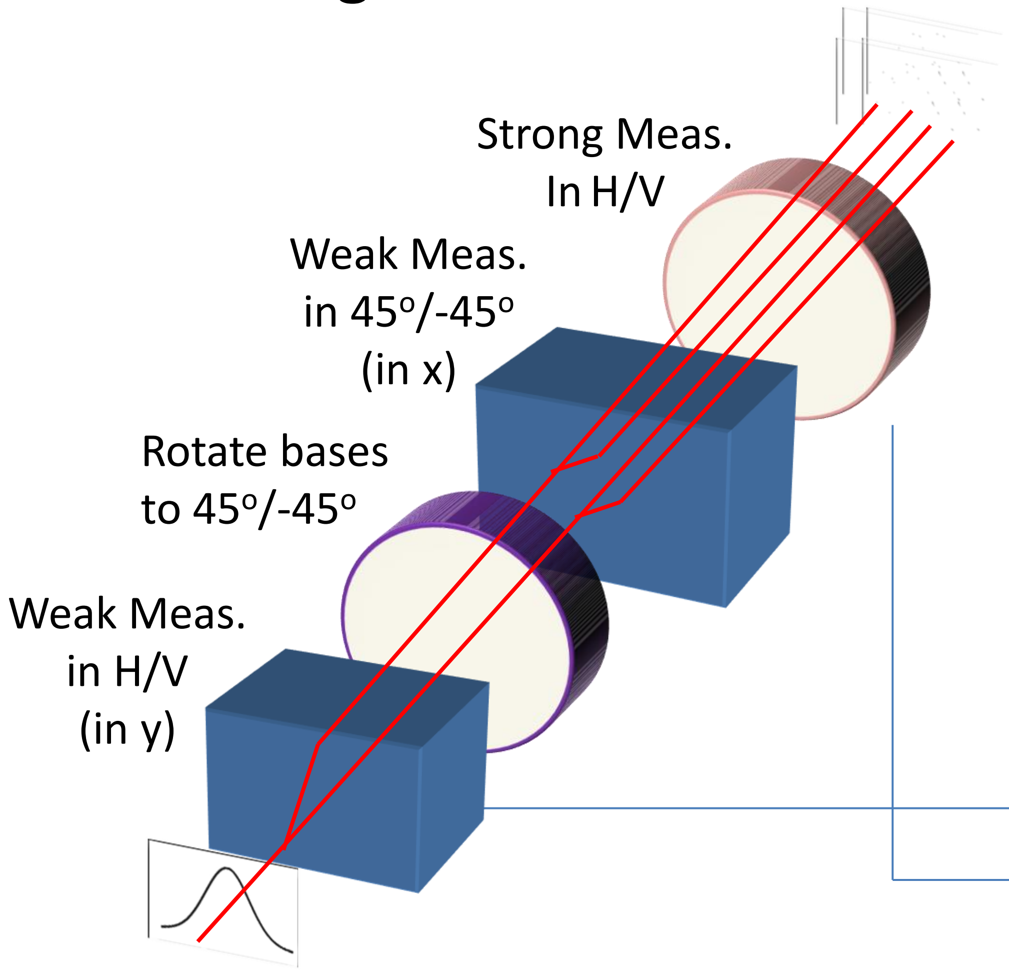
3. Use Eq 1 and the formula for the Dirac Dist to find the propagator:

$$\mathrm{Pq}_{\mathrm{D}}(q,k|x,p) = \frac{\mathrm{Pq}_{\mathrm{D}}(x,q,k,p)}{\mathrm{Pq}_{AS}(x,p)} = \frac{\langle p|k\rangle\,\langle k|q\rangle\,\langle q|x\rangle}{\langle p|x\rangle}$$

- The propagator is a weak conditional probability, made up of state overlaps

# Direct Measurement of the Density Matrix

Use two sequential weak measurements and then a strong one to obtain each density matrix element



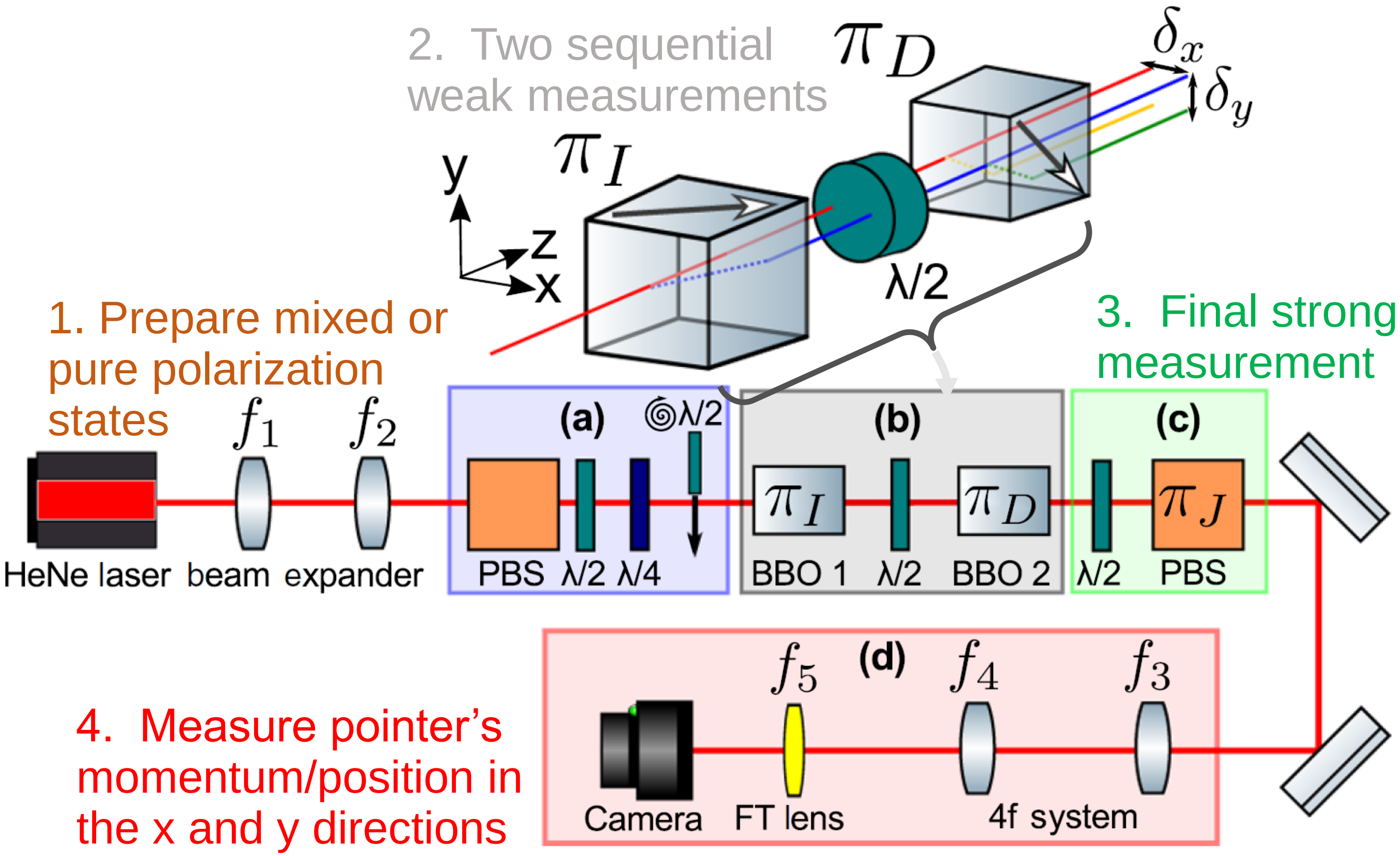

$$\rho = \begin{bmatrix} HH & HV \\ VH & VV \end{bmatrix}$$

Polarization being shifted in first weak measurement

Polarization on which the strong measurement is done

GS Thekkadath,…, JS Lundeen, PRL 117, 120401 (2016)

# Experimental Setup



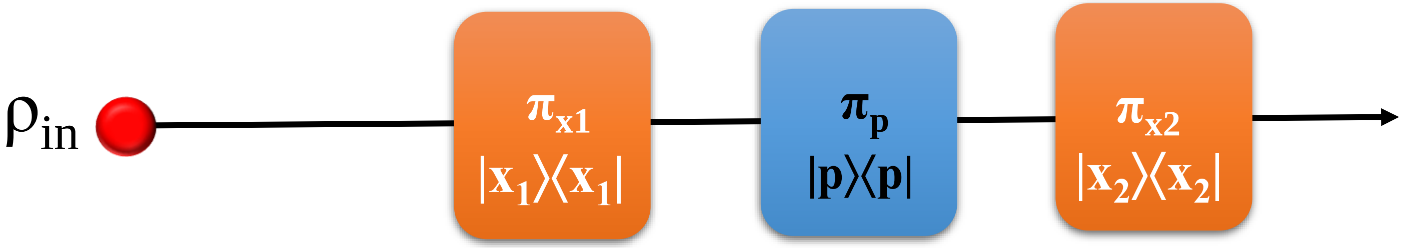

$\rho(I,J) = \langle a_x\, a_y\, \pi_J \rangle /2$, where **a** is the lowering operator

Lundeen & Resch, Phys. Lett. A, 334, 337 (2005)

- Any element of the density matrix is given by this expectation value on the pointers and system

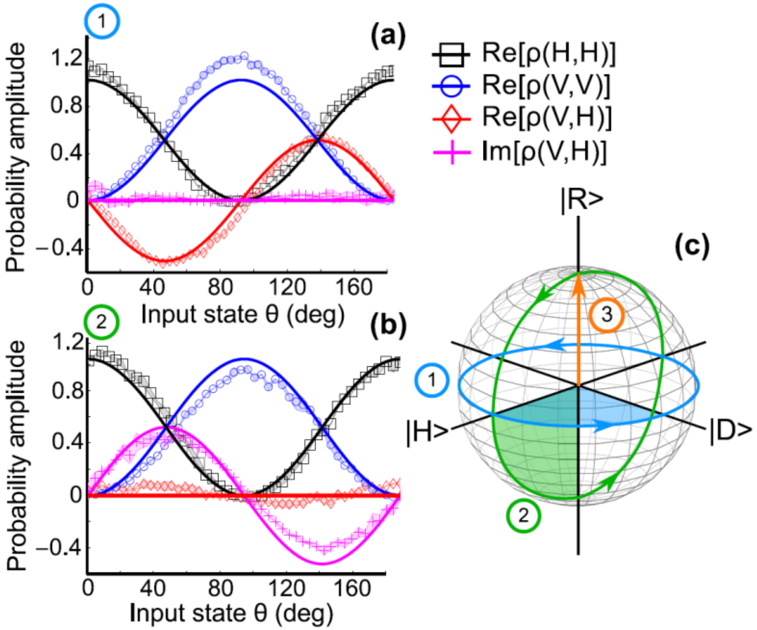
# Directly Measuring the Density Matrix

- Jointly weakly measure X then P then X again




Average result is $\mathrm{Tr}[\pi_{x1}\,\pi_p\,\pi_{x2}\,\rho_{in}] = \rho_{in}(x_1,x_2)$

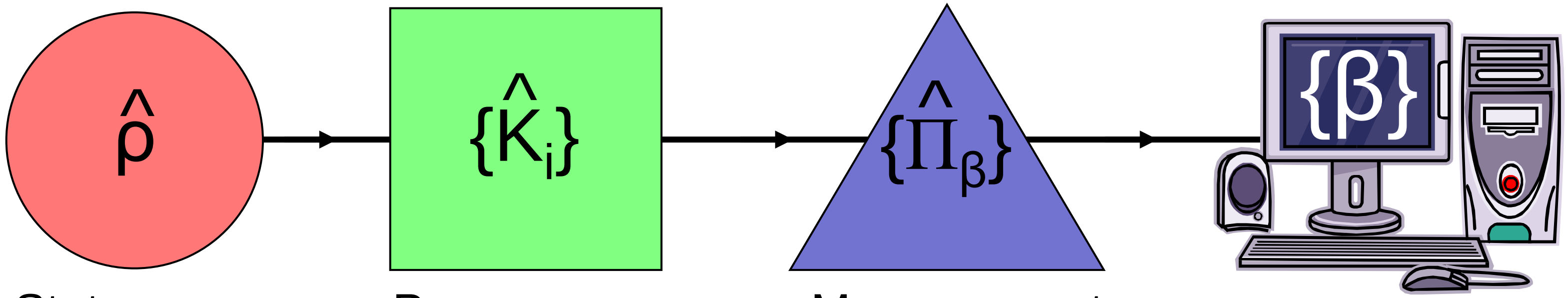
Theory: Lundeen & Bamber PRL 108, 070402 (2012).




- We can know any chosen element $\rho_{in}(x_1,x_2)$ of the density matrix e.g. a particular coherence, entanglement witnesses, etc.

Experiment: GS Thekkadath,…, JS Lundeen, PRL 117, 120401 (2016)

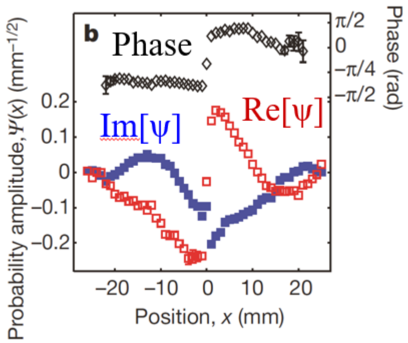
IN CHANNEL DETECTOR RESULTS




State
**Density matrix**

Process
**Kraus Operators**

Measurement
**POVMs**




Lundeen Nature, 474, 188 (2011)

nature COMMUNICATIONS

ARTICLE

DOI: 10.1038/s41467-017-02511-2 OPEN

Corrected: Publisher correction

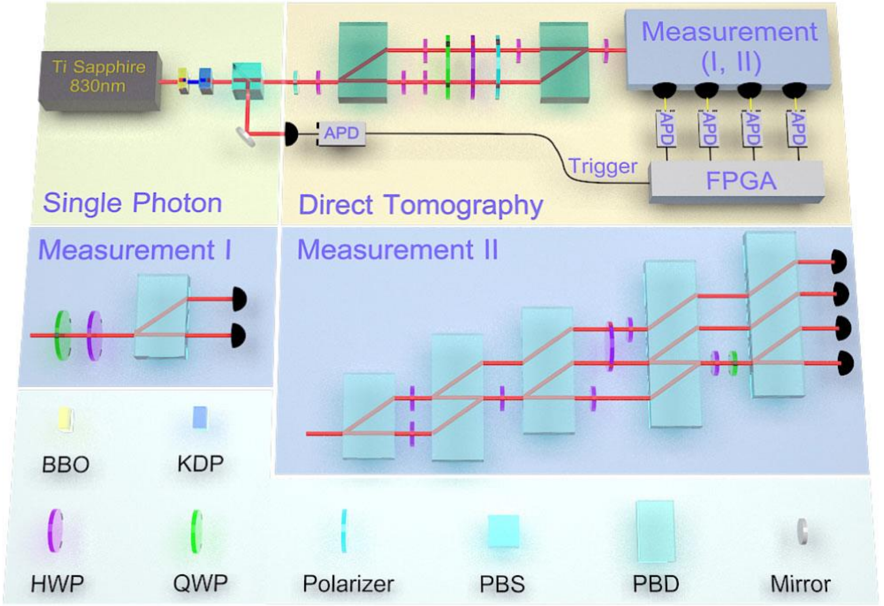
Direct quantum process tomography via measuring sequential weak values of incompatible observables

Yosep Kim[1], Yong-Su Kim[2], Sang-Yun Lee[2], Sang-Wook Han[2], Sung Moon[2], Yoon-Ho Kim[1] & Young-Wook Cho[2]

PHYSICAL REVIEW LETTERS 127, 180401 (2021)

**Direct Characterization of Quantum Measurements Using Weak Values**

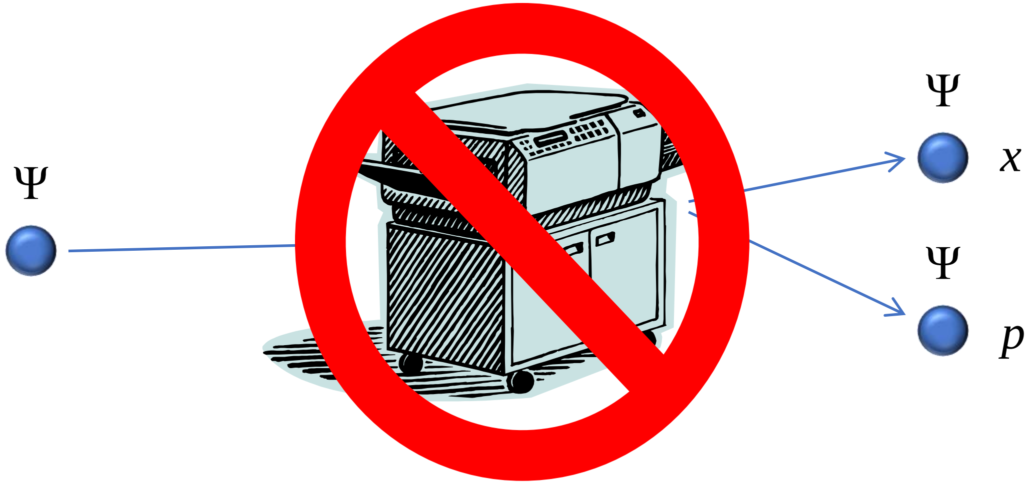
Liang Xu,[1,2] Huichao Xu,[1,3] Tao Jiang,[1] Feixiang Xu,[1] Kaimin Zheng,[1] Ben Wang,[1] Aonan Zhang,[1] and Lijian Zhang[1,*]



# Another strategy to measure x and p

- Measure X on first copy of a particle and P on the second copy.


$$|\Psi\rangle_1 \rightarrow |\Psi\rangle_1 \, |\Psi\rangle_2$$



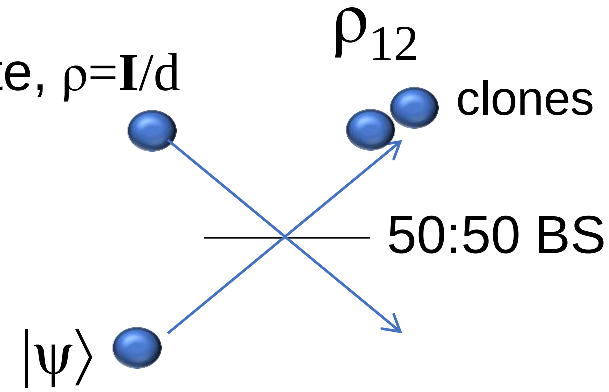

- Perfect copying forbidden by the *No cloning theorem*

Optimal Cloning: Quantum Mechanics only allows imperfect copies.

# X and P Measurements using optimal copies

## Optimal Cloning Device: the beamsplitter

Maximally mixed state, $\rho=\mathbf{I}/d$
d is the dimension




Irvine,…, 92, 047902, Bouwmeester, PRL (2004)

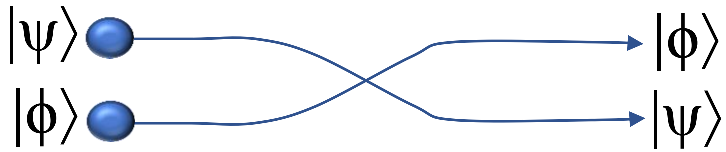
- Consider two photons entering opposite ports of a beamsplitter
- When alike they always bunch, exiting one port together

Mixed state $\rho=\mathbf{I}/2$ is $|\psi\rangle$ 50% of the time (perfect cloning)

And $|\psi^{\perp}\rangle$ 50% of the time (imperfection!)

# Cloning and the SWAP Gate

- The SWAP gate S exchanges the state of two particles




- SWAP, S can be written in terms of symmetric projector (an **optimal cloner**!)

$$\tfrac{1}{2}(I + S) = \Pi^{+}$$

- Square root SWAP is $\sqrt{S} = 1/\sqrt{2}\,(I \pm i\,S) = \Pi^{\pm i}$

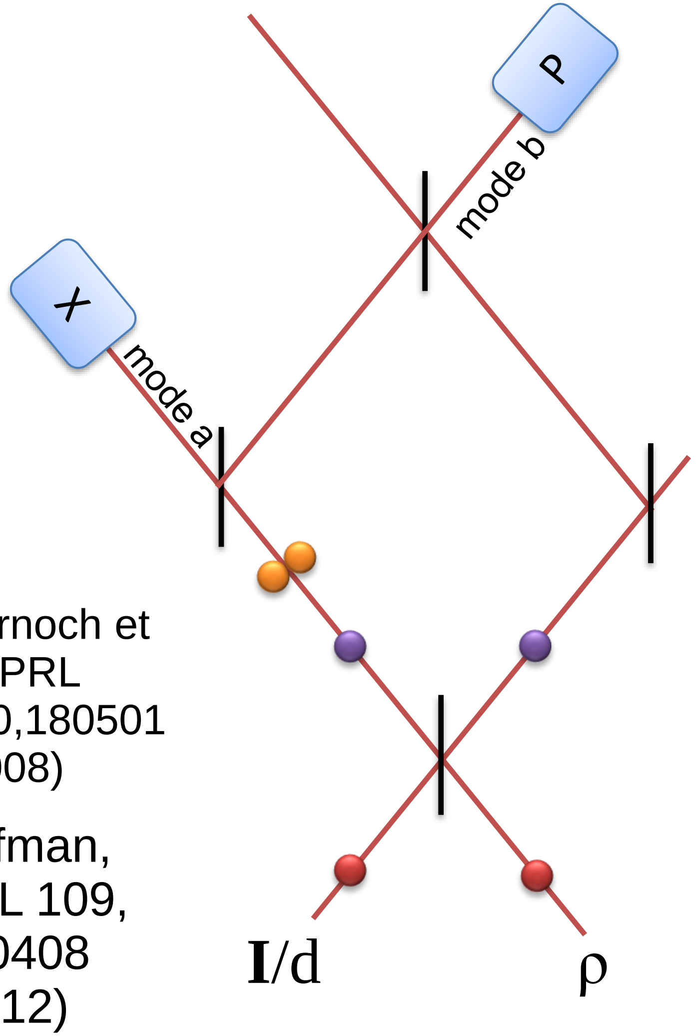
$$\text{or } \Pi^{\pm i} = \Pi^{+} \pm i\,\Pi^{-}$$

$$Re(D(x,p)) = Prob(x1, p2|\,\Pi^{+}) - Prob(x1, p2|\,\Pi^{-})$$
$$Im(D(x,p)) = Prob(x1, p2|\,\Pi^{+i}) - Prob(x1, p2|\,\Pi^{-i})$$

- The Dirac Distribution is intimately related to symmetries in optimal cloning

# Joint Measurements on Optimal Clones

- We **strongly** measure $X_a$ & $P_b$ simultaneously on clones in modes and b




Cernoch et al. PRL 100,180501 (2008)

Hofman, PRL 109, 020408 (2012)

**Case 1:** Optimal Cloning. Measure X and P on optimal clones

$$\mathrm{Prob}_1\,(X_a = x, Pb = p) \; = C + \; \mathrm{Re}(\mathrm{Tr}[\pi_x\,\pi_p\,\rho])$$

**Case 2:** Mix cloning ($\Pi^+$ projection) with $\Pi^-$ projection:

$\Pi^- + i\Pi^+ = \sqrt{}$SWAP gate

$$\mathrm{Prob}_2\,(X_a = x, Pb = p) = C - \mathrm{Im}(\mathrm{Tr}[\pi_x\,\pi_p\,\rho])$$

$$\mathrm{Prob}_1 - \mathrm{Prob}_2 \; = \; \mathrm{Tr}[\pi_x\,\pi_p\,\rho] = D(x,p)$$

**The Dirac Distribution**

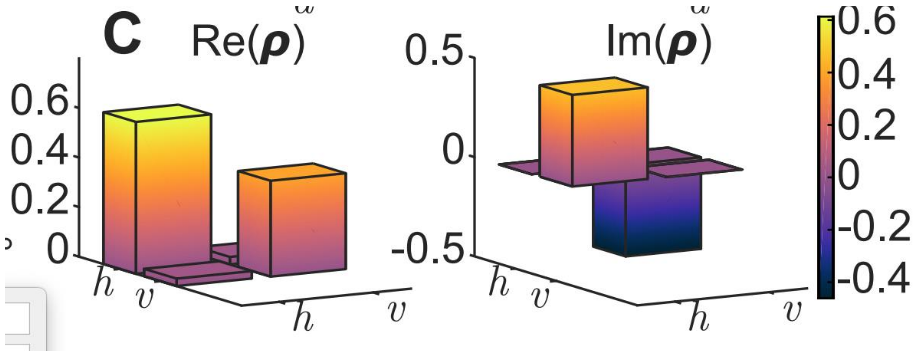
# Joint Measurements on Optimal Clones

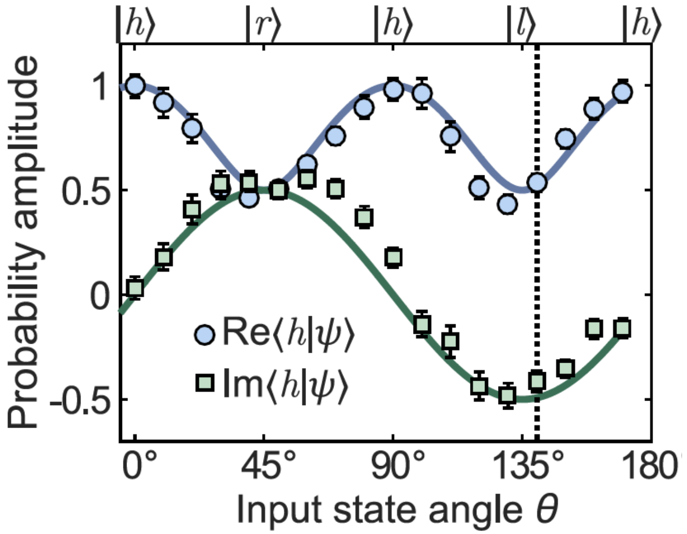
- We do optimal cloning on polarization states

  **Measure $S_z$ on one clone and $S_x$ on its partner**

- Result is a complex 2d distribution (the Dirac distribution) that is rigorously equivalent to quantum state.

- A cross-section of the 2d distribution is the input quantum state:




G. S. Thekkadath et al.
PRL **119**, 050405 (2017)

- The Fourier transform is the density matrix

- Just like in classical physics jointly measuring complementary observables gives the system state

# Learn a bit about both X and P

Balance coupling strength to two pointers for simultaneous measurement of X and P

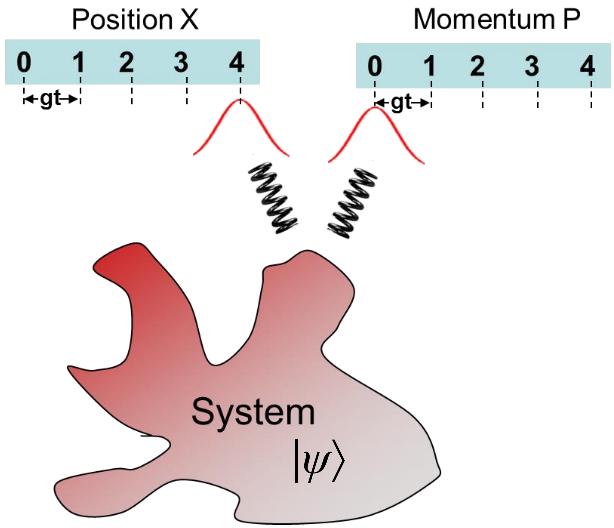
- Uncertainty in x and p are equal: Δx = Δp

B.S.T.J. BRIEFS

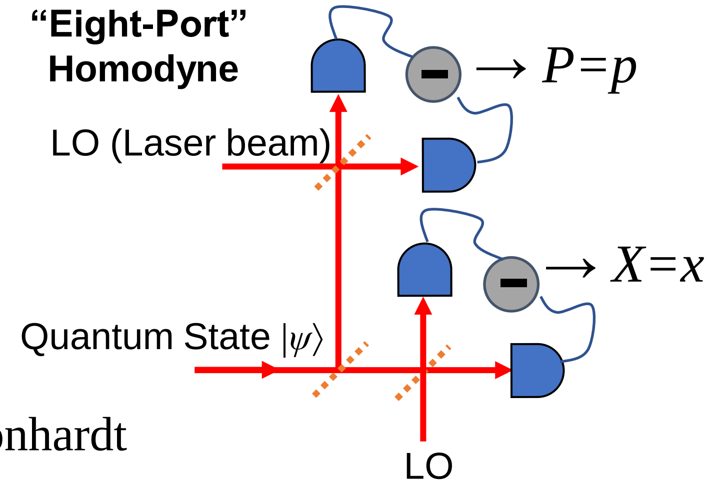
On the Simultaneous Measurement of a Pair of Conjugate Observables

By E. ARTHURS and J. L. KELLY, JR.

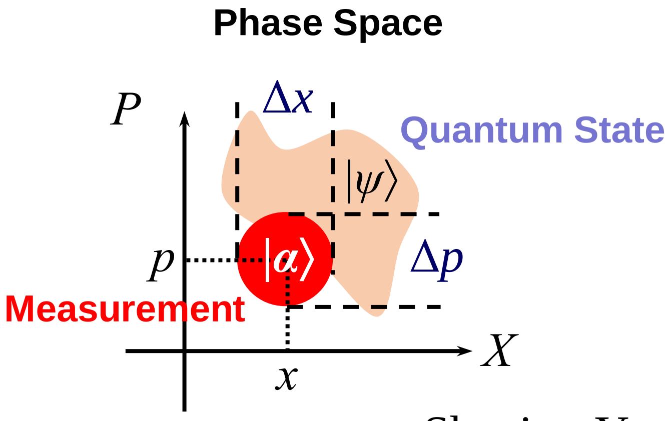
(Manuscript received December 16, 1964)




Same as measurement of Q-function of a quantum state: $Q\,(\alpha = x + iP) = |\langle\psi|\alpha\rangle|^2$




Leonhardt



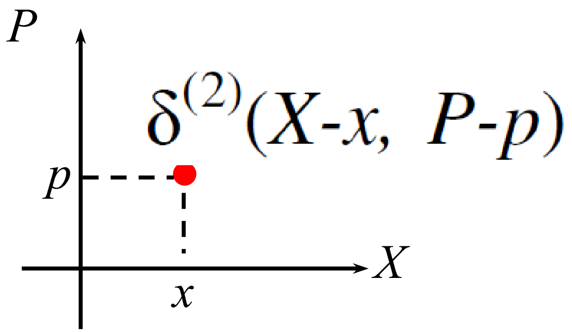

Shapiro, Yuen

Balanced measurement of X and P determines quantum state by its Q-function

# Direct Measurements of Quasi Probability distributions

- Classical measurement of a phase-space point is a Dirac delta
- How does one translate this to a quantum measurement?

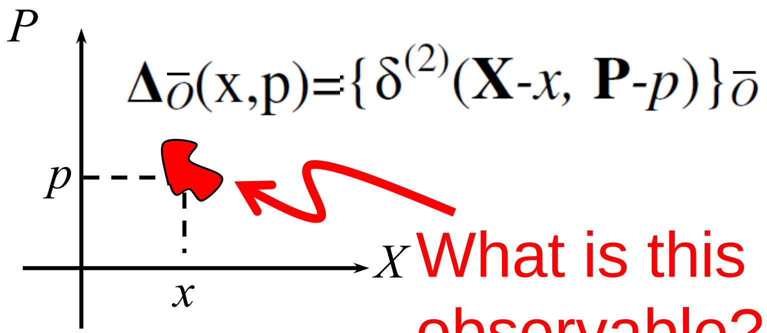
## Classical




Operator anti-ordering $\bar{O}$

$$Pq_O\,(x,p) = \mathrm{Tr}[\Delta_{\bar{O}}\,(x,p)\,\boldsymbol{\rho}]$$

## Quantum




G. S. Agarwal and E. Wolf, *Phys. Rev. D*,**2** (1970) pp. 2161–2186.

| Quasi-Prob, $Pq_O$ | Ordering *O* | Dirac Delta, $\Delta_{\bar{O}}(x,p)$ | Experiments & Theory |
|---|---|---|---|
| **Q** | Normal, N | $\Delta_{AN}(x,p) = \lvert\alpha\rangle\langle\alpha\rvert$ | Shapiro, Yuen, Leonhardt |
| **Wigner** | Symmetric, W | $\Delta_W(x,p) = \mathbf{\Pi}(x,p)$ the parity about point (x,p) | Banaszek, Haroche, Silberhorn, Smith |
| **P** | Anti-N,AN | $\Delta_N(x,p) \neq$ observable | |
| **Kirkwood-Dirac** | Anti-standard | $\Delta_{AS}(x,p) = \lvert p\rangle\langle p\vert\vert x\rangle\langle x\rvert$ | Lundeen, Boyd,… |

# X-P ordered Quasi-Prob Distributions

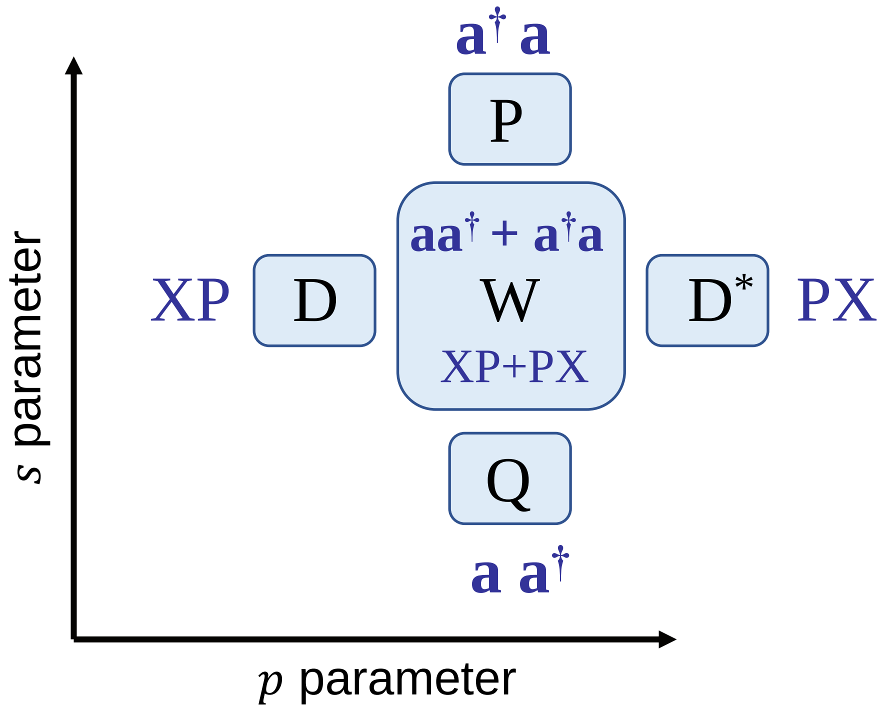
Standard S: **X** to the left of **P**

Anti-Standard AS: **P** to the left of **X**

$$\boldsymbol{\Delta}_{AS}(x,p) = \{\delta^{(2)}(\mathbf{X}-x,\ \mathbf{P}-p)\}_S$$
$$= \delta(\mathbf{P}-p)\delta(\mathbf{X}-x,)$$
$$= |p\rangle\langle p||x\rangle\langle x|$$

$$Pq_S\,(x,p) = \mathrm{Tr}[|p\rangle\langle p|x\rangle\langle x|\,\rho] = \langle p|x\rangle\langle x|\,\rho\,|p\rangle = D_\rho(x,p)$$

## Constellation of Quasi-Prob Distibutions



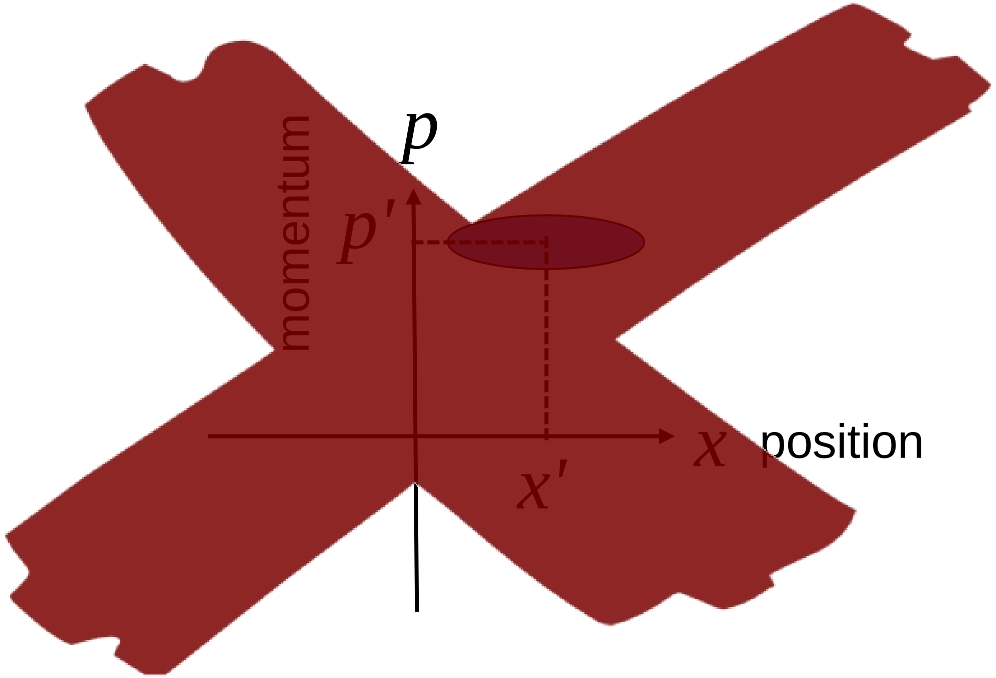

$q \equiv s + ip$ is a complex parameter that moves between all orderings continuously

G. S. Agarwal and E. Wolf, *Phys. Rev. D*,**2** (1970) pp. 2161–2186.

R.F. O'Connell and Lipo Wang, Physics Letters, 107A, p 9 (1985).

# Compatibility with the Heisenberg Uncertainty Relation

- Weak measurements reduce disturbance at the expense of certainty.
- Do they trade precision in $\Delta p$ for imprecision in $\Delta x$?
- What does the POVM $\Pi$ of the measurement look like in phase-space?




GS Thekkadath, F Hufnagel, JS Lundeen, New J Phys 20, 113034 (2018)

# The POVM of Direct Measurement

- What observable is a "direct measurement" measuring?
- On any given trial, it projects on superposition of a position eigenstate and a momentum eigenstate

Weak measurement of $|x'\rangle\langle x'|$

Strong measurement of $|p'\rangle\langle p'|$

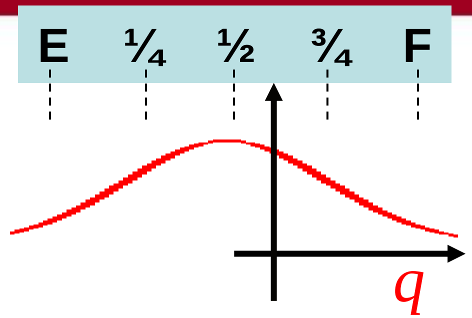
$$|\delta\rangle = |x\rangle + c_{q,x,p}|p\rangle$$

Measured pointer position, $q$

GS Thekkadath, F Hufnagel, JS Lundeen, New J Phys 20, 113034 (2018)

- Measurement is sharp in x and p! What about the Heisenberg Uncertainty Principle?

# Uncertainty and Weak Measurement




- The weak measurement POVM Π is a projector, $|\pi(q)\rangle$.
  - ➢ Superposition of sharp states in $x$ and $p$

Measured pointer position, $q$

Weak measurement of $|x'\rangle\langle x'|$

Strong measurement of $|p'\rangle\langle p'|$

$$|\pi(q)\rangle = |x'\rangle + \mathcal{P}(q)e^{ix'p'}\,|p'\rangle$$

Predictability $\mathcal{P}(q)$ is our ability to predict whether the particle had $x'$ given outcome $q$.

- In the double-slit experiment, predictability $\mathcal{P}$ and visibility $\mathcal{V}$ obey an uncertainty relation:

$$\mathcal{P}^2 + \mathcal{V}^2 \leq 1$$

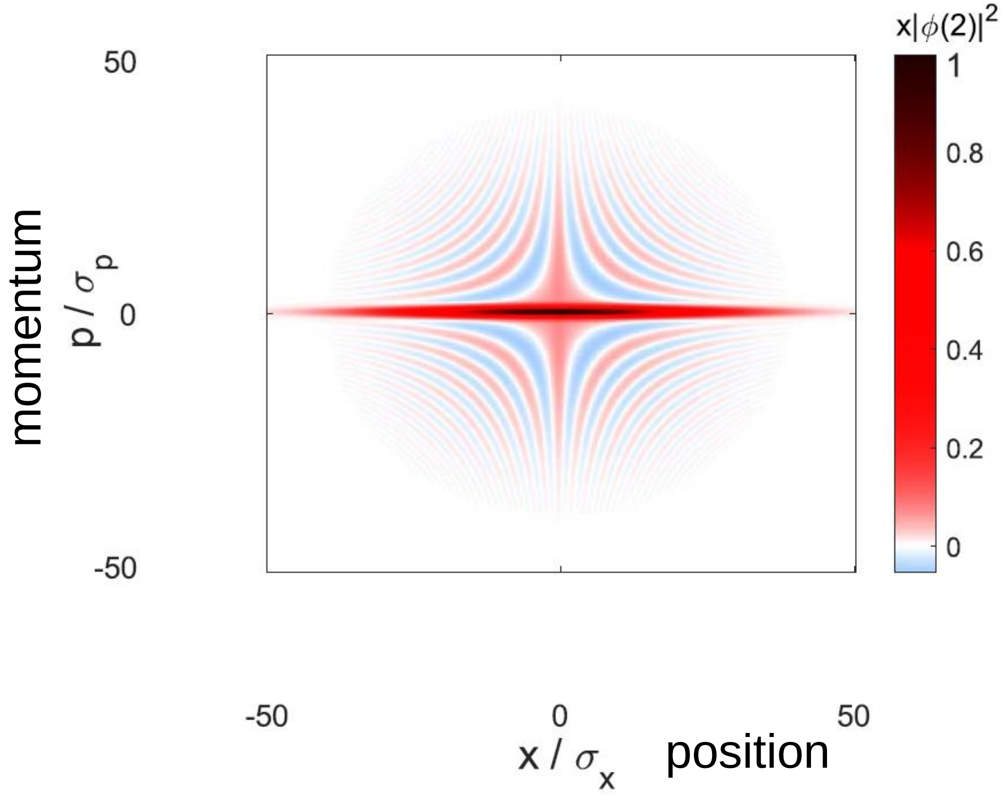
- Weak measurement trades away predictability to reduce disturbance to the quantum coherence (i.e. visibility)

GS Thekkadath, F Hufnagel, JS Lundeen, New J Phys 20, 113034 (2018)

# Compatibility with the Heisenberg Uncertainty Relation

- What does the POVM $\Pi$ of the measurement look like in phase-space?

Wigner Function of $\Pi$ for measurement at $x'=p'=0$

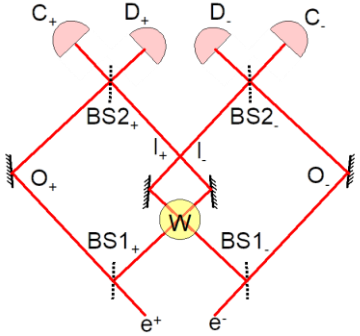



GS Thekkadath, F Hufnagel, JS Lundeen, New J Phys 20, 113034 (2018)

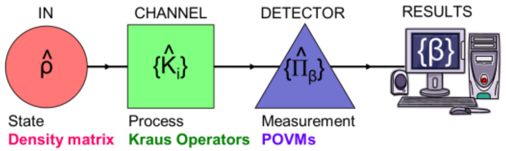
- A single direct measurement trial does contain sharp features in both $x$ and $p$ while keeping $\Delta x\,\Delta p \geq \hbar/2$

# Conclusions

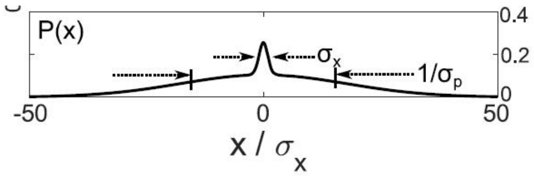
1. Measurements of complementary variables by weak measurement or optimal cloning will directly give the system state.


2. Only requires two bases and works with other photonic degrees of freedom (e.g. OAM, frequency, etc.) and systems (e.g. entangled, electrons, atoms), detectors POVMs, and processes.




3. Cloning and weak x-p measurements project onto superpositions of sharp states: $|\pi(q)\rangle = |x'\rangle + \mathcal{P}(q) e^{ix'p'} |p'\rangle$.



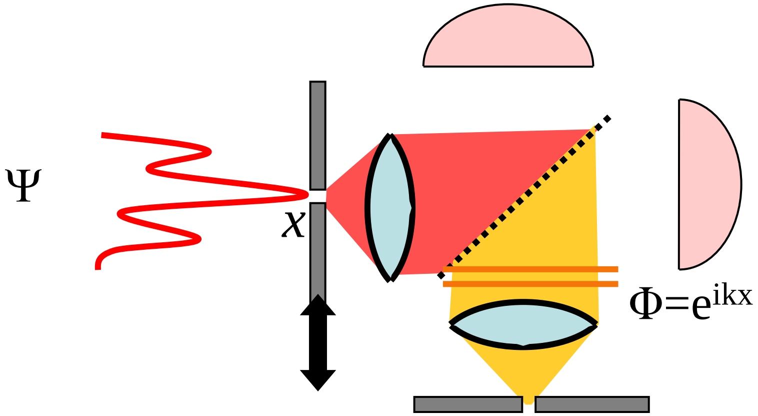

4. What other strategies could give ψ? What uncertainties can we trade-off in joint measurements of x and p? What information do we gain?

**Wavefunction:** Lundeen Nature, 474, 188 (2011)
**Mixed States:** Lundeen PRL 108, 070402 (2012), Bamber PRL 112, 070405 (2014),Thekkadath PRL 117, 120401 (2016), Thekkadath PRL 119, 050405 (2017)
**Heisenberg:** Thekkadath NJP 20, 113034 (2018)

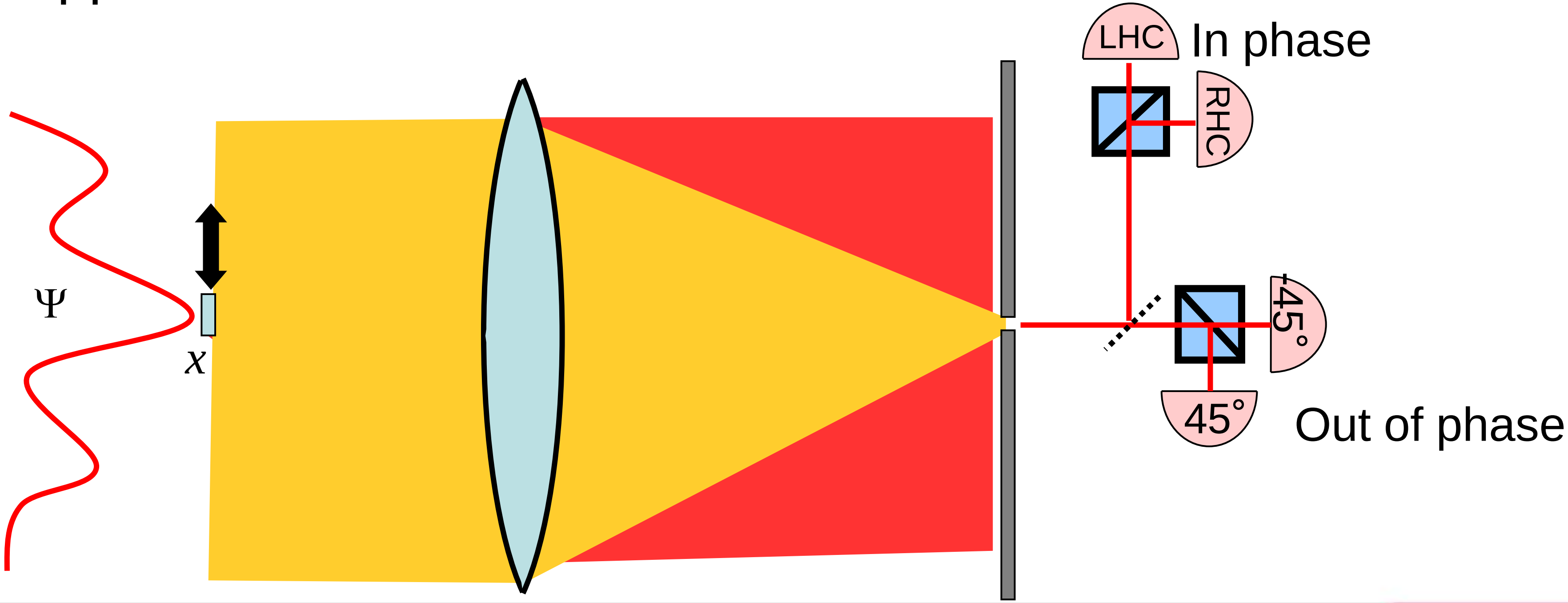
Canada Excellence Research Chairs / Chaires d'excellence en recherche du Canada
CANADA FIRST RESEARCH EXCELLENCE FUND / APOGÉE CANADA FONDS D'EXCELLENCE EN RECHERCHE
NSERC CRSNG
Canada Research Chairs / Chaires de recherche du Canada
Canada
INNOVATION.CA CANADA FOUNDATION FOR INNOVATION / FONDATION CANADIENNE POUR L'INNOVATION

# An Optical Explanation of the Measurement

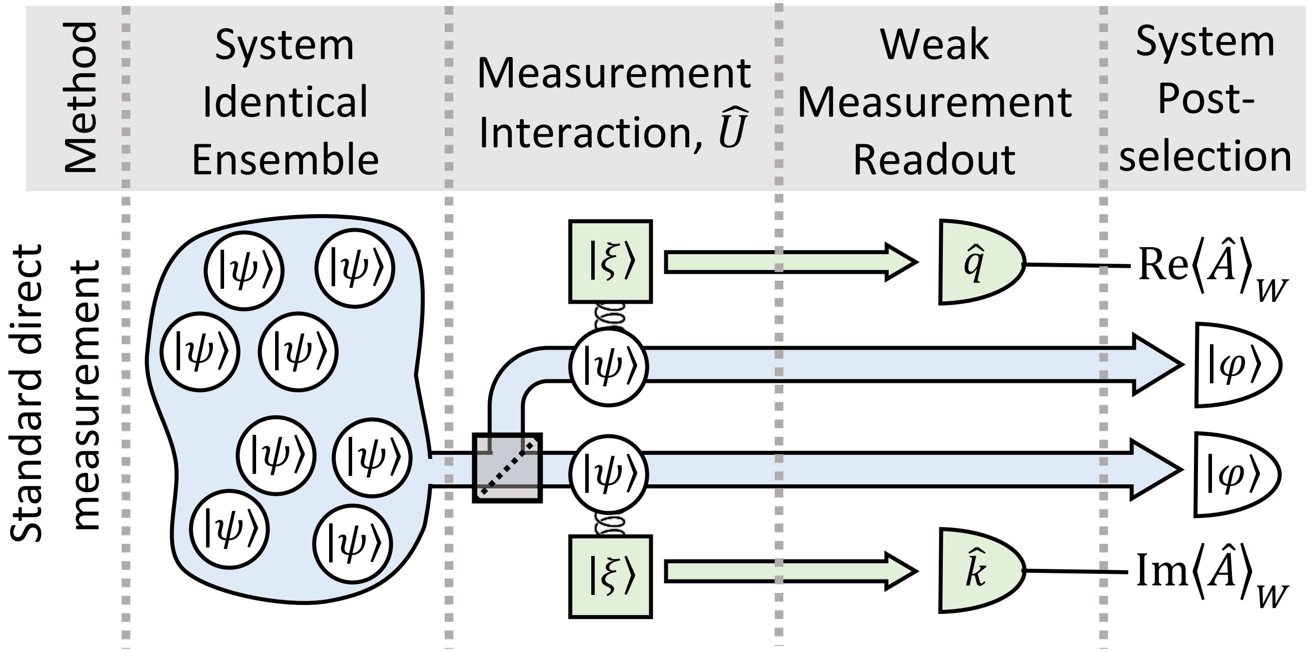
- From interference with a flat reference wavefront one can also determine the wavefunction.




- Look at the apparatus as a self-referenced interferometer



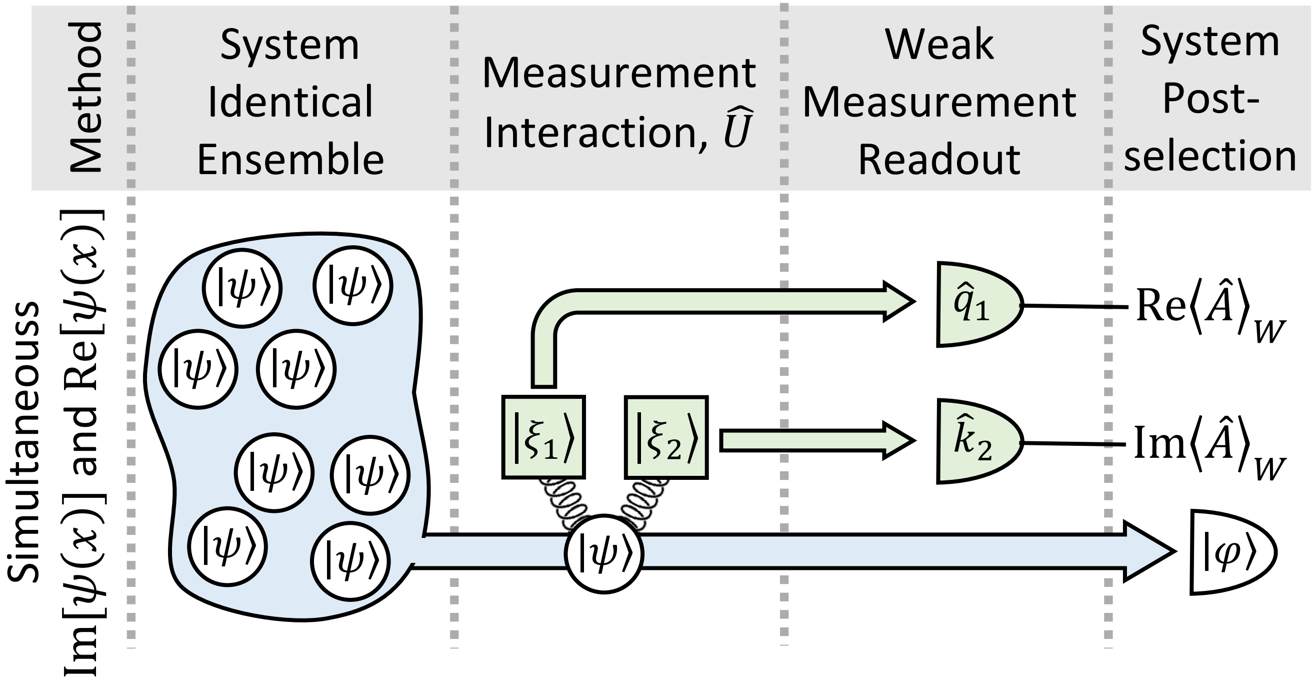
# Even more direct?




- We switch back and forth between measuring $\mathrm{Im}[\psi(x)]$ and $\mathrm{Re}[\psi(x)]$
- Can we measure both in each trial?

# Even more direct: Simultaneous readout

- Solution: Weak measurements do not disturb each other
  ∴ Weakly measure twice in row, once for $\mathrm{Im}[\psi(x)]$ and once $\mathrm{Re}[\psi(x)]$
- Need two readouts (i.e. 'pointers') or a two-dimensional readout.




PHYSICAL REVIEW A **100**, 032119 (2019)

**Experimental simultaneous readout of the real and imaginary parts of the weak value**

A. Hariri, D. Curic, L. Giner, and J. S. Lundeen*



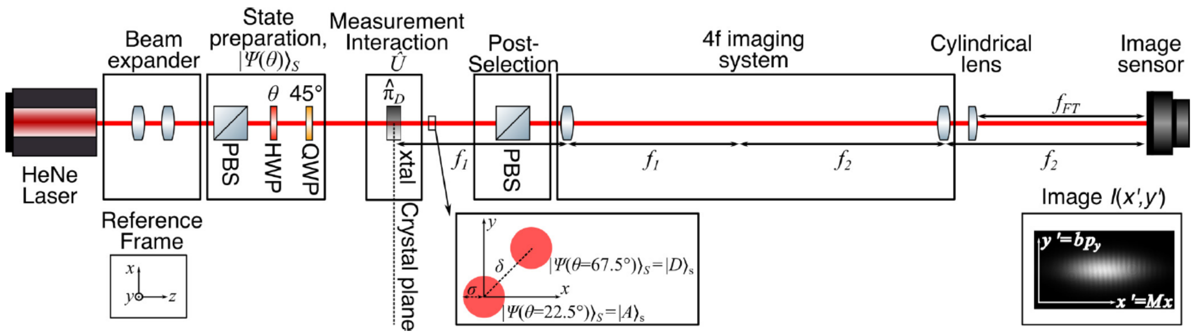
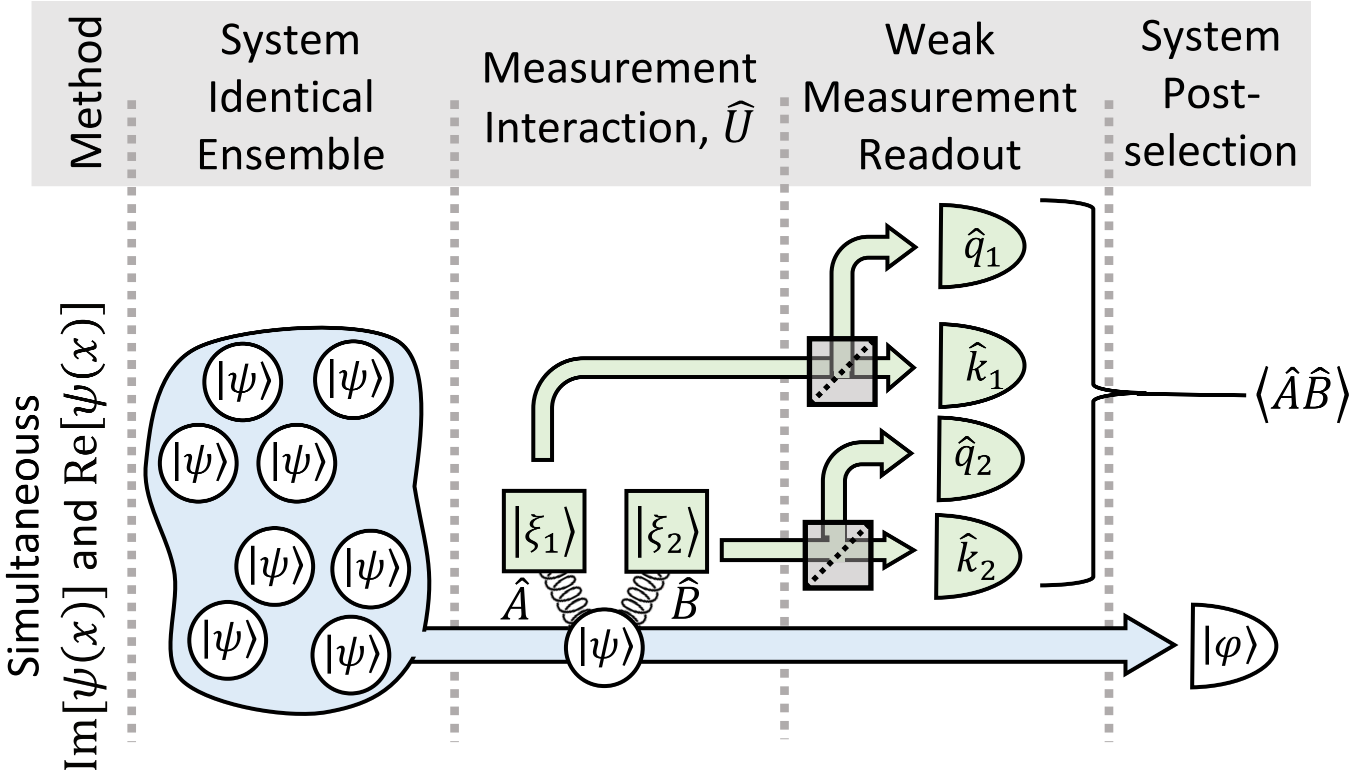



FIG. 2. Experimental setup for simultaneous readout of the real and imaginary parts of the weak value. This setup implements method C in

# Joint measurements of AB

- Measurements of the product of observables AB are used to measure the density matrix
- Normally, this would require a three-system interaction
- Instead, measure A and B separately and look at correlations in the readouts



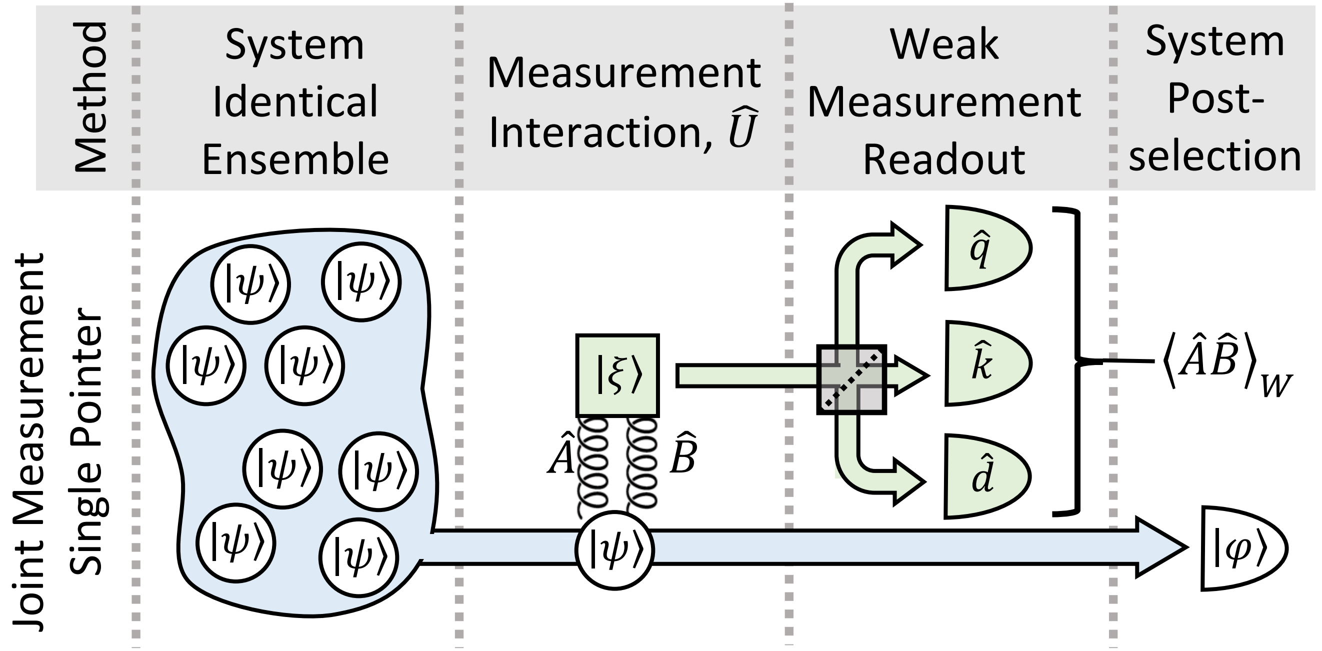

**A and B are observables on two particles:**
Theory: Lundeen, Resch, Physics Letters A, 334, 337-344 (2005)
Experiment: Lundeen, Steinberg, PRL 102, 020404 (2009)

**A and B are observables on the same particle, potentiallly non-commuting:**
Theory: Lundeen, Bamber, PRL 108, 070402 (2012)
Experiment: GS Thekkadath,…, JS Lundeen, PRL 117, 120401 (2016)

- We can measure the average value of two non-commuting observables $\langle \hat{A}\hat{B} \rangle$

# Better Joint Measurements of AB

- Needed one readout system ('pointer') per observable
- Here, only need a single readout system for multiple projectors, $|a\rangle\langle a|, |b\rangle\langle b|$
- But, need to measure more readout system observables




Quantum the open journal for quantum science PAPERS PERSPECTIV

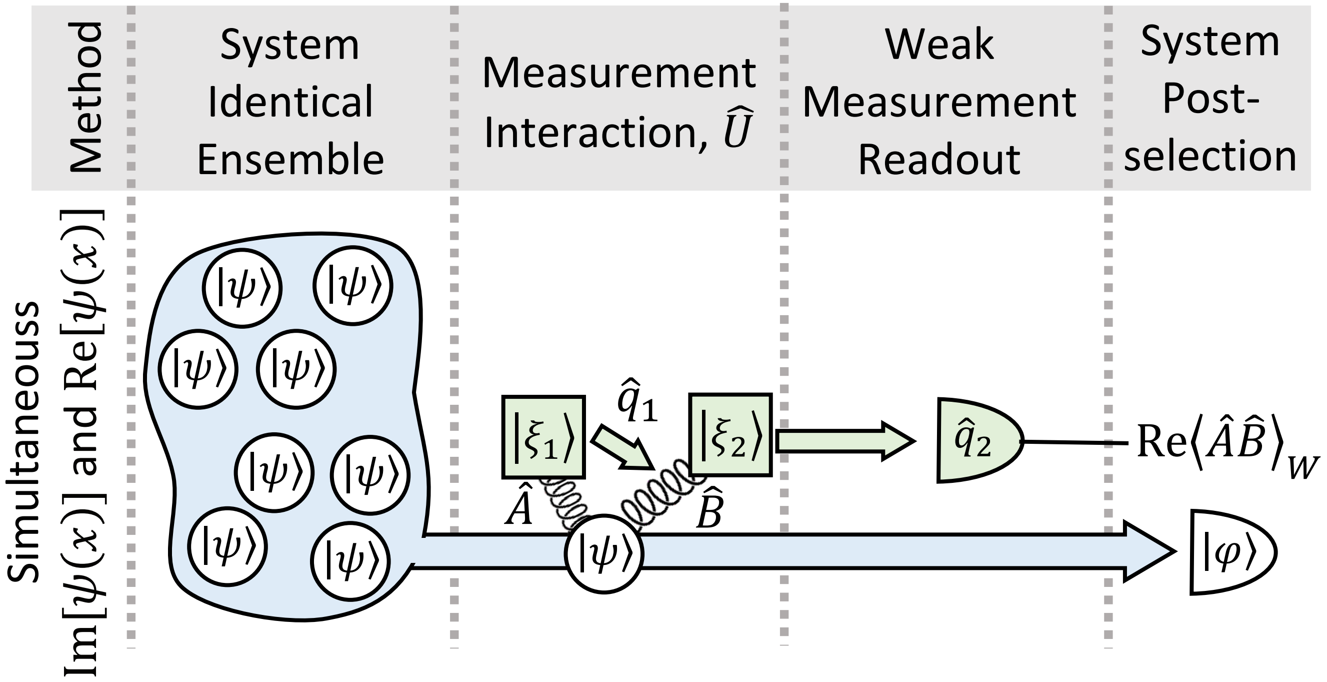
Quantum 5, 599 (2021).

Theory and experiment for resource-efficient joint weak-measurement

Aldo C. Martinez-Becerril[1], Gabriel Bussières[1], Davor Curic[2], Lambert Giner[1,3], Raphael A. Abrahao[1,4], and Jeff S. Lundeen[1,4]

# Even more direct: Simultaneous readout

- Want simple (i.e. 'direct') readout of a single pointer system
- Solution: Measure B scaled by the outcome of the measurement of A
- Condition the strength of the measurement of B on the outcome of A.




Theory: Lundeen, Bamber, PRL 108, 070402 (2012)
Experiment in progress by Thomas Bailey