Probabilistic pure state conversion on the majorization lattice

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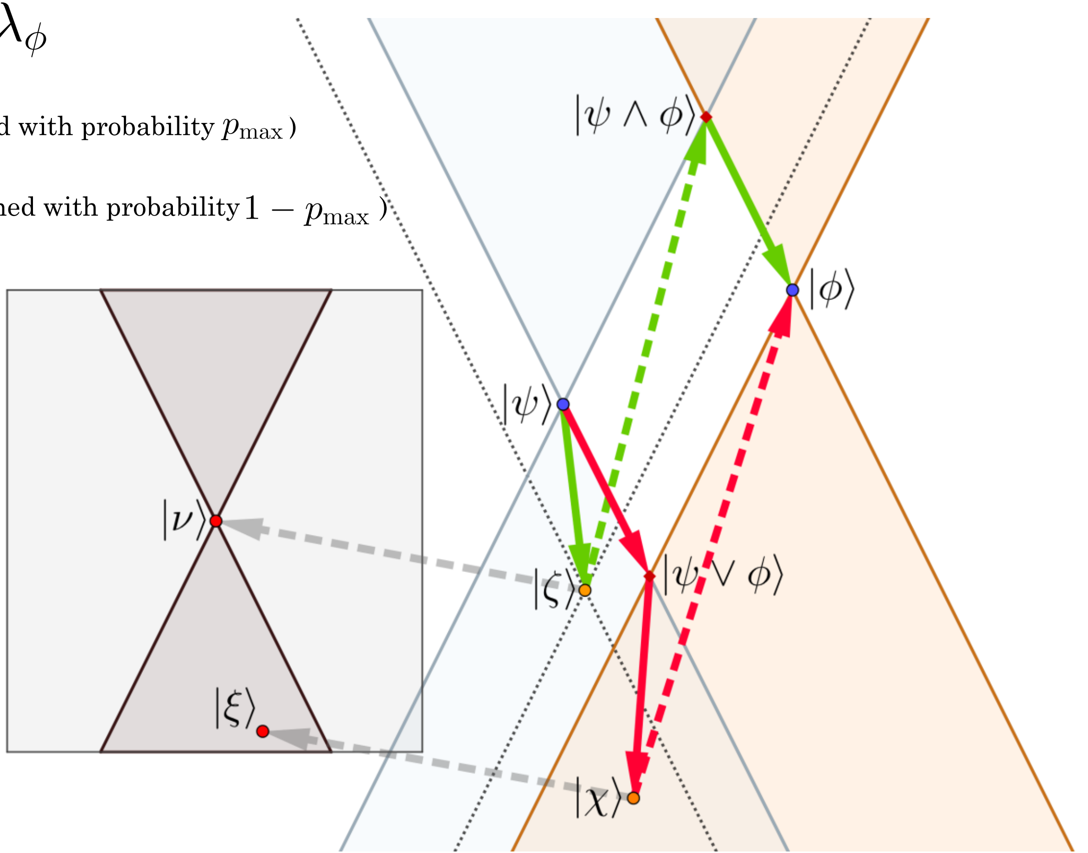
Background

- Nielsen's theorem [1]: $|\psi\rangle_{AB} = \sum_{i=1}^{d} \sqrt{\lambda_\psi^{(i)}} |i\rangle_A |i\rangle_B \xrightarrow{\text{LOCC}} |\phi\rangle_{AB} = \sum_{i=1}^{d} \sqrt{\lambda_\phi^{(i)}} |i\rangle_A |i\rangle_B \iff \lambda_\psi \prec \lambda_\phi$
- Majorization relation $\prec$: preorder on probability distributions
  → Majorization lattice: meet ∧, join ∨
- Vidal's theorem [2]: $|\psi\rangle_{AB} \xrightarrow[\text{proba } p]{\text{LOCC}} |\phi\rangle_{AB} \iff \lambda_\psi \prec^w p\lambda_\phi$
- Optimal protocol: $|\psi\rangle \xrightarrow{\text{LOCC}} |\chi\rangle \xrightarrow[\{\hat{M},\hat{N}\}]{\text{Measurement}}$
  ↗ $|\phi\rangle$ Target state (obtained with probability $p_{\max}$)
  ↘ $|\xi\rangle$ Residual state (obtained with probability $1 - p_{\max}$)

Results

- Greedy protocol & Thrifty protocol
- **Theorem 1:** the optimal conversion probability from $|\psi\rangle$ to $|\phi\rangle$ is equal to that from $|\psi\rangle$ to $|\psi \wedge \phi\rangle$
- **Theorem 2:** the residual state $|\nu\rangle$ of the thrifty protocol is majorized by the residual state $|\xi\rangle$ of the greedy protocol, *i.e.*, $\lambda_\nu \prec \lambda_\xi$



[1] M. A. Nielsen, Conditions for a Class of Entanglement Transformations, Phys. Rev. Lett. 83, 436 (1999).
[2] G. Vidal, Entanglement of Pure States for a Single Copy, Phys. Rev. Lett. 83, 1046 (1999).