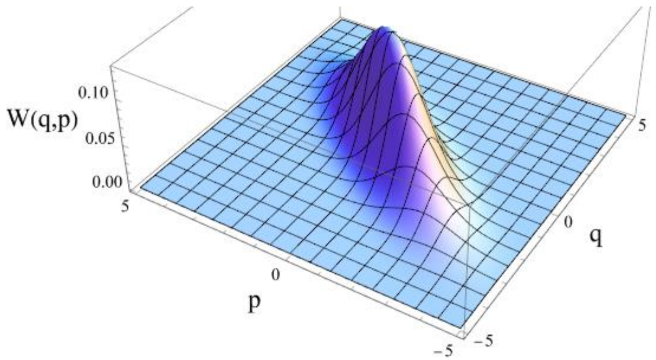
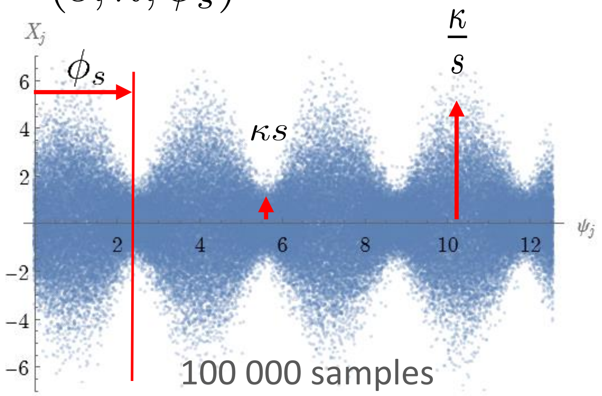
Explicit and efficient estimators for Gaussian state parameters

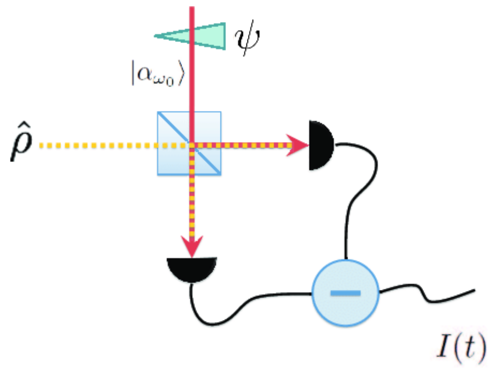
Single-mode squeezed vacuum $\vec{\theta} = (s, \kappa, \phi_s)$





$$A_j = \mathrm{Tr}\left[\hat{X}^2_{\psi}\ \hat{\rho}(\vec{\theta})\right] = \kappa s \cos^2[\phi_s - \psi] + \frac{\kappa}{s}\sin^2[\phi_s - \psi],$$

Homodyne detection



Ways of extracting the parameters:

a. Slow phase scan + min/max of noise (long measurement)

b. Measure $\hat{q}, \hat{p}$ and $\frac{\hat{q}+\hat{p}}{\sqrt{2}}$ (technically difficult)

c. Fast phase scan + MLE fitting (heavy calculations)

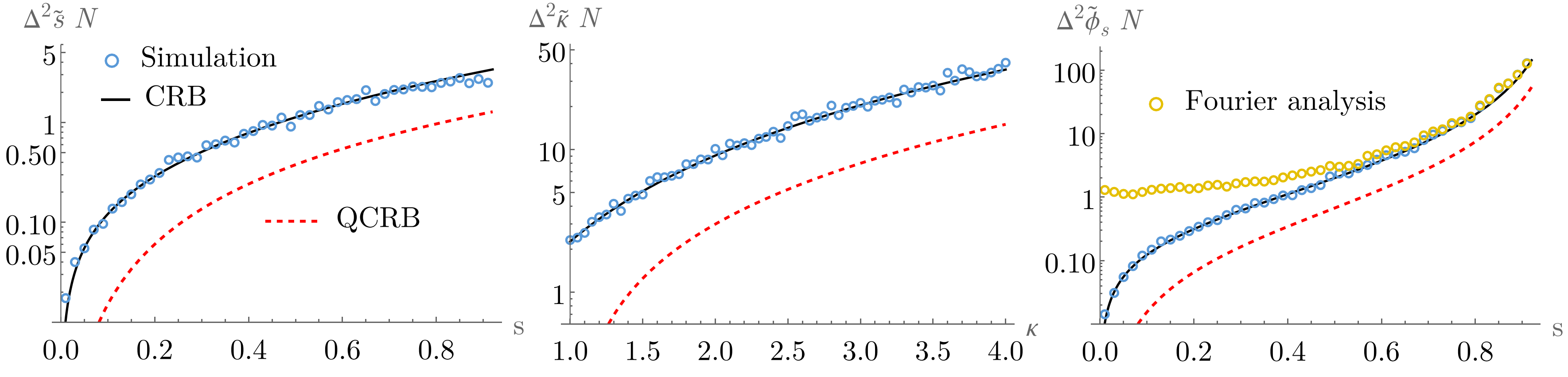
d. Fast phase scan + efficient estimator

Method of moments[1]:

$$Y_k = \frac{1}{N_\psi}\sum_j c_k(\psi_j, \vec{\theta}_0)\ A_j, \quad k = 1,2,3, \quad \text{with} \quad c_k(\psi_j, \vec{\theta}_0) = \left(\frac{1}{2A^2(\psi_j, \vec{\theta})}\frac{\partial A(\psi_j, \vec{\theta})}{\partial \theta_k}\right)\Bigg|_{\vec{\theta} = \vec{\theta}_0}$$

Analytical transformation

$$\tilde{\vec{\theta}}: \ \mathrm{cov}(\tilde{\theta}_k, \tilde{\theta}_l) \geq \mathbf{M}^{-1} = \frac{1}{N_\psi}\begin{pmatrix} s(1+s)^2 & \kappa(1-s^2) & 0 \\ \kappa(1-s^2) & \kappa^2\frac{1+s^2}{s} & 0 \\ 0 & 0 & \frac{s}{(1-s)^2} \end{pmatrix}$$



[1] M. Gessner, A. Smerzi, and L. Pezz`e, Multiparameter squeezing for optimal quantum enhancements in sensor networks, Nature Communications 11, 3817 (2020)