The Kirkwood-Dirac Distribution and Quantum Metrology

University of Lille – 8 Nov 2023

David Arvidsson-Shukur
Hitachi Cambridge Laboratory

Thanking Organisers

Université de Lille

Laboratoire Paul Painlevé

PhLAM
Physique des Lasers Atomes et Molécules

CEMPI CENTRE EUROPÉEN POUR LES MATHÉMATIQUES, LA PHYSIQUE ET LEURS INTERACTIONS

Inria

cnrs

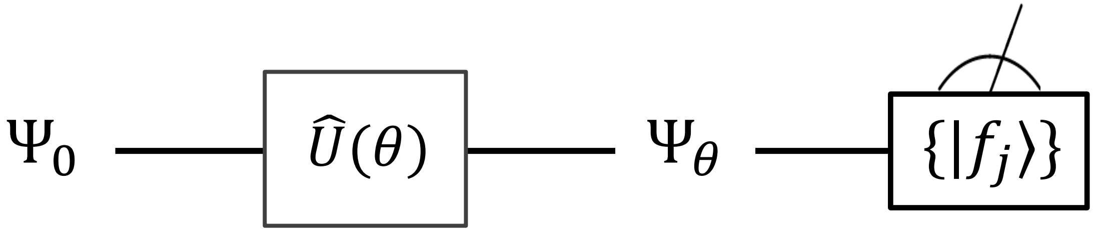
Thanking Organisers

Metrology

Metrology

The scientific study of Measurements and Estimation.

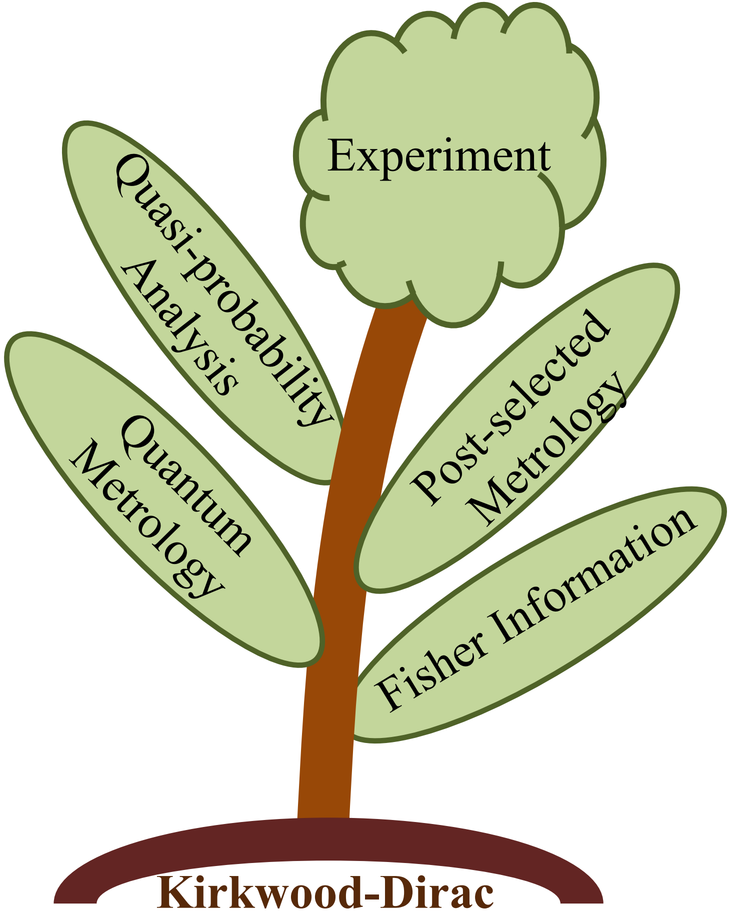
Take-home message



Ability to guess θ depends on a Kirkwood-Dirac distribution.

Talk outline

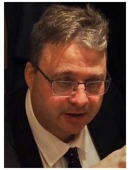
HITACHI
Inspire the Next



N. Yunger-Halpern

S. Lloyd

C. Barnes



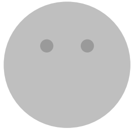
A. Lasek

H. Lepage

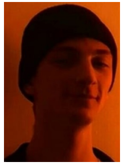
F. Salvati

J. Jenne

A. Pang



J. Chevalier Drori



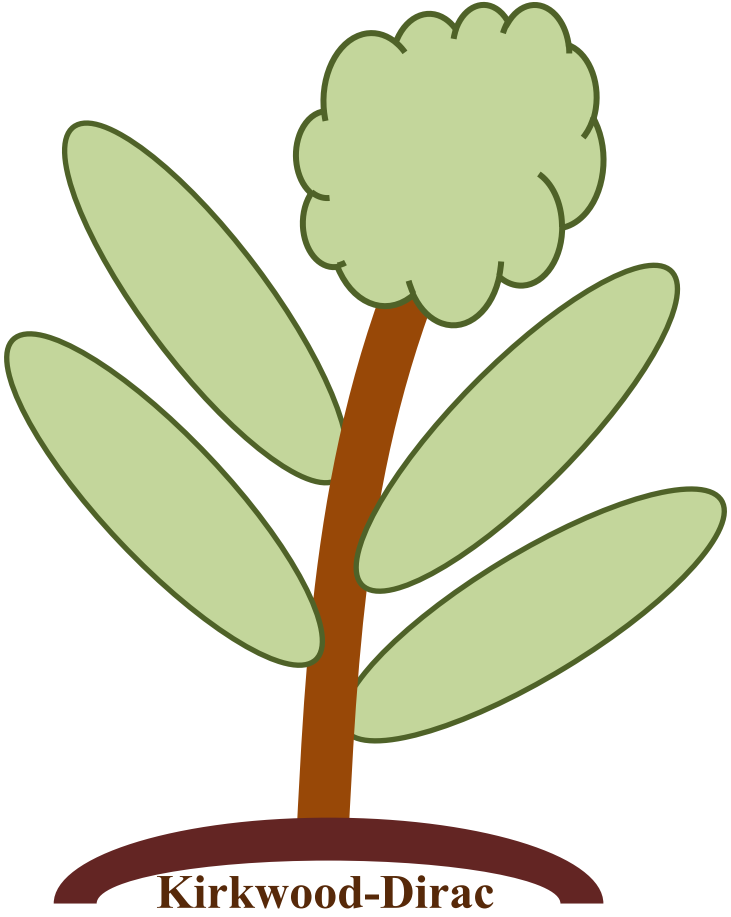
Æ. Steinberg

N. Lupu-Gladstein

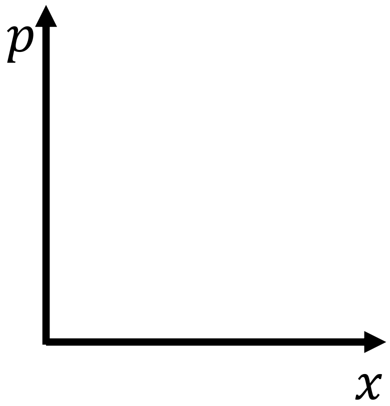
Y. Batuhan

A. Brodutch

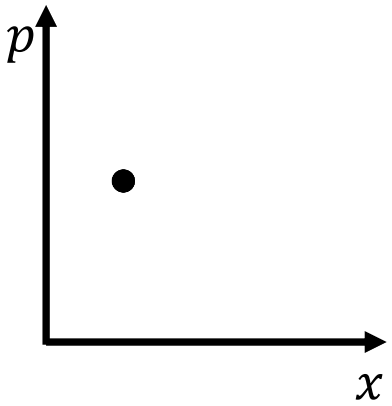
Talk outline



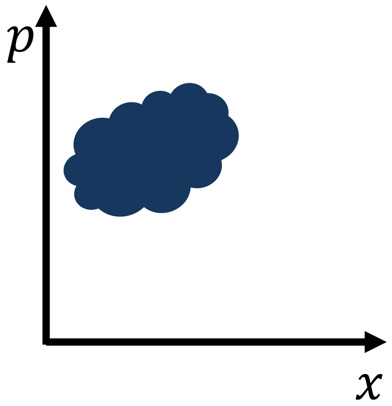
Kirkwood-Dirac Distribution



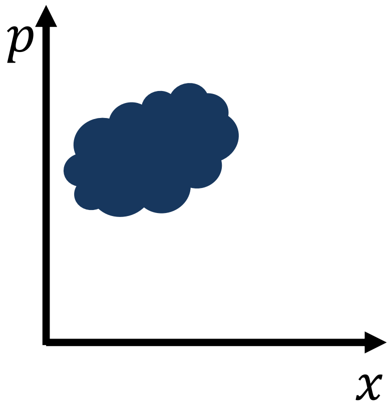
Kirkwood-Dirac Distribution



Kirkwood-Dirac Distribution

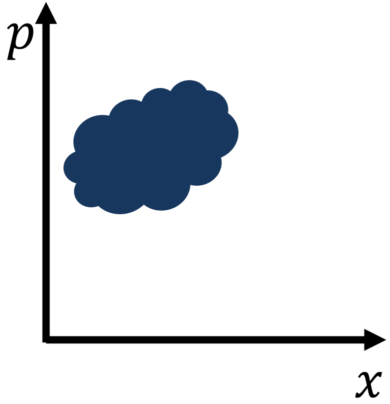


Kirkwood-Dirac Distribution



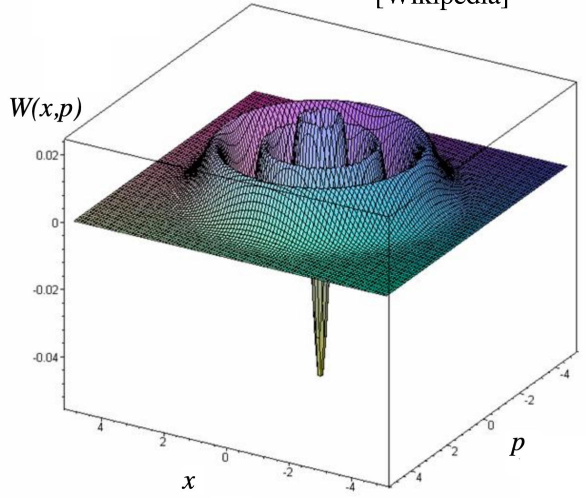
$$[\hat{x}, \hat{p}] \neq 0$$

Kirkwood-Dirac Distribution



$$[\hat{x}, \hat{p}] \neq 0$$

[Wikipedia]



Kirkwood-Dirac Distribution

Kirkwood-Dirac Distribution

Properties

Observables

$$\hat{A} = \sum_i a_i |a_i\rangle\langle a_i|$$

$$\hat{F} = \sum_j f_j |f_j\rangle\langle f_j|$$

Open Question

Kirkwood-Dirac Distribution

Kirkwood-Dirac Distribution

$$Q_{i,j}(\hat{\rho}) = \mathrm{Tr}\left(|f_j\rangle\langle f_j|a_i\rangle\langle a_i|\hat{\rho}\right)$$

Properties

Observables

$$\hat{A} = \sum_i a_i |a_i\rangle\langle a_i|$$

$$\hat{F} = \sum_j f_j |f_j\rangle\langle f_j|$$

Open Question

Kirkwood-Dirac Distribution

Kirkwood-Dirac Distribution

$$Q_{i,j}(\hat{\rho}) = \mathrm{Tr}\left(|f_j\rangle\langle f_j|a_i\rangle\langle a_i|\hat{\rho}\right)$$

$$Q_{i,j}(\hat{\rho}) \in \mathbb{C}$$

Properties

Observables

$$\hat{A} = \sum_i a_i|a_i\rangle\langle a_i|$$

$$\hat{F} = \sum_j f_j|f_j\rangle\langle f_j|$$

Open Question

Kirkwood-Dirac Distribution

Kirkwood-Dirac Distribution

$$Q_{i,j}(\hat{\rho}) = \mathrm{Tr}\left(|f_j\rangle\langle f_j|a_i\rangle\langle a_i|\hat{\rho}\right)$$

$$Q_{i,j}(\hat{\rho}) \in \mathbb{C}$$

Properties

$$\sum_{i,j} Q_{i,j}(\hat{\rho}) = 1$$

$$\sum_{j} Q_{i,j}(\hat{\rho}) = p_i$$

$$\sum_{i} Q_{i,j}(\hat{\rho}) = p_j$$

Observables

$$\hat{A} = \sum_i a_i |a_i\rangle\langle a_i|$$

$$\hat{F} = \sum_j f_j |f_j\rangle\langle f_j|$$

Open Question

Kirkwood-Dirac Distribution

Kirkwood-Dirac Distribution

$$Q_{i,j}(\hat{\rho}) = \mathrm{Tr}\big(|f_j\rangle\langle f_j|a_i\rangle\langle a_i|\hat{\rho}\big)$$

$$Q_{i,j}(\hat{\rho}) \in \mathbb{C}$$

Properties

$$\sum_{i,j} Q_{i,j}(\hat{\rho}) = 1$$

$$\sum_{j} Q_{i,j}(\hat{\rho}) = p_i$$

$$\sum_{i} Q_{i,j}(\hat{\rho}) = p_j$$

Observables

$$\hat{A} = \textstyle\sum_i a_i|a_i\rangle\langle a_i|$$

$$\hat{F} = \textstyle\sum_j f_j|f_j\rangle\langle f_j|$$

Open Question

Other (less elegant) functions satisfy the conditions.
What extra conditions would uniquely determine Q?

Kirkwood-Dirac Distribution

Kirkwood-Dirac Distribution

Observables

$$\hat{A} = \sum_i a_i |a_i\rangle\langle a_i|$$

$$\hat{B} = \sum_j b_j |b_j\rangle\langle b_j|$$

$$\hat{F} = \sum_k f_k |f_k\rangle\langle f_k|$$

Kirkwood-Dirac Distribution

HITACHI
Inspire the Next

Kirkwood-Dirac Distribution

$$Q_{i,j,k}(\hat{\rho}) = \mathrm{Tr}\big(|f_k\rangle\langle f_k|b_j\rangle\langle b_j|a_i\rangle\langle a_i|\hat{\rho}\big)$$

$$\sum_{i,j,k} Q_{i,j,k}(\hat{\rho}) = 1$$

Observables

$$\hat{A} = \sum_i a_i |a_i\rangle\langle a_i|$$

$$\hat{B} = \sum_j b_j |b_j\rangle\langle b_j|$$

$$\hat{F} = \sum_k f_k |f_k\rangle\langle f_k|$$

Kirkwood-Dirac Distribution

Kirkwood-Dirac Distribution

$$Q_{i,j,k}(\hat{\rho}) = \mathrm{Tr}\big(|f_k\rangle\langle f_k|b_j\rangle\langle b_j|a_i\rangle\langle a_i|\hat{\rho}\big)$$

$$\sum_{i,j,k} Q_{i,j,k}(\hat{\rho}) = 1$$

"Negative energies and probabilities should not be considered as nonsense. They are well-defined concepts mathematically, like a negative of money."

Observables

$$\hat{A} = \sum_i a_i |a_i\rangle\langle a_i|$$

$$\hat{B} = \sum_j b_j |b_j\rangle\langle b_j|$$

$$\hat{F} = \sum_k f_k |f_k\rangle\langle f_k|$$

Kirkwood-Dirac Distribution

Kirkwood-Dirac Distribution

$$Q_{i,j,k}(\hat{\rho}) = \mathrm{Tr}\big(|f_k\rangle\langle f_k|b_j\rangle\langle b_j|a_i\rangle\langle a_i|\hat{\rho}\big)$$

$$\sum_{i,j,k} Q_{i,j,k}(\hat{\rho}) = 1$$

When is $Q(\hat{\rho})$ non-classical?

"Negative energies and probabilities should not be considered as nonsense. They are well-defined concepts mathematically, like a negative of money."

Observables

$$\hat{A} = \sum_i a_i |a_i\rangle\langle a_i|$$

$$\hat{B} = \sum_j b_j |b_j\rangle\langle b_j|$$

$$\hat{F} = \sum_k f_k |f_k\rangle\langle f_k|$$

Kirkwood-Dirac Distribution

Kirkwood-Dirac Distribution

$$Q_{i,j,k}(\hat{\rho}) = \mathrm{Tr}\big(|f_k\rangle\langle f_k|b_j\rangle\langle b_j|a_i\rangle\langle a_i|\hat{\rho}\big)$$

$$\sum_{i,j,k} Q_{i,j,k}(\hat{\rho}) = 1$$

"Negative energies and probabilities should not be considered as nonsense. They are well-defined concepts mathematically, like a negative of money."

Observables

$$\hat{A} = \sum_i a_i |a_i\rangle\langle a_i|$$

$$\hat{B} = \sum_j b_j |b_j\rangle\langle b_j|$$

$$\hat{F} = \sum_k f_k |f_k\rangle\langle f_k|$$

When is $Q(\hat{\rho})$ non-classical?

IOP Publishing Journal of Physics A: Mathematical and Theoretical
J. Phys. A: Math. Theor. **54** (2021) 284001 (20pp) https://doi.org/10.1088/1751-8121/ac0289

Conditions tighter than noncommutation needed for nonclassicality

David R M Arvidsson-Shukur[1,2,3,*],
Jacob Chevalier Drori[4] and
Nicole Yunger Halpern[3,5,6,7,8,9]

[or Stephan's superior article]

Kirkwood-Dirac Distribution

Kirkwood-Dirac Distribution

$$Q_{i,j}(\hat{\rho}) = \mathrm{Tr}\left(|f_j\rangle\langle f_j|a_i\rangle\langle a_i|\hat{\rho}\right)$$

Formula of interest

Modus Operandi

Kirkwood-Dirac Distribution

Kirkwood-Dirac Distribution

$$Q_{i,j}(\hat{\rho}) = \mathrm{Tr}\left(|f_j\rangle\langle f_j|a_i\rangle\langle a_i|\hat{\rho}\right)$$

Formula of interest

$$f\left(\hat{A}, \hat{B}, \ldots, \hat{F}; \hat{\rho}\right) \rightarrow f[Q(\hat{\rho})]$$

Modus Operandi

Kirkwood-Dirac Distribution

HITACHI
Inspire the Next

Kirkwood-Dirac Distribution

$$Q_{i,j}(\hat{\rho}) = \mathrm{Tr}\left(|f_j\rangle\langle f_j|a_i\rangle\langle a_i|\hat{\rho}\right)$$

Formula of interest

$$f\left(\hat{A}, \hat{B}, \ldots, \hat{F}; \hat{\rho}\right) \rightarrow f[Q(\hat{\rho})]$$

Modus Operandi

$$f^{\mathrm{p}} = \mathrm{opt}_{Q(\hat{\rho})\in[0,1]}\{f[Q(\hat{\rho})]\} \overset{?}{=} f^{\mathrm{np}} = \mathrm{opt}_{Q(\hat{\rho})}\{f[Q(\hat{\rho})]\}$$

Kirkwood-Dirac Distribution

Kirkwood-Dirac Distribution

$$Q_{i,j}(\hat{\rho}) = \mathrm{Tr}\big(|f_j\rangle\langle f_j|a_i\rangle\langle a_i|\hat{\rho}\big)$$

Formula of interest

$$f\big(\hat{A}, \hat{B}, \dots, \hat{F}; \hat{\rho}\big) \to f[Q(\hat{\rho})]$$

Modus Operandi

optimal experiment from: $Q^{\star}(\hat{\rho}) = \arg \mathrm{opt}_{Q(\hat{\rho})}\{f[Q(\hat{\rho})]\}$

Kirkwood-Dirac Distribution

Kirkwood-Dirac Distribution

$$Q_{i,j}(\hat{\rho}) = \mathrm{Tr}\left(|f_j\rangle\langle f_j|a_i\rangle\langle a_i|\hat{\rho}\right)$$

Total Non-positivity

Classical Distribution

Quantum Distribution

Kirkwood-Dirac Distribution

Kirkwood-Dirac Distribution

$$Q_{i,j}(\hat{\rho}) = \mathrm{Tr}\big(|f_j\rangle\langle f_j|a_i\rangle\langle a_i|\hat{\rho}\big)$$

Total Non-positivity

$$\mathcal{N}[Q(\hat{\rho})] = \sum_{i,j}\left|Q_{i,j}(\hat{\rho})\right|$$

Classical Distribution

Quantum Distribution

Kirkwood-Dirac Distribution

Kirkwood-Dirac Distribution

$$Q_{i,j}(\hat{\rho}) = \mathrm{Tr}\big(|f_j\rangle\langle f_j|a_i\rangle\langle a_i|\hat{\rho}\big)$$

Total Non-positivity

$$\mathcal{N}[Q(\hat{\rho})] = \sum_{i,j} \left|Q_{i,j}(\hat{\rho})\right|$$

Classical Distribution

$$\mathcal{N}[Q(\hat{\rho})] = 1$$

Quantum Distribution

$$\mathcal{N}[Q(\hat{\rho})] \geq 1$$

Intermezzo: Kirkwood-Dirac Distribution

Scrambling/out-of-time-ordered correlators:

Out-of-Time-Ordered-Correlator Quasiprobabilities Robustly Witness Scrambling

José Raúl González Alonso,[1,*] Nicole Yunger Halpern,[2] and Justin Dressel[1,3]

Entropic uncertainty relations for quantum information scrambling

Nicole Yunger Halpern [1,2,5], Anthony Bartolotta[3] & Jason Pollack[4]

The quasiprobability behind the out-of-time-ordered correlator

Nicole Yunger Halpern,[1] Brian Swingle,[2,3,4] and Justin Dressel[5,6]

Optimizing measurement strengths for qubit quasiprobabilities behind out-of-time-ordered correlators

Razieh Mohseninia,[1,2] José Raúl González Alonso,[3] and Justin Dressel[1,3]

Thermodynamics:

A quasiprobability distribution for heat fluctuations in the quantum regime

Amikam Levy[1,2,3] and Matteo Lostaglio[4,5,*]

Jarzynski-like equality for the out-of-time-ordered correlator

Nicole Yunger Halpern[*]

Metrology:

Quantum Advantage in Postselected Metrology

David R. M. Arvidsson-Shukur,[1,2,3] Nicole Yunger Halpern,[4,5,3] Hugo V. Lepage,[6] Aleksander A. Lasek,[6] Crispin H. W. Barnes,[6] and Seth Lloyd[2,3]

Foundations

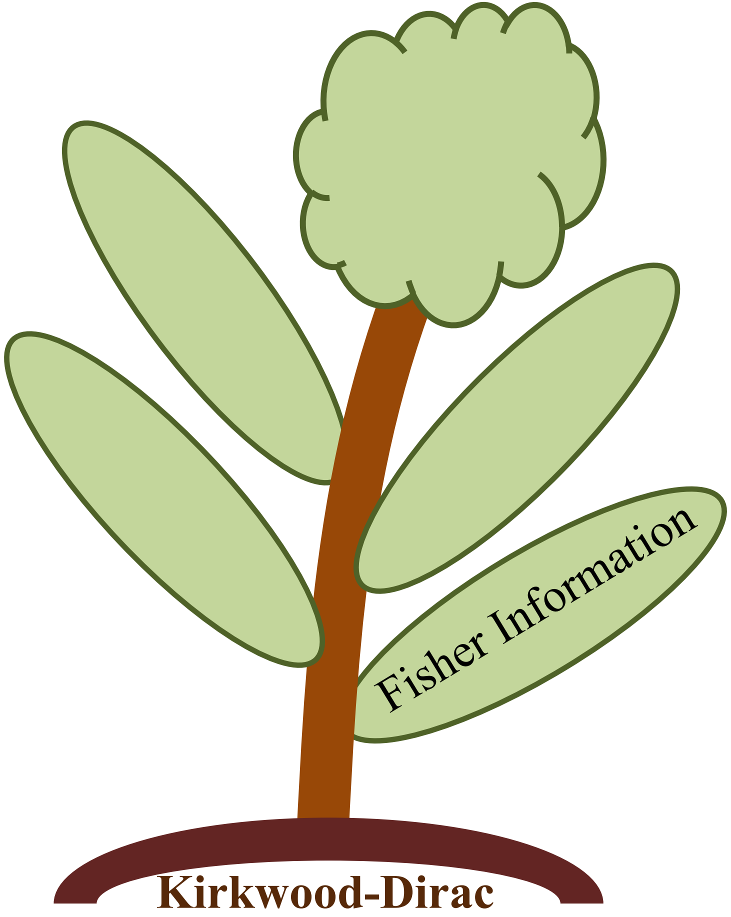
Linear positivity and virtual probability

James B. Hartle[*]

Complex joint probabilities as expressions of reversible transformations in quantum mechanics

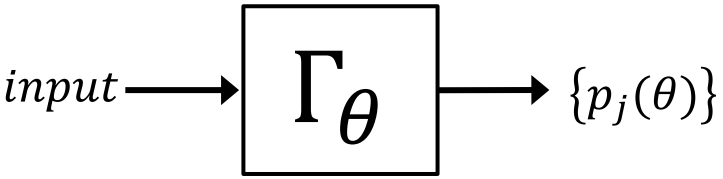
Holger F Hofmann

Talk outline



Fisher information

Any Experiment



Experiment Outcomes

Information Analysis

Fisher information

HITACHI Inspire the Next

Any Experiment

$$input \rightarrow \boxed{\Gamma_\theta} \rightarrow \{p_j(\theta)\}$$

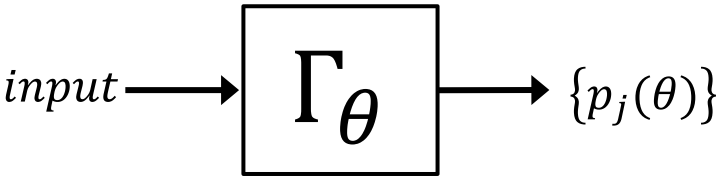
Experiment Outcomes

$$\left.\begin{matrix} p_1(\theta) \\ p_2(\theta) \\ p_3(\theta) \\ \cdots \\ p_M(\theta) \end{matrix}\right. \longrightarrow \theta_e$$

Information Analysis

Fisher information

Any Experiment



Experiment Outcomes

$p_1(\theta)$
$p_2(\theta)$
$p_3(\theta)$ → θ_e
$\cdots$
$p_M(\theta)$

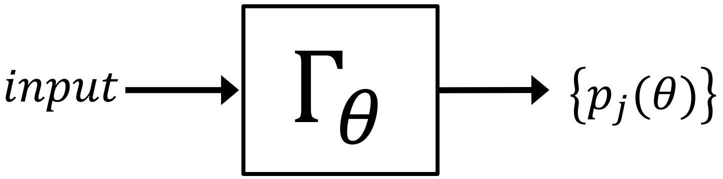
$$\mathrm{Var}(\theta_e) = ?$$

Information Analysis

Fisher information

HITACHI
Inspire the Next

Any Experiment



Experiment Outcomes

$p_1(\theta)$
$p_2(\theta)$
$p_3(\theta)$ → θ_e
…
$p_M(\theta)$

$$\mathrm{Var}(\theta_\mathrm{e}) = ?$$

Information Analysis

No Bias: $\mathrm{E}[\theta_\mathrm{e} - \theta | \theta] = 0$

$$\sum_j p_j(\theta)\,(\theta_\mathrm{e} - \theta) = 0$$

Fisher information

HITACHI
Inspire the Next

Any Experiment

$$\textit{input} \longrightarrow \boxed{\Gamma_\theta} \longrightarrow \{p_j(\theta)\}$$

Experiment Outcomes

$$\left.\begin{array}{l} p_1(\theta) \\ p_2(\theta) \\ p_3(\theta) \\ \cdots \\ p_M(\theta) \end{array}\right. \longrightarrow \theta_\mathrm{e}$$

$$\mathrm{Var}(\theta_\mathrm{e}) = ?$$

Information Analysis

No Bias: $\mathrm{E}[\theta_\mathrm{e} - \theta | \theta] = 0$

$$\partial_\theta \sum_j p_j(\theta)\,(\theta_\mathrm{e} - \theta) = 0$$

Fisher information

Any Experiment

$$\textit{input} \longrightarrow \boxed{\Gamma_\theta} \longrightarrow \{p_j(\theta)\}$$

Experiment Outcomes

$$\begin{matrix} p_1(\theta) \\ p_2(\theta) \\ p_3(\theta) \\ \cdots \\ p_M(\theta) \end{matrix} \longrightarrow \theta_{\mathrm{e}}$$

$$\mathrm{Var}(\theta_{\mathrm{e}}) = ?$$

Information Analysis

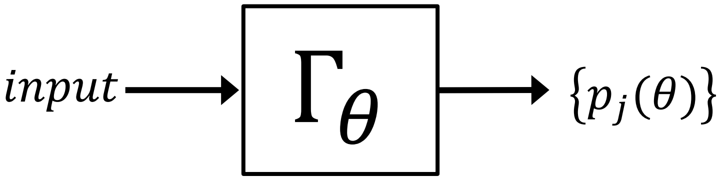
No Bias: $\mathrm{E}[\theta_{\mathrm{e}} - \theta|\theta] = 0$

$$\sum_j \left[\partial_\theta p_j(\theta) \right] (\theta_{\mathrm{e}} - \theta) - 1 = 0$$

Fisher information

HITACHI
Inspire the Next

Any Experiment



Experiment Outcomes

$$\begin{array}{l} p_1(\theta) \\ p_2(\theta) \\ p_3(\theta) \\ \cdots \\ p_M(\theta) \end{array} \longrightarrow \theta_\mathrm{e}$$

$$\mathrm{Var}(\theta_\mathrm{e}) = ?$$

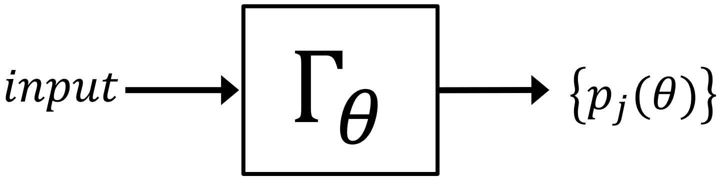
Information Analysis

No Bias: $\mathrm{E}[\theta_\mathrm{e} - \theta | \theta] = 0$

$$\left[\sum_j p_j(\theta)\, \partial_\theta \ln\left[p_j(\theta)\right] (\theta_\mathrm{e} - \theta)\right]^2 = 1$$

Fisher information

Any Experiment



Experiment Outcomes

$p_1(\theta)$
$p_2(\theta)$
$p_3(\theta)$ → θ_e
$\dots$
$p_M(\theta)$

$\mathrm{Var}(\theta_e) = ?$

Information Analysis

No Bias: $\mathrm{E}[\theta_e - \theta | \theta] = 0$

$$\sum_j p_j(\theta)\{\partial_\theta \ln[p_j(\theta)]\}^2$$
$$\times \sum_j p_j(\theta)[\theta_e - \theta]^2 \geq 1$$

Fisher information

Any Experiment

$$input \longrightarrow \boxed{\Gamma_\theta} \longrightarrow \{p_j(\theta)\}$$

Experiment Outcomes

$$\begin{matrix} p_1(\theta) \\ p_2(\theta) \\ p_3(\theta) \\ \cdots \\ p_M(\theta) \end{matrix} \longrightarrow \theta_e$$

$$\mathrm{Var}(\theta_e) = ?$$

Information Analysis

No Bias: $\mathrm{E}[\theta_e - \theta|\theta] = 0$

$$\sum_j p_j(\theta)\{\partial_\theta \ln[p_j(\theta)]\}^2 \times \sum_j p_j(\theta)[\theta_e - \theta]^2 \geq 1$$

$$\mathrm{Var}(\theta_e) \geq 1/\mathcal{I}(\theta)$$

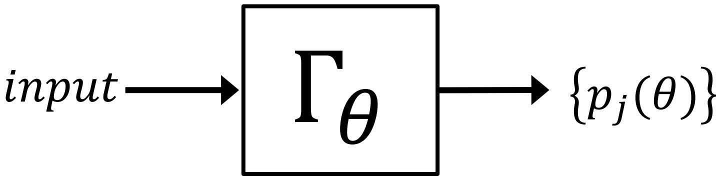
[Fisher information]

$$\mathcal{I}(\theta) = \sum_j p_j(\theta)\{\partial_\theta \ln[p_j(\theta)]\}^2$$

Fisher information

HITACHI
Inspire the Next

Any Experiment



Experiment Outcomes

$p_1(\theta)$
$p_2(\theta)$
$p_3(\theta)$ $\longrightarrow$ θ_e
…
$p_M(\theta)$

[Cramèr-Rao Bound]

$$\mathrm{Var}(\theta_e) \geq \frac{1}{N\mathcal{I}(\theta)}$$

Information Analysis

No Bias: $\mathrm{E}[\theta_e - \theta|\theta] = 0$

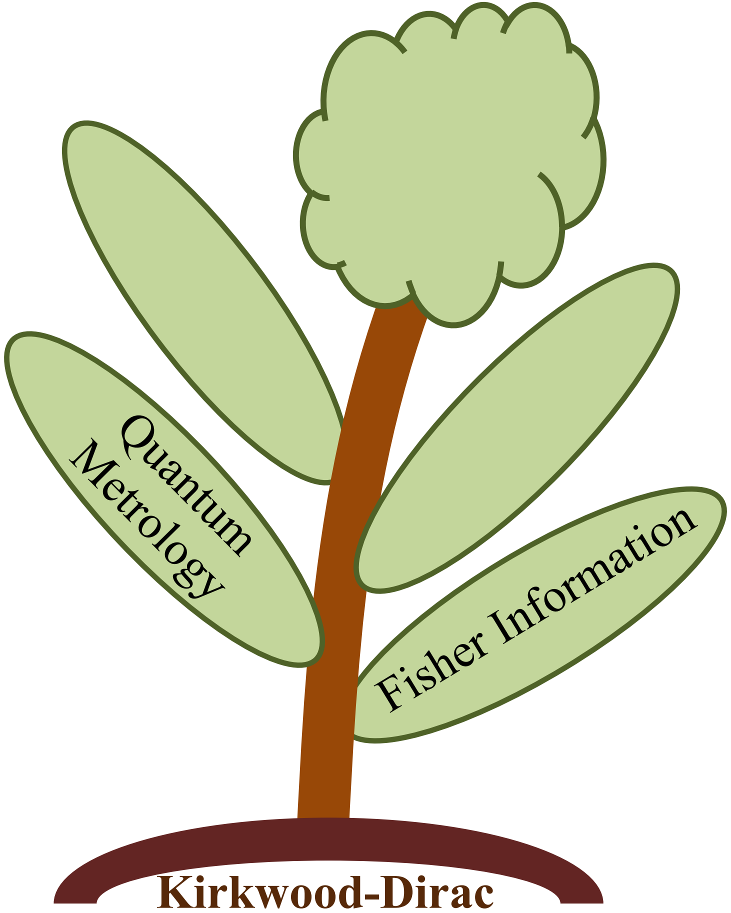
$$\sum_j p_j(\theta)\{\partial_\theta \ln[p_j(\theta)]\}^2 \times \sum_j p_j(\theta)[\theta_e - \theta]^2 \geq 1$$

$$\mathrm{Var}(\theta_e) \geq 1/\mathcal{I}(\theta)$$

[Fisher information]

$$\mathcal{I}(\theta) = \sum_j p_j(\theta)\{\partial_\theta \ln[p_j(\theta)]\}^2$$

Talk outline



Quantum metrology

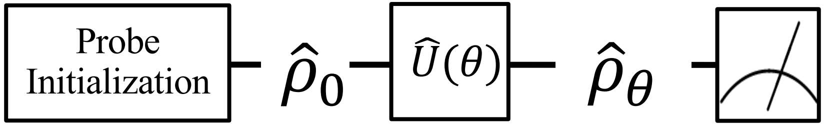
Quantum Experiment

Output

Information

Quantum metrology

Quantum Experiment



$$\hat{U}(\theta) = e^{i\theta\hat{A}}$$

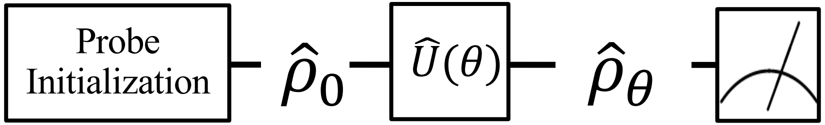
Output

Information

Quantum metrology

HITACHI
Inspire the Next

Quantum Experiment



$$\hat{U}(\theta) = e^{i\theta\hat{A}}$$

Output

$$\{p_j(\theta) = \langle f_j|\hat{\rho}_\theta|f_j\rangle\} \longrightarrow \theta_{\mathrm{e}}$$

[Cramèr-Rao Bound]

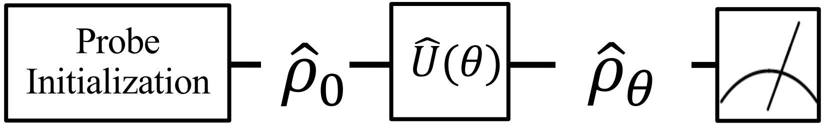
$$\mathrm{Var}(\theta_{\mathrm{e}}) \geq \frac{1}{N\mathcal{I}(\theta)}$$

$$\mathcal{I}\,(\theta) = \frac{\left[\partial_\theta p_j(\theta)\right]^2}{p_j(\theta)}$$

Information

Quantum metrology

Quantum Experiment



$$\hat{U}(\theta) = e^{i\theta\hat{A}}$$

Output

$$\{p_j(\theta) = \langle f_j|\hat{\rho}_\theta|f_j\rangle\} \longrightarrow \theta_\mathrm{e}$$

[Cramèr-Rao Bound]

$$\mathrm{Var}(\theta_\mathrm{e}) \geq \frac{1}{N\mathcal{J}(\theta)}$$

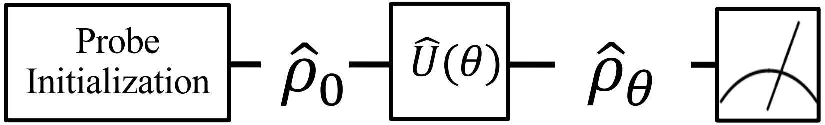
$$\mathcal{J}\,(\theta) = \frac{\left[\partial_\theta p_j(\theta)\right]^2}{p_j(\theta)}$$

Information

$$\partial_\theta p\left(f_j|\Psi_\theta\right) = \partial_\theta \langle f_j|\Psi_\theta\rangle\,\langle\Psi_\theta|f_j\rangle$$

Quantum metrology

Quantum Experiment



$$\hat{U}(\theta) = e^{i\theta\hat{A}}$$

Output

$$\{p_j(\theta) = \langle f_j|\hat{\rho}_\theta|f_j\rangle\} \longrightarrow \theta_{\mathrm{e}}$$

[Cramèr-Rao Bound]

$$\mathrm{Var}(\theta_{\mathrm{e}}) \geq \frac{1}{N\mathcal{I}(\theta)}$$

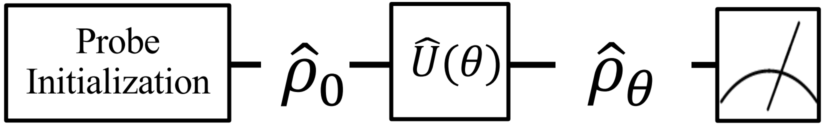
$$\mathcal{I}\,(\theta) = \frac{\left[\partial_\theta p_j(\theta)\right]^2}{p_j(\theta)}$$

Information

$$\partial_\theta p(f_j|\Psi_\theta) = i\langle f_j|\hat{A}|\Psi_\theta\rangle\langle\Psi_\theta|f_j\rangle - i\langle f_j|\Psi_\theta\rangle\langle\Psi_\theta|\hat{A}|f_j\rangle$$

Quantum metrology

Quantum Experiment



$$\hat{U}(\theta) = e^{i\theta\hat{A}}$$

Output

$$\{p_j(\theta) = \langle f_j|\hat{\rho}_\theta|f_j\rangle\} \longrightarrow \theta_\mathrm{e}$$

[Cramèr-Rao Bound]

$$\mathrm{Var}(\theta_\mathrm{e}) \geq \frac{1}{N\mathcal{J}(\theta)}$$

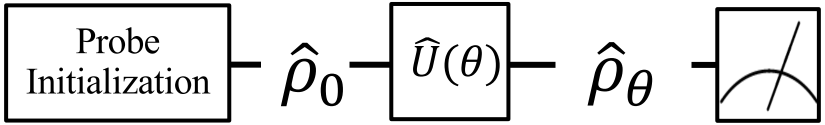
$$\mathcal{J}\,(\theta) = \frac{[\partial_\theta p_j(\theta)]^2}{p_j(\theta)}$$

Information

$$\partial_\theta p(f_j|\Psi_\theta) = 2\mathrm{Im}\langle f_j|\Psi_\theta\rangle\langle\Psi_\theta|\hat{A}|f_j\rangle$$

Quantum metrology

Quantum Experiment



$$\hat{U}(\theta) = e^{i\theta\hat{A}}$$

Output

$$\{p_j(\theta) = \langle f_j|\hat{\rho}_\theta|f_j\rangle\} \longrightarrow \theta_\mathrm{e}$$

[Cramèr-Rao Bound]

$$\mathrm{Var}(\theta_\mathrm{e}) \geq \frac{1}{N\mathcal{J}(\theta)}$$

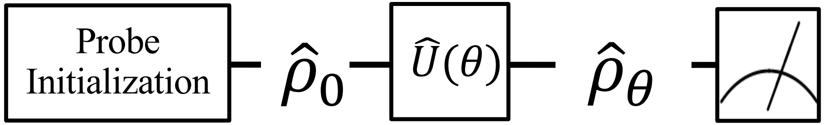
$$\mathcal{J}\,(\theta) = \frac{\left[\partial_\theta p_j(\theta)\right]^2}{p_j(\theta)}$$

Information

$$\partial_\theta p\left(f_j|\Psi_\theta\right) = 2\sum_i a_i \cdot \mathrm{Im}\left[Q_{i,j}(\hat{\rho})\right]$$

Quantum metrology

Quantum Experiment



$$\hat{U}(\theta) = e^{i\theta\hat{A}}$$

Output

$$\{p_j(\theta) = \langle f_j|\hat{\rho}_\theta|f_j\rangle\} \longrightarrow \theta_\mathrm{e}$$

[Cramèr-Rao Bound]

$$\mathrm{Var}(\theta_\mathrm{e}) \geq \frac{1}{N\mathcal{J}(\theta)}$$

$$\mathcal{J}\,(\theta) = \frac{\left[\partial_\theta p_j(\theta)\right]^2}{p_j(\theta)}$$

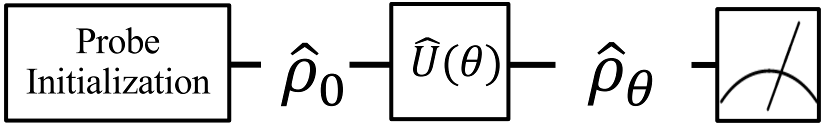
Information

$$\partial_\theta p\left(f_j|\Psi_\theta\right) = 2\sum_i a_i \cdot \mathrm{Im}\left[Q_{i,j}(\hat{\rho})\right]$$

$$Q_{i,j}(\hat{\rho}) = \langle f_j|\Psi_\theta\rangle\langle\Psi_\theta|a_i\rangle\langle a_i|f_j\rangle$$

Quantum metrology

Quantum Experiment



$$\hat{U}(\theta) = e^{i\theta\hat{A}}$$

Output

$$\{p_j(\theta) = \langle f_j|\hat{\rho}_\theta|f_j\rangle\} \longrightarrow \theta_\mathrm{e}$$

[Cramèr-Rao Bound]

$$\mathrm{Var}(\theta_\mathrm{e}) \geq \frac{1}{N\mathcal{J}(\theta)}$$

$$\mathcal{J}\,(\theta) = \frac{[\partial_\theta p_j(\theta)]^2}{p_j(\theta)} = 4\sum_j \frac{\left(\sum_i a_i \cdot \mathrm{Im}\,[Q_{i,j}(\hat{\rho})]\right)^2}{p_j(\theta)}$$

Information

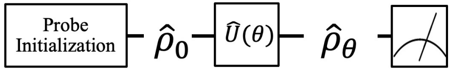
$$\partial_\theta p(f_j|\Psi_\theta) = 2\textstyle\sum_i a_i \cdot \mathrm{Im}\,[Q_{i,j}(\hat{\rho})]$$

$$Q_{i,j}(\hat{\rho}) = \langle f_j|\Psi_\theta\rangle\langle\Psi_\theta|a_i\rangle\langle a_i|f_j\rangle$$

Kirkwood-Dirac Distribution

Kirkwood-Dirac Distribution

$$Q_{i,j}(\hat{\rho}) = \langle f_j|\Psi_\theta\rangle\langle\Psi_\theta|a_i\rangle\langle a_i|f_j\rangle$$



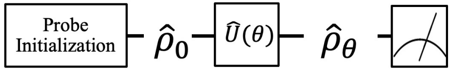
Formula of interest

Modus Operandi

Kirkwood-Dirac Distribution

Kirkwood-Dirac Distribution

$$Q_{i,j}(\hat{\rho}) = \langle f_j|\Psi_\theta\rangle\langle\Psi_\theta|a_i\rangle\langle a_i|f_j\rangle$$



Formula of interest

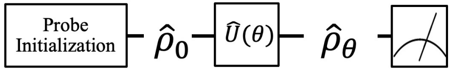
$$\mathcal{J}(\theta) = 4\sum_j \frac{\left(\sum_i a_i \cdot \mathrm{Im}\left[Q_{i,j}(\hat{\rho})\right]\right)^2}{p_j(\theta)}$$

Modus Operandi

Kirkwood-Dirac Distribution

Kirkwood-Dirac Distribution

$$Q_{i,j}(\hat{\rho}) = \langle f_j | \Psi_\theta \rangle \langle \Psi_\theta | a_i \rangle \langle a_i | f_j \rangle$$



Formula of interest

$$\mathcal{J}(\theta) = 4 \sum_j \frac{\left(\sum_i a_i \cdot \mathrm{Im}\left[Q_{i,j}(\hat{\rho})\right]\right)^2}{p_j(\theta)}$$

Modus Operandi

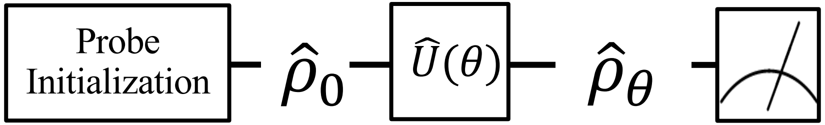
$$\mathcal{J}^{\mathrm{p}}(\theta) = \max_{Q(\hat{\rho}) \in [0,1]} \{\mathcal{J}(\theta)\} = 0$$

$$\mathcal{J}^{\mathrm{np}}(\theta) = \max_{Q(\hat{\rho})} \{\mathcal{J}(\theta)\} \geq 0$$

Quantum metrology

HITACHI
Inspire the Next

Quantum Experiment



$$\hat{U}(\theta) = e^{i\theta\hat{A}}$$

Output

$$\{p_j(\theta) = \mathrm{Tr}(\hat{E}_j\hat{\rho}_\theta)\} \longrightarrow \theta_{\mathrm{e}}$$

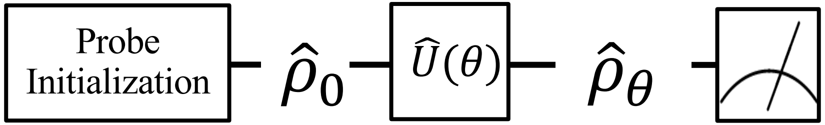
[Cramèr-Rao Bound]

$$\mathrm{Var}(\theta_{\mathrm{e}}) \geq \frac{1}{N\mathcal{J}(\theta)}$$

Information

Quantum metrology

Quantum Experiment



$$\hat{U}(\theta) = e^{i\theta\hat{A}}$$

Output

$$\{p_j(\theta) = \mathrm{Tr}(\hat{E}_j\hat{\rho}_\theta)\} \longrightarrow \theta_\mathrm{e}$$

[Cramèr-Rao Bound]

$$\mathrm{Var}(\theta_\mathrm{e}) \geq \frac{1}{N\mathcal{J}(\theta)}$$

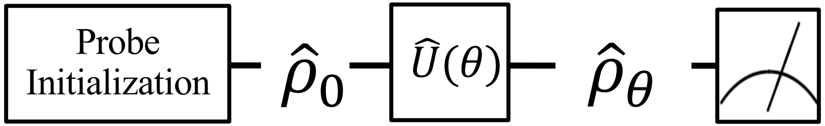
Information

$$\mathcal{J}(\theta) = \sum_j p_j(\theta)\{\partial_\theta \ln[p_j(\theta)]\}^2$$

Quantum metrology

HITACHI
Inspire the Next

Quantum Experiment



$$\hat{U}(\theta) = e^{i\theta\hat{A}}$$

Output

$$\{p_j(\theta) = \mathrm{Tr}(\hat{E}_j\hat{\rho}_\theta)\} \longrightarrow \theta_{\mathrm{e}}$$

[Cramèr-Rao Bound]

$$\mathrm{Var}(\theta_{\mathrm{e}}) \geq \frac{1}{N\mathcal{I}(\theta)}$$

Information

$$\mathcal{I}(\theta) = \sum_j p_j(\theta)\{\partial_\theta \ln[p_j(\theta)]\}^2$$

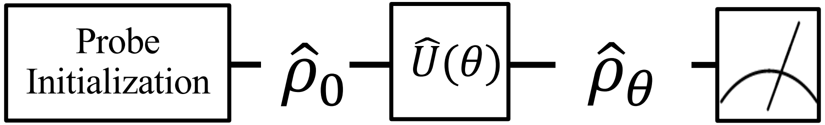
[Braunstein & Caves - 94]

$$\mathcal{I}_Q(\theta|\hat{\rho}_\theta) \equiv \max_{\{E_j\}}\{\mathcal{I}(\theta)\} = \mathrm{Tr}(\hat{\rho}_\theta\hat{\Lambda}^2_{\hat{\rho}_\theta})$$

$$\partial_\theta\hat{\rho}_\theta = \frac{1}{2}(\hat{\Lambda}_{\hat{\rho}_\theta}\hat{\rho}_\theta + \hat{\rho}_\theta\,\hat{\Lambda}_{\hat{\rho}_\theta})$$

Quantum metrology

Quantum Experiment



$$\hat{U}(\theta) = e^{i\theta\hat{A}}$$

Output

$$\{p_j(\theta) = \mathrm{Tr}(\hat{E}_j\hat{\rho}_\theta)\} \longrightarrow \theta_\mathrm{e}$$

[Cramèr-Rao Bound]

$$\mathrm{Var}(\theta_\mathrm{e}) \geq \frac{1}{N\mathcal{J}(\theta)}$$

Information

$$\mathcal{J}(\theta) = \sum_j p_j(\theta)\{\partial_\theta \ln[p_j(\theta)]\}^2$$

[Braunstein & Caves - 94]

$$\mathcal{J}_Q(\theta|\hat{\rho}_\theta) \equiv \max_{\{E_j\}}\{\mathcal{J}(\theta)\} = \mathrm{Tr}\left(\hat{\rho}_\theta\hat{\Lambda}^2_{\hat{\rho}_\theta}\right)$$

$$\partial_\theta\hat{\rho}_\theta = \frac{1}{2}\left(\hat{\Lambda}_{\hat{\rho}_\theta}\hat{\rho}_\theta + \hat{\rho}_\theta\,\hat{\Lambda}_{\hat{\rho}_\theta}\right)$$

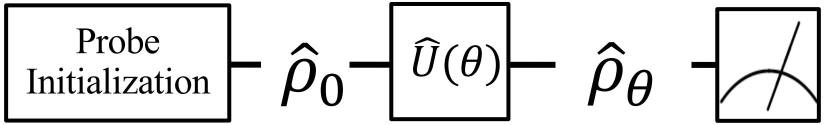
Pure states: $\hat{\rho}_\theta = |\Psi_\theta\rangle\langle\Psi_\theta|$

$$\mathcal{J}_Q(\theta|\hat{\rho}_\theta) = 4\langle\Psi_\theta|\hat{A}^2|\Psi_\theta\rangle - 4\left|\langle\Psi_\theta|\hat{A}|\Psi_\theta\rangle\right|^2$$

Quantum metrology

HITACHI Inspire the Next

Quantum Experiment



$$\hat{U}(\theta) = e^{i\theta\hat{A}}$$

Output

$$\{p_j(\theta) = \mathrm{Tr}(\hat{E}_j\hat{\rho}_\theta)\} \longrightarrow \theta_\mathrm{e}$$

[Cramèr-Rao Bound]

$$\mathrm{Var}(\theta_\mathrm{e}) \geq \frac{1}{N\mathcal{I}(\theta)}$$

Information

$$\mathcal{I}(\theta) = \sum_j p_j(\theta)\{\partial_\theta \ln[p_j(\theta)]\}^2$$

[Braunstein & Caves - 94]

$$\mathcal{I}_Q(\theta|\hat{\rho}_\theta) \equiv \max_{\{E_j\}}\{\mathcal{I}(\theta)\} = \mathrm{Tr}(\hat{\rho}_\theta\hat{\Lambda}^2_{\hat{\rho}_\theta})$$

$$\partial_\theta\hat{\rho}_\theta = \frac{1}{2}(\hat{\Lambda}_{\hat{\rho}_\theta}\hat{\rho}_\theta + \hat{\rho}_\theta\,\hat{\Lambda}_{\hat{\rho}_\theta})$$

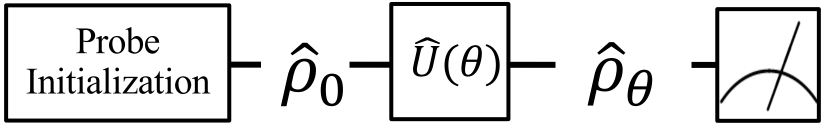
Pure states: $\hat{\rho}_\theta = |\Psi_\theta\rangle\langle\Psi_\theta|$

$$\mathcal{I}_Q(\theta|\hat{\rho}_\theta) = 4\langle\Psi_\theta|\hat{A}^2|\Psi_\theta\rangle - 4|\langle\Psi_\theta|\hat{A}|\Psi_\theta\rangle|^2$$

$$\max_{\{\hat{\rho}_0\}}\{\mathcal{I}_Q(\theta|\hat{\rho}_\theta)\} = (a_{\max} - a_{\min})^2 \equiv (\Delta a)^2$$

Quantum metrology

Quantum Experiment



$$\hat{U}(\theta) = e^{i\theta\hat{A}}$$

Output

$$\{p_j(\theta) = \mathrm{Tr}(\hat{E}_j\hat{\rho}_\theta)\} \longrightarrow \theta_\mathrm{e}$$

[Cramèr-Rao Bound]

$$\mathrm{Var}(\theta_\mathrm{e}) \geq \frac{1}{N\mathcal{J}(\theta)}$$

$$\mathrm{Var}(\theta_\mathrm{e}) = \frac{1}{N(\Delta a)^2}$$

Information

$$\mathcal{J}(\theta) = \sum_j p_j(\theta)\{\partial_\theta \ln[p_j(\theta)]\}^2$$

[Braunstein & Caves - 94]

$$\mathcal{J}_Q(\theta|\hat{\rho}_\theta) \equiv \max_{\{E_j\}}\{\mathcal{J}(\theta)\} = \mathrm{Tr}\left(\hat{\rho}_\theta\hat{\Lambda}^2_{\hat{\rho}_\theta}\right)$$

$$\partial_\theta\hat{\rho}_\theta = \frac{1}{2}\left(\hat{\Lambda}_{\hat{\rho}_\theta}\hat{\rho}_\theta + \hat{\rho}_\theta\,\hat{\Lambda}_{\hat{\rho}_\theta}\right)$$

Pure states: $\hat{\rho}_\theta = |\Psi_\theta\rangle\langle\Psi_\theta|$

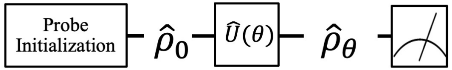
$$\mathcal{J}_Q(\theta|\hat{\rho}_\theta) = 4\langle\Psi_\theta|\hat{A}^2|\Psi_\theta\rangle - 4\left|\langle\Psi_\theta|\hat{A}|\Psi_\theta\rangle\right|^2$$

$$\max_{\{\hat{\rho}_0\}}\{\mathcal{J}_Q(\theta|\hat{\rho}_\theta)\} = (a_{\max} - a_{\min})^2 \equiv (\Delta a)^2$$

Kirkwood-Dirac Distribution

Kirkwood-Dirac Distribution

$$Q_{i,j}(\hat{\rho}) = \mathrm{Tr}\left(|f_j\rangle\langle f_j|a_i\rangle\langle a_i|\hat{\rho}\right)$$



Formula of interest

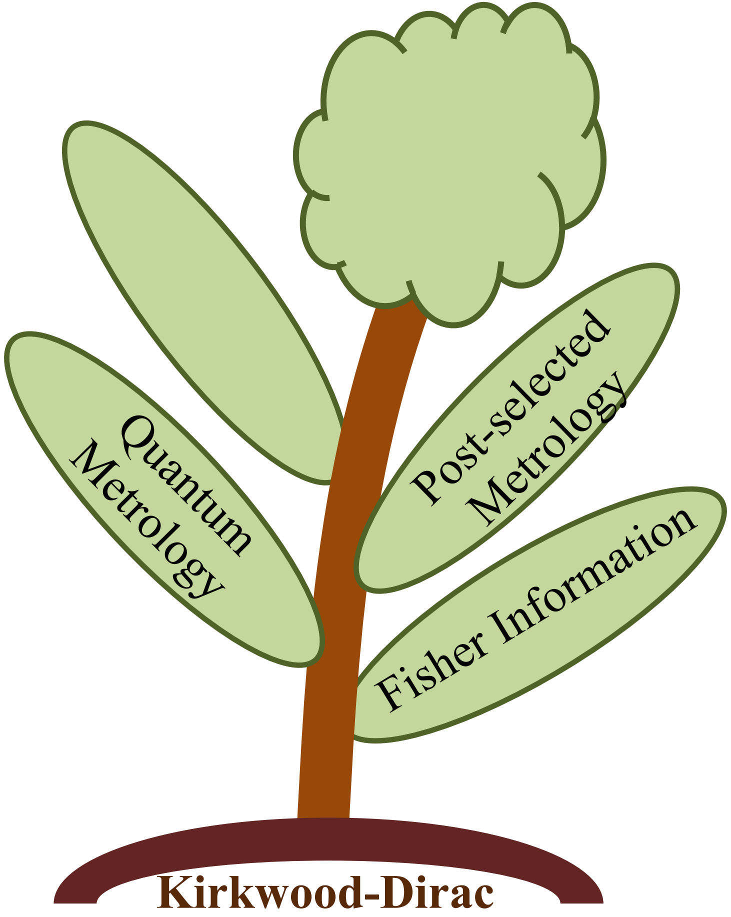
$$\mathcal{J}(\theta) = 4\sum_j \frac{\left(\sum_i a_i \cdot \mathrm{Im}\left[Q_{i,j}(\hat{\rho})\right]\right)^2}{p_j(\theta)}$$

Modus Operandi

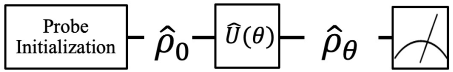
$$\mathcal{J}^{\mathrm{p}}(\theta) = \max_{Q(\hat{\rho})\in[0,1]}\{\mathcal{J}(\theta)\} = 0$$

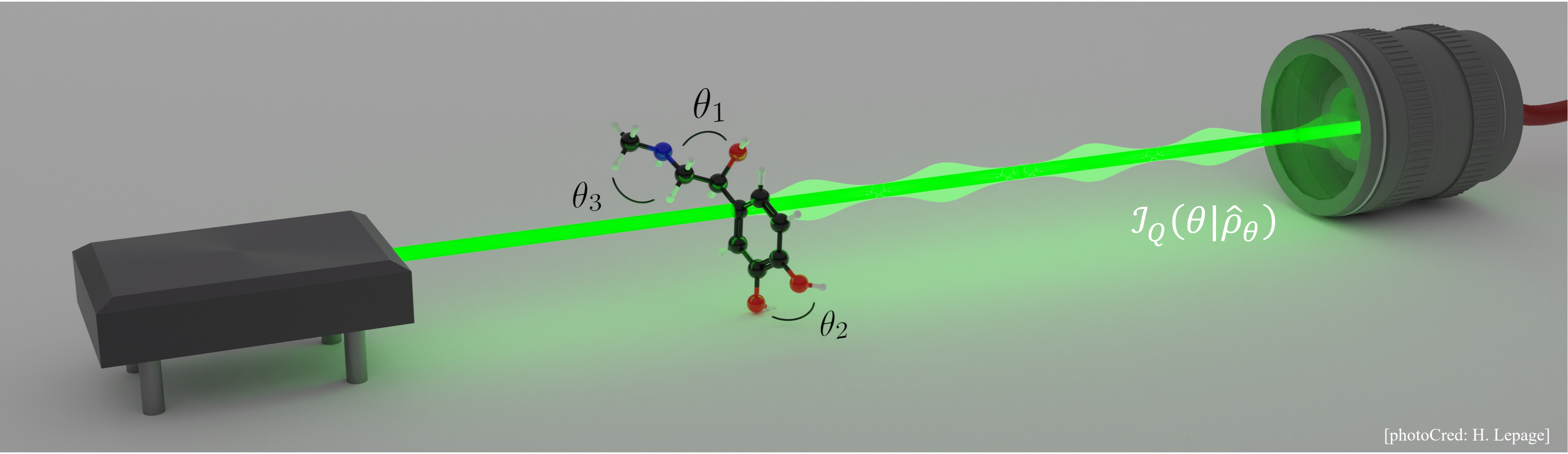
$$0 \le \mathcal{J}^{\mathrm{np}}(\theta) = \max_{Q(\hat{\rho})}\{\mathcal{J}(\theta)\} \le (\Delta a)^2$$

Talk outline



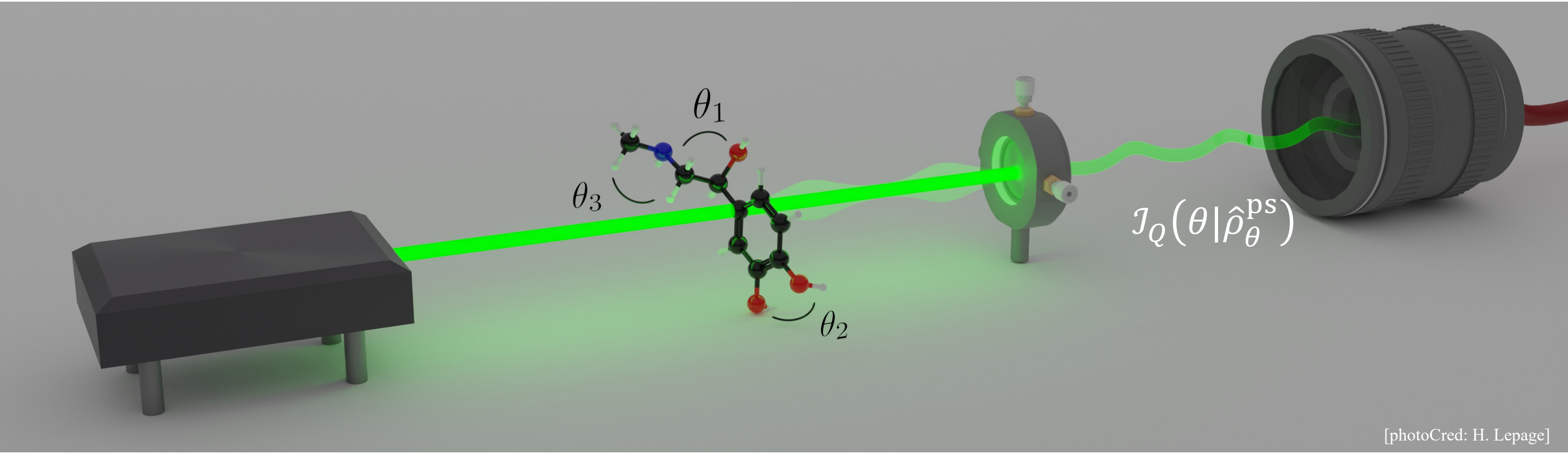
Quantum metrology



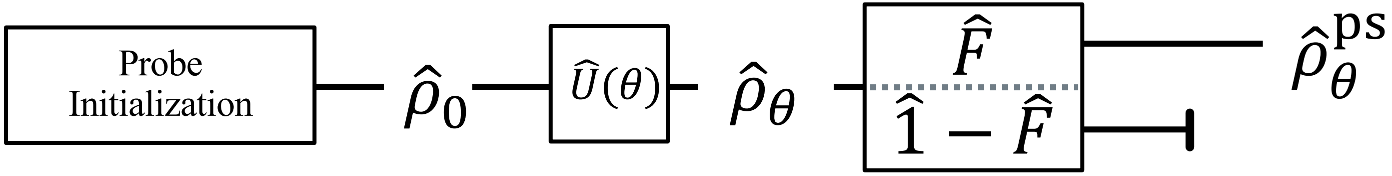


Post-selected quantum metrology

HITACHI
Inspire the Next

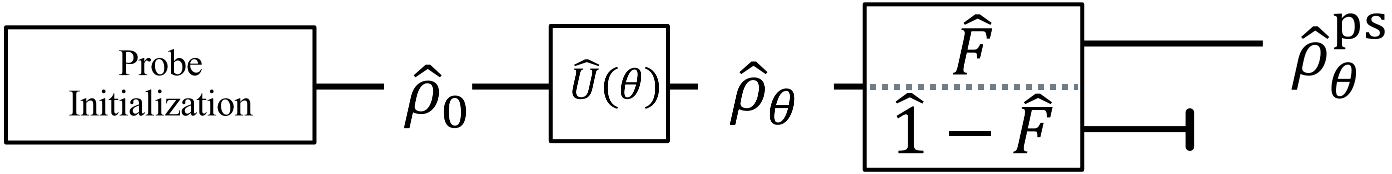


Post-selected quantum metrology



Post-selected quantum metrology

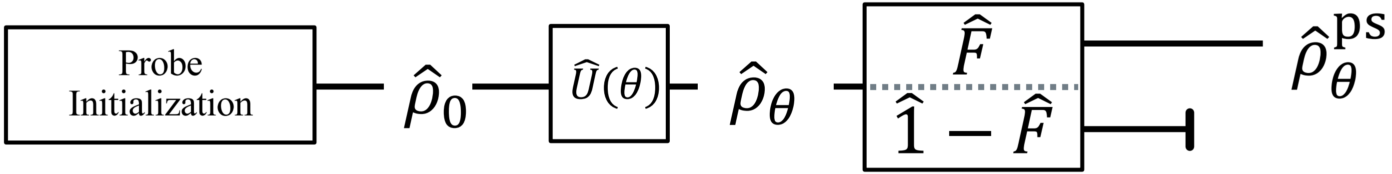
HITACHI
Inspire the Next



$$\mathcal{I}_Q\left(\theta|\hat{\rho}_\theta^{\mathrm{ps}}\right) \leq (a_{\max}-a_{\min})^2,$$

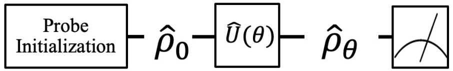
In a classically non-negative theory

Post-selected quantum metrology

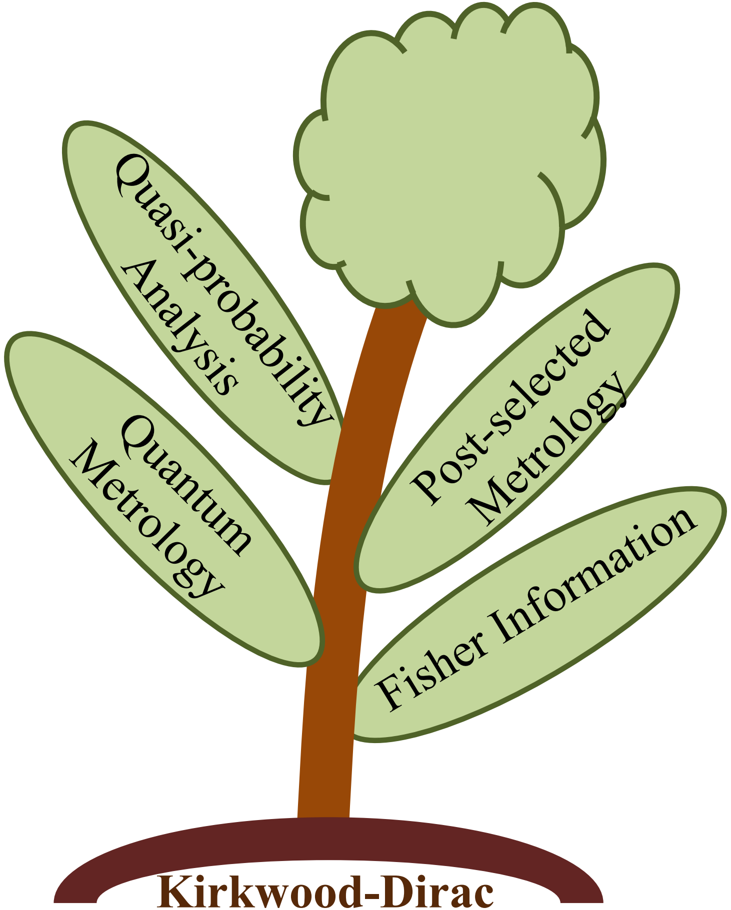


$$\mathcal{I}_Q\left(\theta|\hat{\rho}_\theta^{\text{ps}}\right) \le (a_{\max} - a_{\min})^2,$$ *In a classically non-negative theory*

Same as:

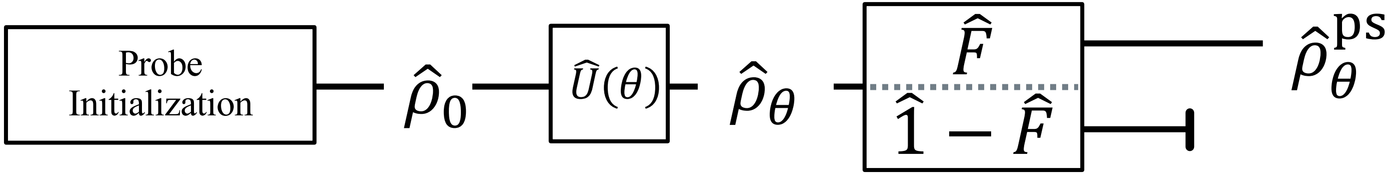


Talk outline



Post-selected quantum metrology - Bounds

HITACHI
Inspire the Next



Formula of Interest

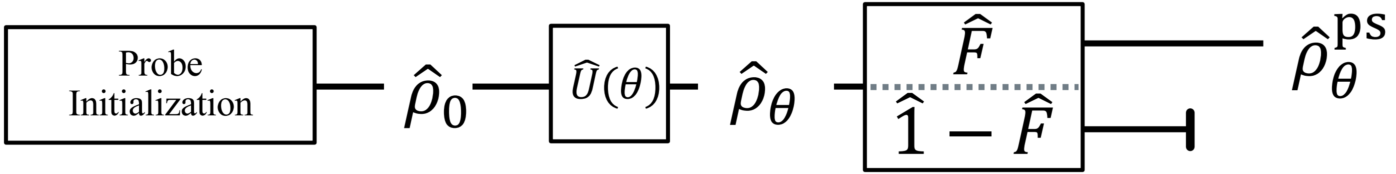
Quasi-probabilistic Analysis

Remember

$$\hat{U}(\theta) = e^{i\theta\hat{A}}$$

Post-selected quantum metrology - Bounds

HITACHI
Inspire the Next



Formula of Interest

$$\mathcal{I}_Q\left(\theta|\hat{\rho}_\theta^{\mathrm{ps}}\right) = \frac{4}{p_\theta^{\mathrm{ps}}}\mathrm{Tr}\left[\hat{F}\hat{A}\hat{\rho}_\theta\hat{A}\right] - \frac{4}{\left(p_\theta^{\mathrm{ps}}\right)^2}\left|\mathrm{Tr}\left[\hat{F}\hat{\rho}_\theta\hat{A}\right]\right|^2 , \qquad p_\theta^{\mathrm{ps}} = \mathrm{Tr}\left[\hat{F}\hat{\rho}_\theta\right]$$

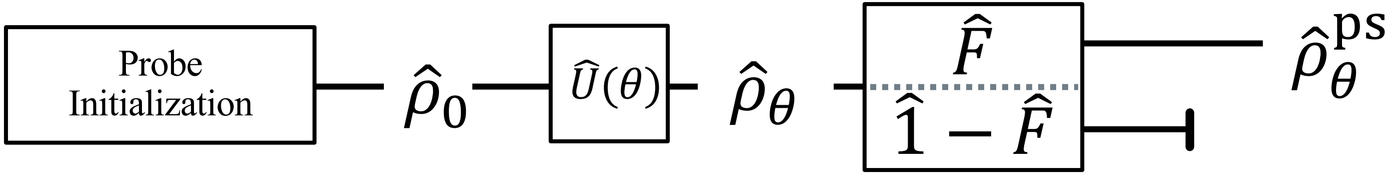
Quasi-probabilistic Analysis

Remember

$$\hat{U}(\theta) = e^{i\theta\hat{A}}$$

Post-selected quantum metrology - Bounds

HITACHI
Inspire the Next



Formula of Interest

$$\mathcal{I}_Q\left(\theta|\hat{\rho}_\theta^{\mathrm{ps}}\right) = \frac{4}{p_\theta^{\mathrm{ps}}}\mathrm{Tr}\left[\hat{F}\hat{A}\hat{\rho}_\theta\hat{A}\right] - \frac{4}{\left(p_\theta^{\mathrm{ps}}\right)^2}\left|\mathrm{Tr}\left[\hat{F}\hat{\rho}_\theta\hat{A}\right]\right|^2 \quad , \quad p_\theta^{\mathrm{ps}} = \mathrm{Tr}\left[\hat{F}\hat{\rho}_\theta\right]$$

Quasi-probabilistic Analysis

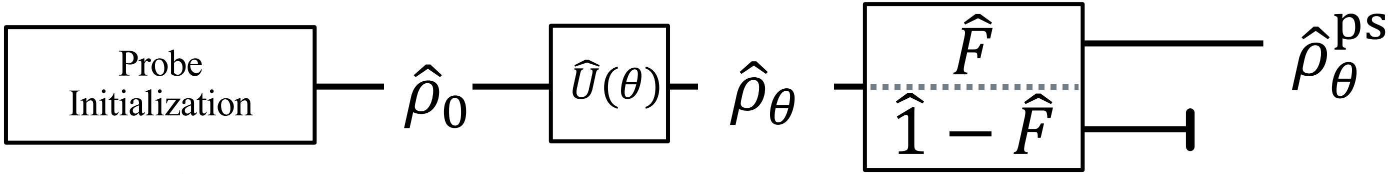
$$\tilde{Q}_{i,j}(\hat{\rho}) = \langle a_i|\hat{\rho}|a_j\rangle\langle a_j|\hat{F}|a_i\rangle/\mathrm{Tr}\left(\hat{F}\hat{\rho}_\theta\right)$$

Remember

$$\hat{U}(\theta) = e^{i\theta\hat{A}}$$

Post-selected quantum metrology - Bounds

HITACHI
Inspire the Next



Formula of Interest

$$\mathcal{I}_Q\left(\theta|\hat{\rho}_\theta^{\mathrm{ps}}\right) = \frac{4}{p_\theta^{\mathrm{ps}}}\mathrm{Tr}\left[\hat{F}\hat{A}\hat{\rho}_\theta\hat{A}\right] - \frac{4}{\left(p_\theta^{\mathrm{ps}}\right)^2}\left|\mathrm{Tr}\left[\hat{F}\hat{\rho}_\theta\hat{A}\right]\right|^2 \quad , \quad p_\theta^{\mathrm{ps}} = \mathrm{Tr}\left[\hat{F}\hat{\rho}_\theta\right]$$

Quasi-probabilistic Analysis

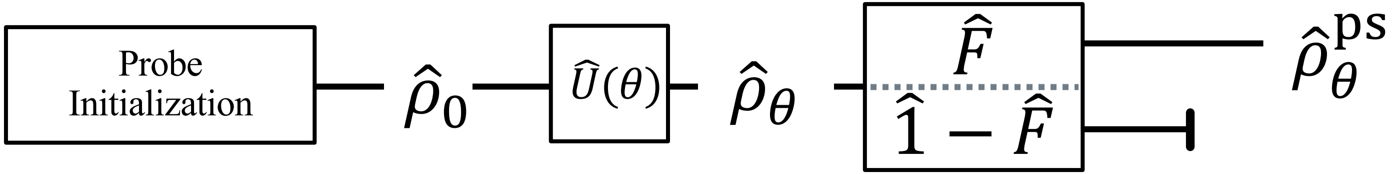
$$\tilde{Q}_{i,j}(\hat{\rho}) = \langle a_i|\hat{\rho}|a_j\rangle\langle a_j|\hat{F}|a_i\rangle/\mathrm{Tr}\left(\hat{F}\hat{\rho}_\theta\right)$$

$$\mathcal{I}_Q\left(\theta|\hat{\rho}_\theta^{\mathrm{ps}}\right) = 4\sum_{i,j}\tilde{Q}_{i,j}(\hat{\rho})a_i a_j - 4\left|\sum_{i,j}\tilde{Q}_{i,j}(\hat{\rho})a_i\right|^2$$

Remember

$$\hat{U}(\theta) = e^{i\theta\hat{A}}$$

Post-selected quantum metrology - Bounds



Formula of Interest

$$\mathcal{I}_Q\left(\theta|\hat{\rho}_\theta^{\mathrm{ps}}\right) = \frac{4}{p_\theta^{\mathrm{ps}}}\mathrm{Tr}\left[\hat{F}\hat{A}\hat{\rho}_\theta\hat{A}\right] - \frac{4}{\left(p_\theta^{\mathrm{ps}}\right)^2}\left|\mathrm{Tr}\left[\hat{F}\hat{\rho}_\theta\hat{A}\right]\right|^2 \quad , \quad p_\theta^{\mathrm{ps}} = \mathrm{Tr}\left[\hat{F}\hat{\rho}_\theta\right]$$

Quasi-probabilistic Analysis

$$\tilde{Q}_{i,j}(\hat{\rho}) = \langle a_i|\hat{\rho}|a_j\rangle\langle a_j|\hat{F}|a_i\rangle/\mathrm{Tr}\left(\hat{F}\hat{\rho}_\theta\right)$$

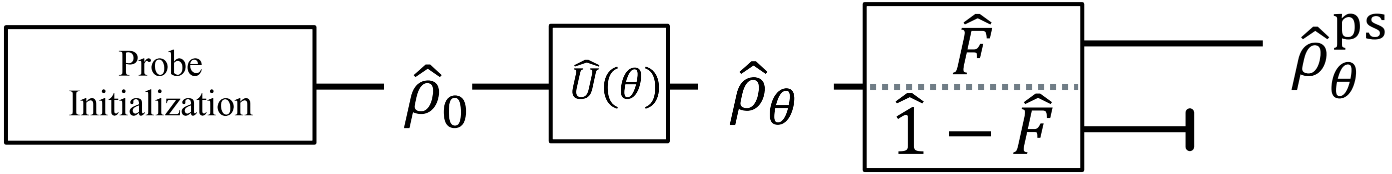
$$\mathcal{I}_Q\left(\theta|\hat{\rho}_\theta^{\mathrm{ps}}\right) = 4\sum_{i,j}\tilde{Q}_{i,j}(\hat{\rho})a_i a_j - 4\left|\sum_{i,j}\tilde{Q}_{i,j}(\hat{\rho})a_i\right|^2$$

$$\max_{\tilde{Q}_{i,j}(\hat{\rho})\in[0,1]}\mathcal{I}_Q\left(\theta|\hat{\rho}_\theta^{\mathrm{ps}}\right) = \left(a_{\max} - a_{\min}\right)^2$$

Remember

$$\hat{U}(\theta) = e^{i\theta\hat{A}}$$

Post-selected quantum metrology - Bounds



Formula of Interest

$$\mathcal{I}_Q\left(\theta|\hat{\rho}_\theta^{\mathrm{ps}}\right) = \frac{4}{p_\theta^{\mathrm{ps}}}\mathrm{Tr}\left[\hat{F}\hat{A}\hat{\rho}_\theta\hat{A}\right] - \frac{4}{\left(p_\theta^{\mathrm{ps}}\right)^2}\left|\mathrm{Tr}\left[\hat{F}\hat{\rho}_\theta\hat{A}\right]\right|^2 \quad , \quad p_\theta^{\mathrm{ps}} = \mathrm{Tr}\left[\hat{F}\hat{\rho}_\theta\right]$$

Quasi-probabilistic Analysis

$$\tilde{Q}_{i,j}(\hat{\rho}) = \langle a_i|\hat{\rho}|a_j\rangle\langle a_j|\hat{F}|a_i\rangle/\mathrm{Tr}\left(\hat{F}\hat{\rho}_\theta\right)$$

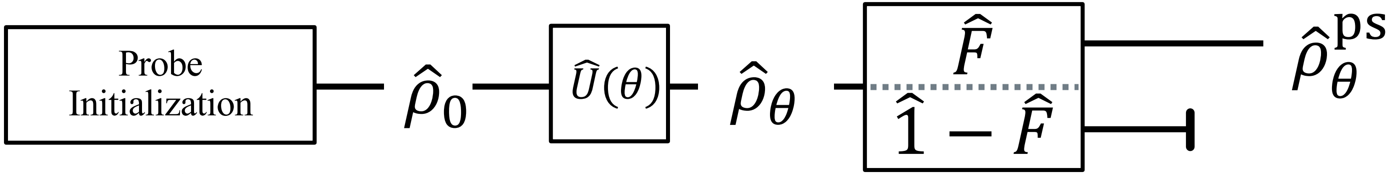
$$\mathcal{I}_Q\left(\theta|\hat{\rho}_\theta^{\mathrm{ps}}\right) = 4\sum_{i,j}\tilde{Q}_{i,j}(\hat{\rho})a_i a_j - 4\left|\sum_{i,j}\tilde{Q}_{i,j}(\hat{\rho})a_i\right|^2$$

$\mathcal{I}_Q\left(\theta|\hat{\rho}_\theta^{\mathrm{ps}}\right) \leq (\Delta a)^2$, *In a classically non-negative theory*

Remember

$$\hat{U}(\theta) = e^{i\theta\hat{A}}$$

Post-selected quantum metrology - Bounds



Formula of Interest

$$\mathcal{I}_Q\left(\theta|\hat{\rho}_\theta^{\mathrm{ps}}\right) = \frac{4}{p_\theta^{\mathrm{ps}}}\mathrm{Tr}\left[\hat{F}\hat{A}\hat{\rho}_\theta\hat{A}\right] - \frac{4}{\left(p_\theta^{\mathrm{ps}}\right)^2}\left|\mathrm{Tr}\left[\hat{F}\hat{\rho}_\theta\hat{A}\right]\right|^2 \quad , \quad p_\theta^{\mathrm{ps}} = \mathrm{Tr}\left[\hat{F}\hat{\rho}_\theta\right]$$

Quasi-probabilistic Analysis

$$\tilde{Q}_{i,j}(\hat{\rho}) = \langle a_i|\hat{\rho}|a_j\rangle\langle a_j|\hat{F}|a_i\rangle/\mathrm{Tr}\left(\hat{F}\hat{\rho}_\theta\right)$$

$$\mathcal{I}_Q\left(\theta|\hat{\rho}_\theta^{\mathrm{ps}}\right) = 4\sum_{i,j}\tilde{Q}_{i,j}(\hat{\rho})a_i a_j - 4\left|\sum_{i,j}\tilde{Q}_{i,j}(\hat{\rho})a_i\right|^2$$

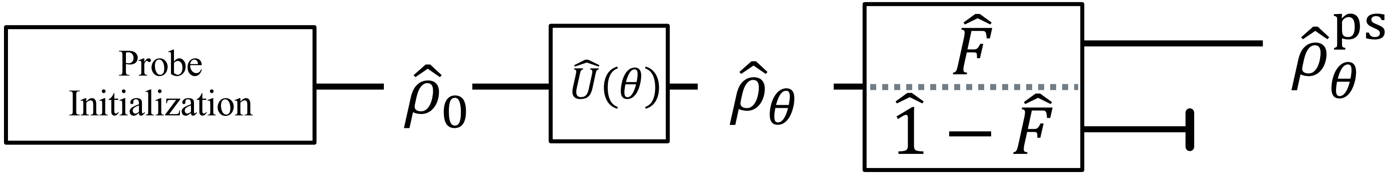
$$\max_{\tilde{Q}_{i,j}(\hat{\rho})}\mathcal{I}_Q\left(\theta|\hat{\rho}_\theta^{\mathrm{ps}}\right) = (\Delta a)^2\times\mathcal{N}[Q(\hat{\rho})] = (\Delta a)^2\times\sum_{i,j}\left|\tilde{Q}_{i,j}(\hat{\rho})\right|$$

Remember

$$\hat{U}(\theta) = e^{i\theta\hat{A}}$$

Post-selected quantum metrology - Bounds

HITACHI
Inspire the Next



Formula of Interest

$$\mathcal{I}_Q\left(\theta|\hat{\rho}_\theta^{\mathrm{ps}}\right) = \frac{4}{p_\theta^{\mathrm{ps}}}\mathrm{Tr}\left[\hat{F}\hat{A}\hat{\rho}_\theta\hat{A}\right] - \frac{4}{\left(p_\theta^{\mathrm{ps}}\right)^2}\left|\mathrm{Tr}\left[\hat{F}\hat{\rho}_\theta\hat{A}\right]\right|^2 \quad , \quad p_\theta^{\mathrm{ps}} = \mathrm{Tr}\left[\hat{F}\hat{\rho}_\theta\right]$$

Quasi-probabilistic Analysis

$$\tilde{Q}_{i,j}(\hat{\rho}) = \langle a_i|\hat{\rho}|a_j\rangle\langle a_j|\hat{F}|a_i\rangle/\mathrm{Tr}\left(\hat{F}\hat{\rho}_\theta\right)$$

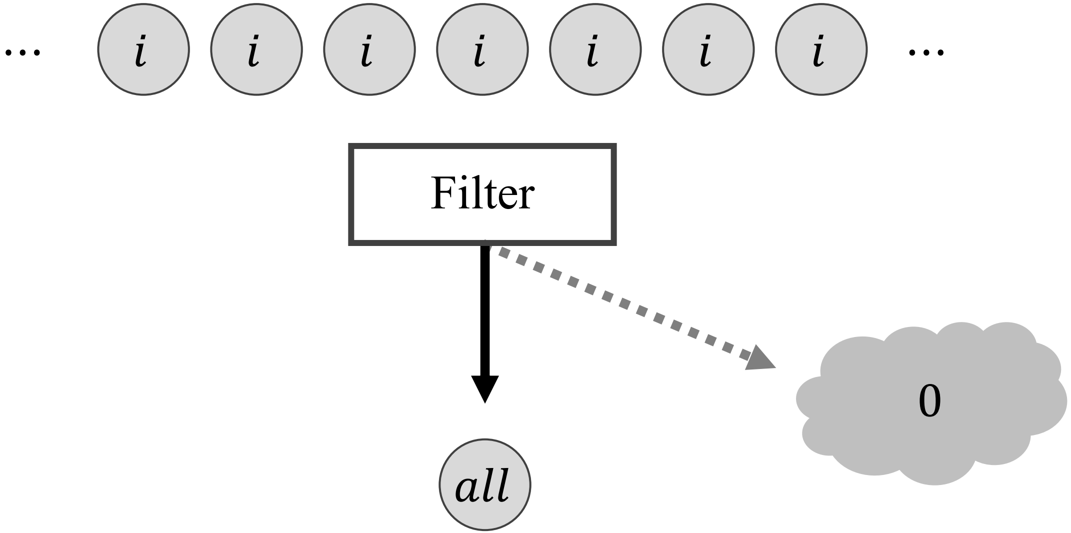
$$\mathcal{I}_Q\left(\theta|\hat{\rho}_\theta^{\mathrm{ps}}\right) = 4\sum_{i,j}\tilde{Q}_{i,j}(\hat{\rho})a_i a_j - 4\left|\sum_{i,j}\tilde{Q}_{i,j}(\hat{\rho})a_i\right|^2$$

$$\mathcal{I}_Q\left(\theta|\hat{\rho}_\theta^{\mathrm{ps}}\right) \leq \infty$$, *In a non-positive theory*

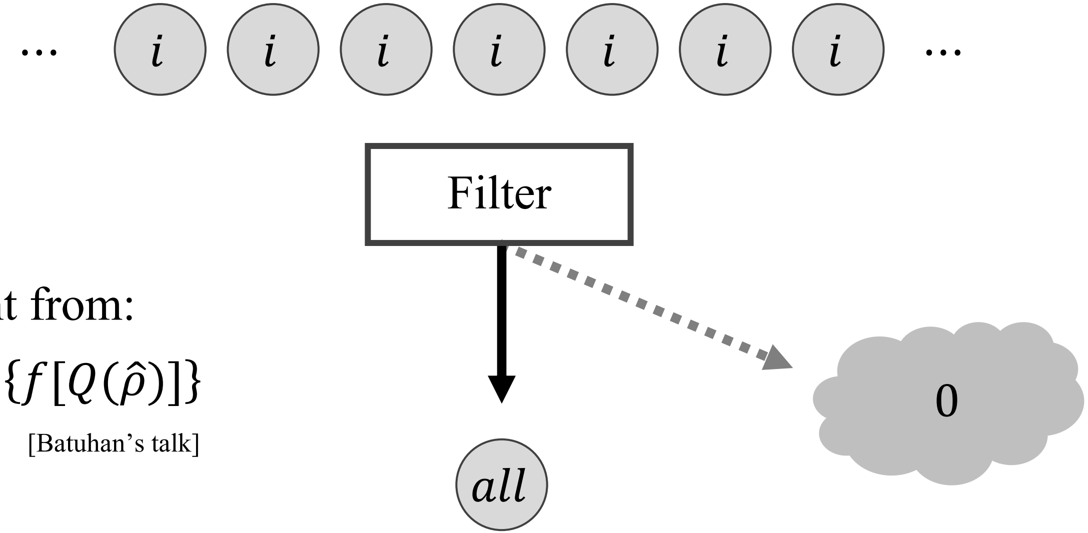
Remember

$$\hat{U}(\theta) = e^{i\theta\hat{A}}$$

Summary



Summary



optimal experiment from:

$$Q^{\star}(\hat{\rho}) \;= \arg \operatorname{opt}_{Q(\hat{\rho})}\{f[Q(\hat{\rho})]\}$$

[Batuhan's talk]

Papers

HITACHI
Inspire the Next

Quantum Learnability is Arbitrarily Distillable

Joe H. Jenne[1] and David R. M. Arvidsson-Shukur[2,1,3]
[1] Cavendish Laboratory, Department of Physics, University of Camb… …ambridge CB3 0HE
[2] Hitachi Cambridge Laboratory, J. J. Thomson Avenue, C… …mbridge, Unite…
[3] Research Laboratory of Electronics, Massachusetts Institute of Techn… …idge, Massach…

PHYSICAL REVIEW LETTERS 131, 150202 (2023)

Nonclassical Advantage in Metrology Established via Quantum Simulations of Hypothetical Closed Timelike Curves

David R. M. Arvidsson-Shukur,[1] Aidan G. McConnell,[2,3,4] and Nicole Yunger …
[1] Hitachi Cambridge Laboratory, J. J. Thomson Avenue, Cambridge CB3 0HE, United …
[2] Cavendish Laboratory, Department of Physics, University of Cambridge, Cambridge CB3 0HE, …
[3] Laboratory for X-ray Nanoscience and Technologies, Paul Scherrer Institut, 5232 Villigen, S…
[4] Department of Physics and Quantum Center, Eidgenössische Technische Hochschule Zürich, CH-8093 Z…
[5] Joint Center for Quantum Information and Computer Science, NIST and University of Maryland, College Park, M…
[6] Institute for Physical Science and Technology, University of Maryland, College Park, Maryland 2074…

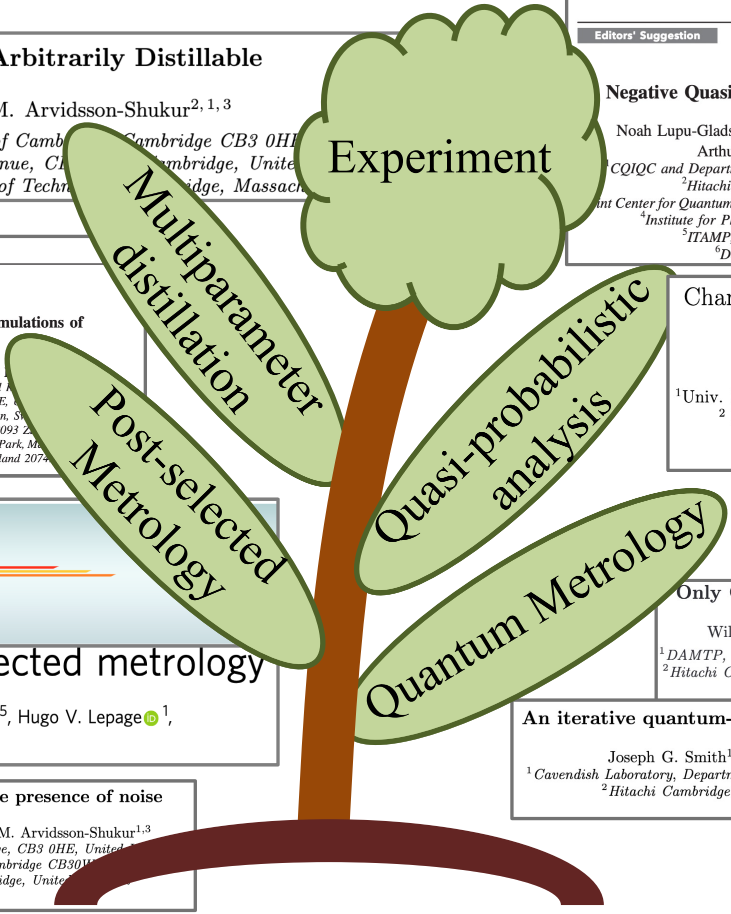
nature COMMUNICATIONS

Quantum advantage in postselected metrology

David R. M. Arvidsson-Shukur[1,2,3], Nicole Yunger Halpern[3,4,5], Hugo V. Lepage[1], Aleksander A. Lasek[1], Crispin H. W. Barnes[1] & Seth Lloyd[2,3]

Compression of metrological quantum information in the presence of noise

Flavio Salvati[1], Wilfred Salmon[2,3], Crispin H.W. Barnes[1], and David R.M. Arvidsson-Shukur[1,3]
[1] Cavendish Laboratory, Department of Physics, University of Cambridge, Cambridge, CB3 0HE, United …
[2] DAMTP, Centre for Mathematical Sciences, University of Cambridge, Cambridge CB30…
[3] Hitachi Cambridge Laboratory, J. J. Thomson Avenue, CB3 0HE, Cambridge, Unite…
(Dated: July 18, 2023)



PHYSICAL REVIEW LETTERS 128, 220504 (2022)
Editors' Suggestion

Negative Quasiprobabilities Enhance Phase Estimation in Quantum-Optics Experiment

Noah Lupu-Gladstein,[1,*] Y. Batuhan Yilmaz,[1,†] David R. M. Arvidsson-Shukur,[2,‡] Aharon Brodutch,[1,§] Arthur O. T. Pang,[1,‖] Aephraim M. Steinberg,[1,¶] and Nicole Yunger Halpern[3,4,5,6]
[1] CQIQC and Department of Physics, University of Toronto, 60 Saint George Street, Toronto, Ontario M5S 1A7, Canada
[2] Hitachi Cambridge Laboratory, J. J. Thomson Avenue, Cambridge CB3 0HE, United Kingdom
…int Center for Quantum Information and Computer Science, NIST and University of Maryland, College Park, Maryland 20742, USA
[4] Institute for Physical Science and Technology, University of Maryland, College Park, Maryland 20742, USA
[5] ITAMP, Harvard-Smithsonian Center for Astrophysics, Cambridge, Massachusetts 02138, USA
[6] Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA

Characterizing the geometry of the Kirkwood-Dirac positive states

C. Langrenez[1,*] D.R.M. Arvidsson-Shukur[2,†] S. De Bièvre[1,‡]
[1] Univ. Lille, CNRS, Inria, UMR 85…
[2] Hitachi Cambridge Lab., J.…

Journal of Physics A: Mathematical and Theoretical

PAPER

Conditions tighter than noncommutation needed for nonclassicality

David R M Arvidsson-Shukur[10,1,2,3], Jacob Chevalier Drori[4] and Nicole Yunger Halpern[3,5,6,7,8,9]

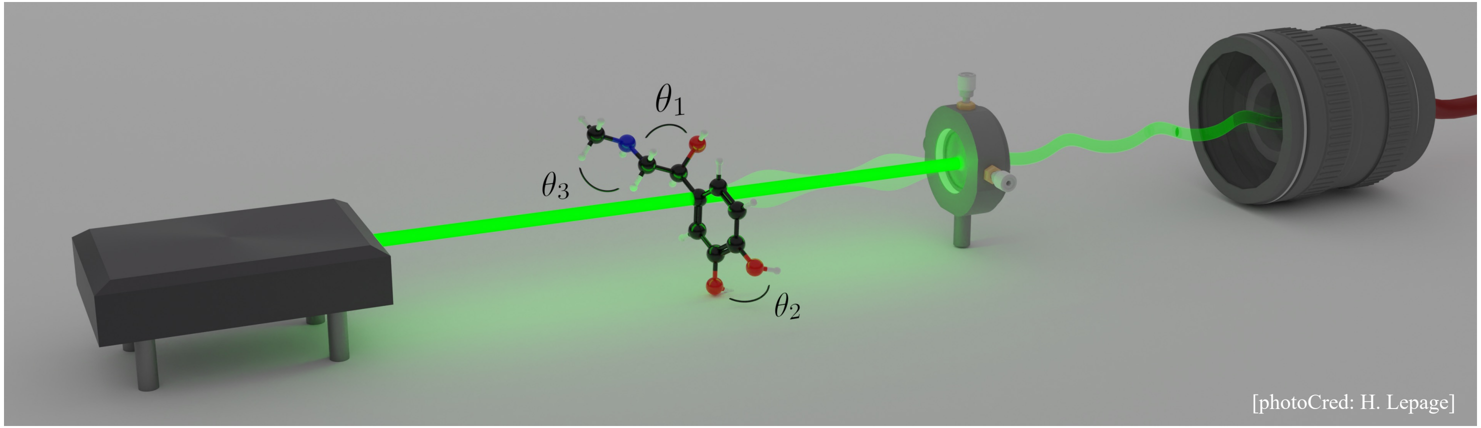
Only Classical Parametrised States have Optimal Measurements

Wilfred Salmon,[1,2,*] Sergii Strelchuk,[1] and David R. M. Arvidsson-Shukur[2]
[1] DAMTP, Centre for Mathematical Sciences, University of Cambridge, Cambridge CB30WA, UK
[2] Hitachi Cambridge Laboratory, J. J. Thomson Avenue, CB3 0HE, Cambridge, United Kingdom
(Dated: May 30, 2022)

An iterative quantum-phase-estimation protocol for near-term quantum hardware

Joseph G. Smith[1,2,*] Crispin H. W. Barnes[1,†] and David R. M. Arvidsson-Shukur[2,‡]
[1] Cavendish Laboratory, Department of Physics, University of Cambridge, Cambridge, CB3 0HE, United Kingdom and
[2] Hitachi Cambridge Laboratory, J. J. Thomson Ave., Cambridge, CB3 0HE, United Kingdom
(Dated: June 15, 2022)

Closing Slide



Merci!