Conference talk in Lille, France, November 8, 2023

Weak Values and pseudo-distributions

QuiDiQua Conference

<u>Andrew N. Jordan</u>

Chapman University
+ Univ. Rochester

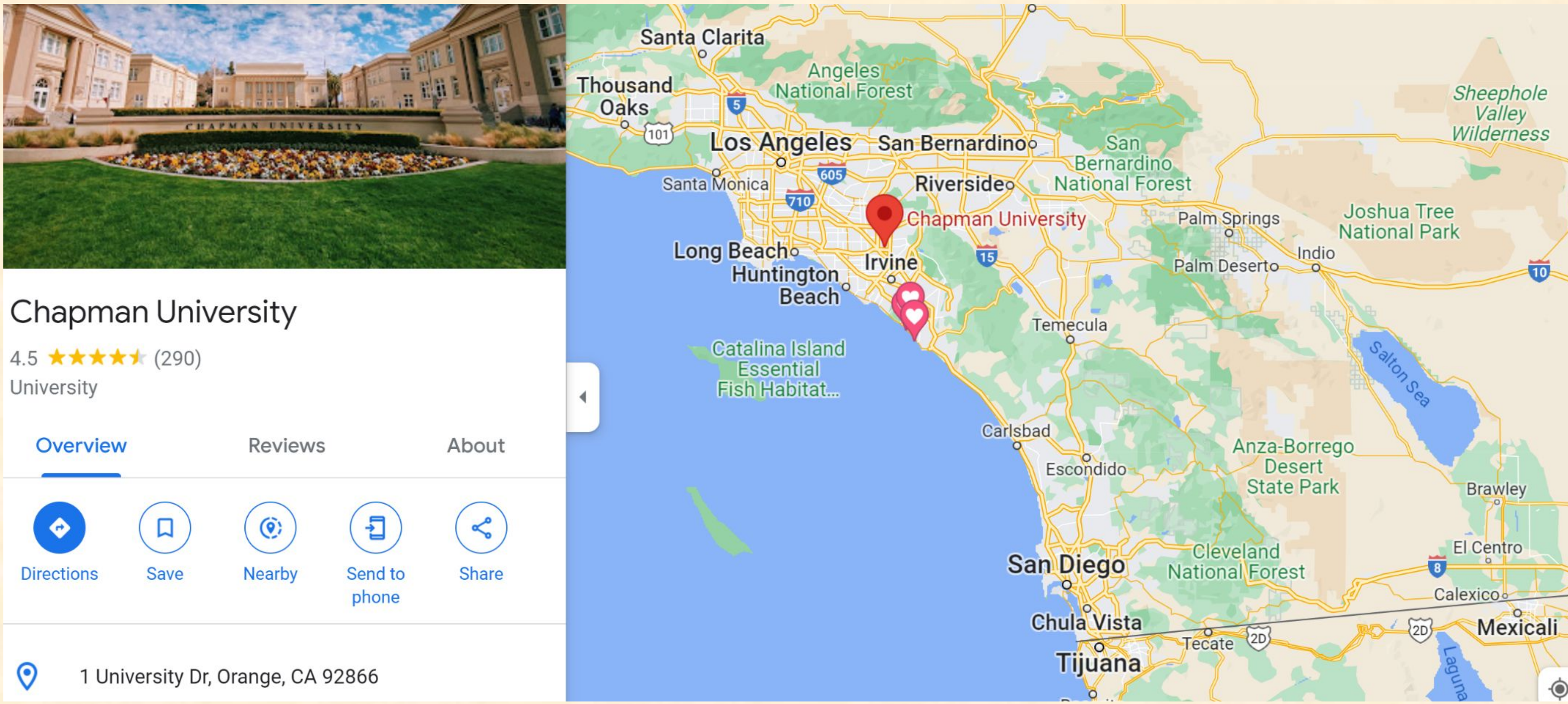
Funding for this work:

National Science Foundation
WHERE DISCOVERIES BEGIN

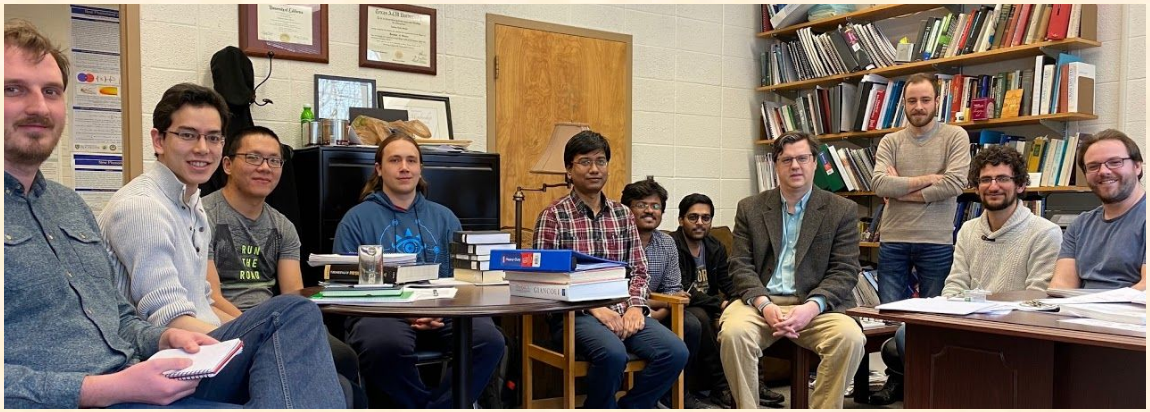
SCIENCE TO SHAPE THE FUTURE OF THE ARMY
ARMY RESEARCH OFFICE
1951

Where I am from

Where is Chapman?

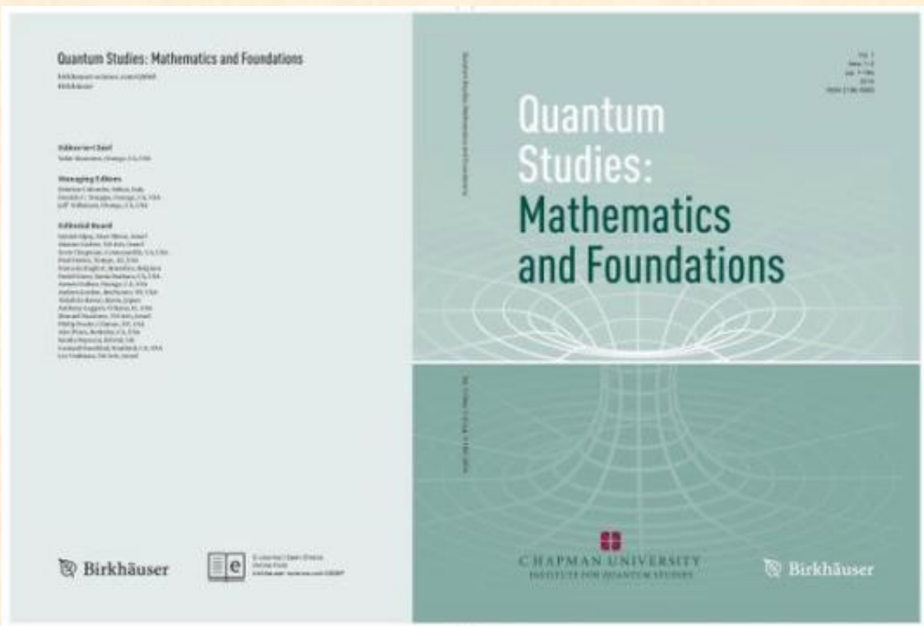


Group photo
- 2020



Group photo
- 2022

Plug for Quantum Studies submissions



- Journal dedicated to quantum physics
- Quantum Foundations contributions welcome

Plug for my forthcoming book (fall 2023)

Quantum Measurement:
Theory and Practice

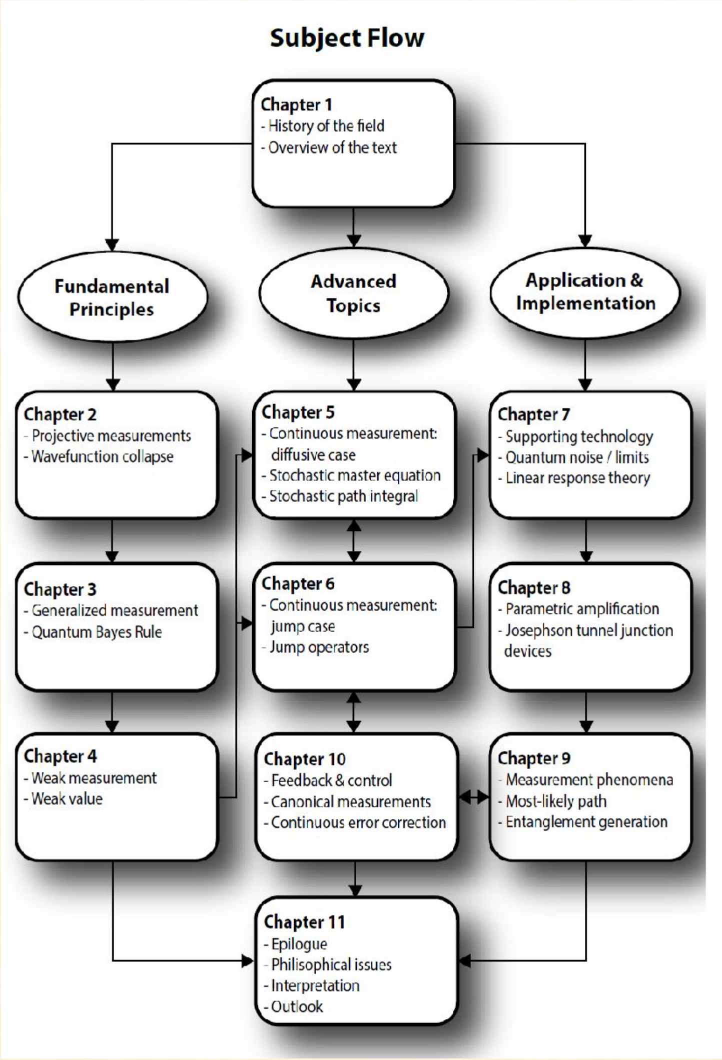
Andrew N. Jordan & Irfan Siddiqi

CAMBRIDGE UNIVERSITY PRESS

Laguna Beach, CA *May 13, 2022*

Table of Contents

- How long does a measurement take?
- Can a measurement be reversed?
- Is it possible to track the quantum wavefunction collapse in time?
- How can one entangle separated objects via measurement?
- When does a state jump versus diffuse?
- What is the most likely path of a quantum trajectory?
- How can one describe joint unitary and non-unitary processes as a dynamical system?
- What limits does quantum mechanics put on amplification?
- How does one build a quantum limited amplifier?



In collaboration with ….

Irfan Siddiqi + group

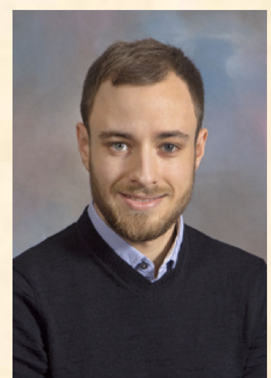
Benjamin Huard

Phil Lewalle

Areeya Chantasri

Kater Murch + group

Pierre Rouchon

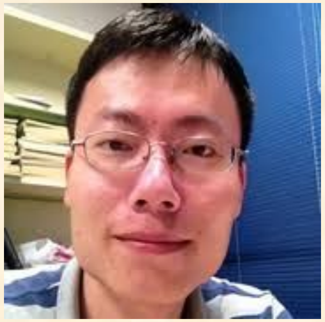


John Steinmetz

Justin Dressel

More Collaborators

Kevin Lyons



Shengshi Pang

John Howell

Courtney Byard

Paul Kwiat

Jaime Cardenas

Overview

- Weak Values & Pseudo-distributions
  - Brief review of AAV's idea
  - Connection to pseudo-distributions
  - Physical Implementations
  - Connection the Leggett-Garg inequality
  - Contextual values as a way to get back to real probabilities
  - Weak values as a novel amplification technique
  - Beam deflection & Phase detection

- Recent advances
  - Weak value amplification on chip

Weak value definition

Aharanov, Albert, and Vaidman (1988)

$$A_w = \frac{\langle \psi_f | \mathbf{A} | \psi_i \rangle}{\langle \psi_f | \psi_i \rangle}$$

"How the Result of Measurement of a Component of the Spin of a Spin- 1/2 Particle Can Turn Out to Be 100"

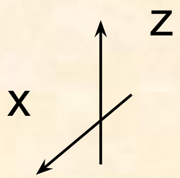
Ingredients:

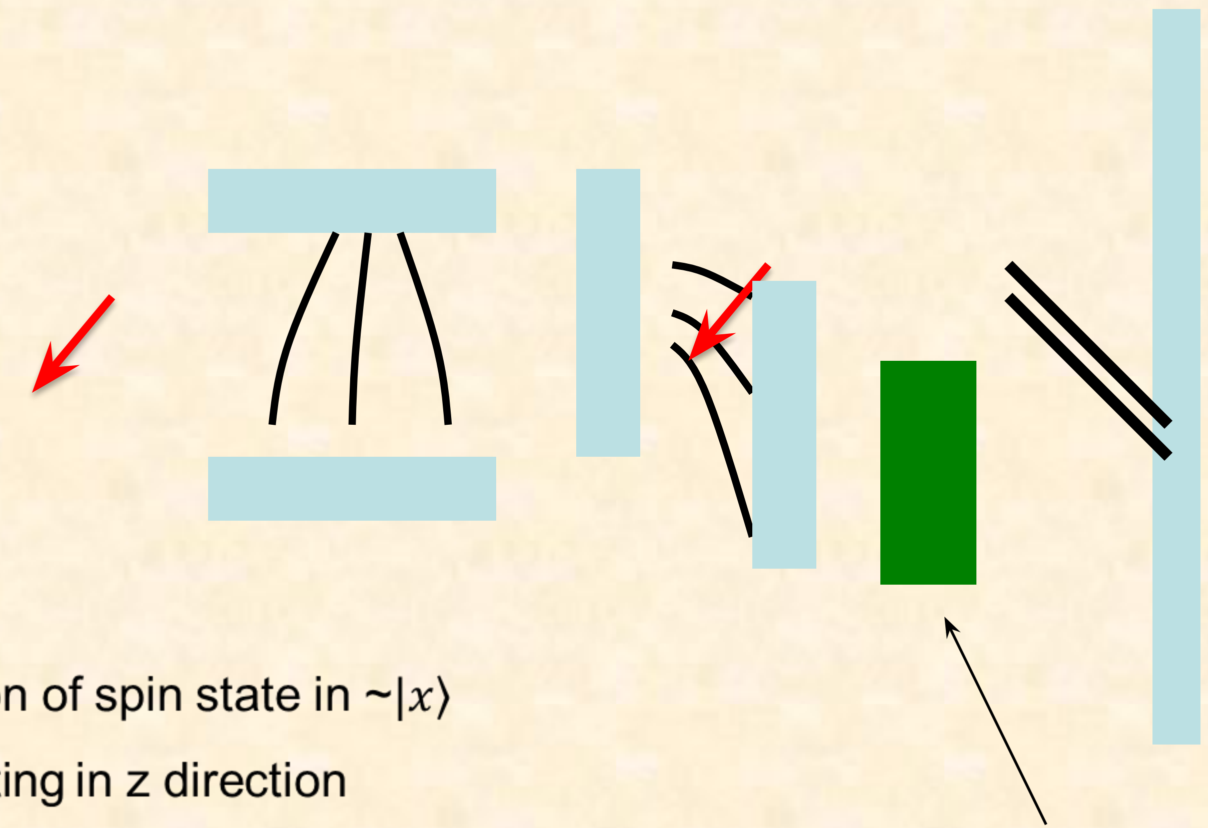
1) Pre-selection of system
2) Weak measurement w/ a meter
3) Post-selection of system
4) Measure (conditioned) meter shift.

Properties:

1) Time symmetric
2) Formally similar to the expectation value
3) Can exceed the eigenvalue range
4) Generally complex

Stern-Gerlach



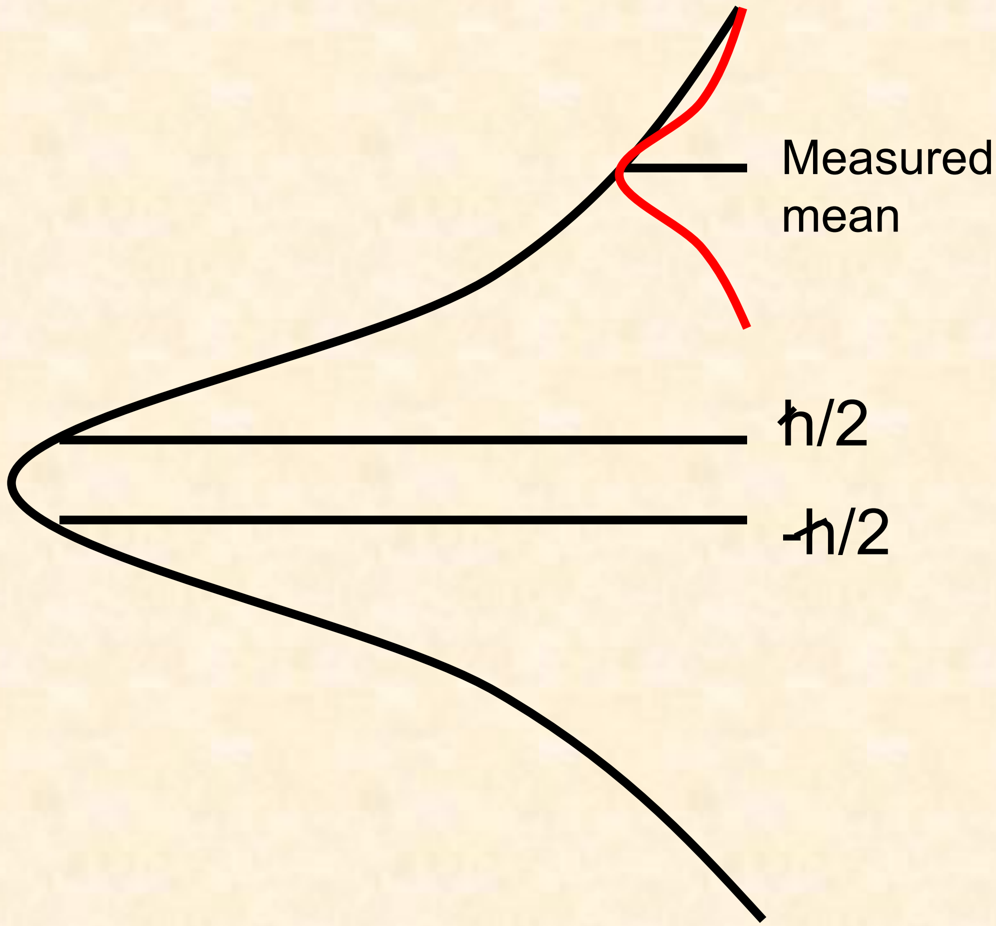


Contains

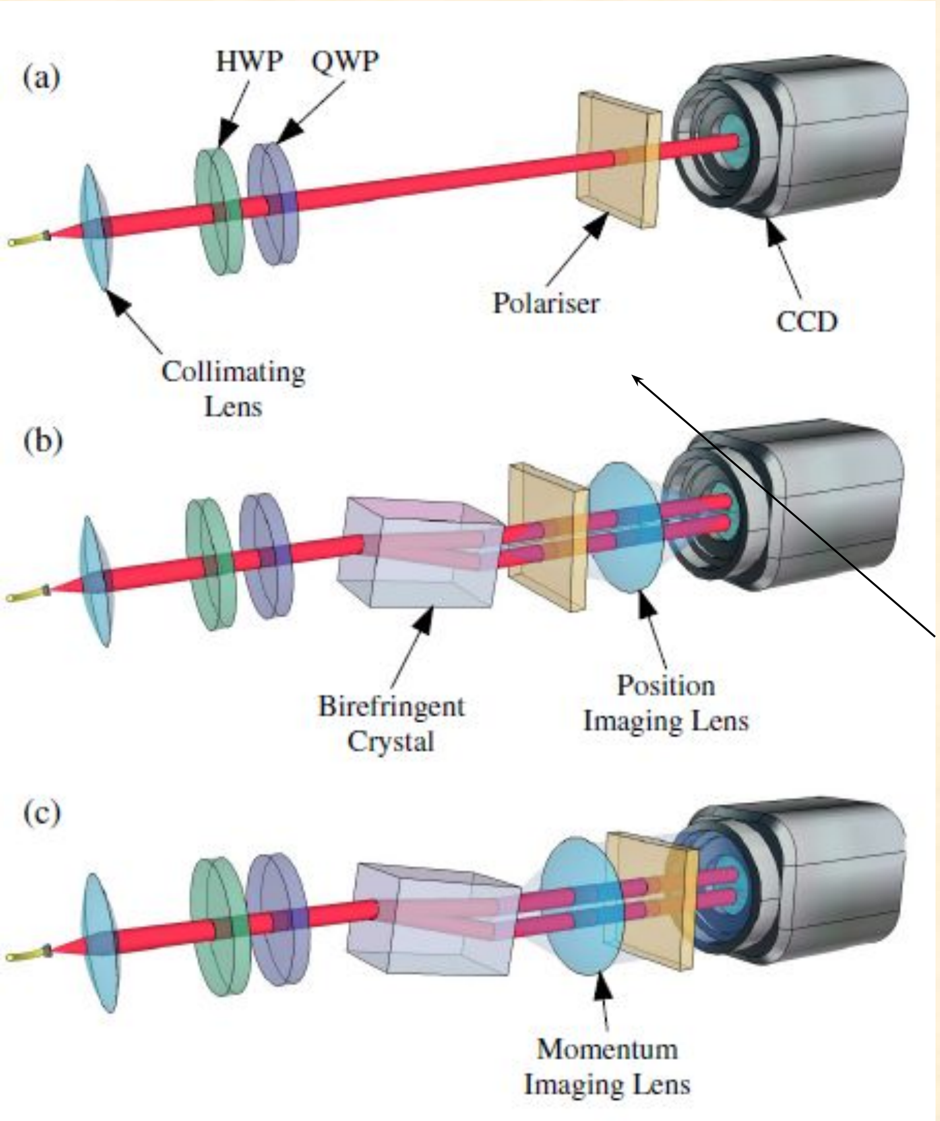
- Preselection of spin state in ~$|x\rangle$
- Weak splitting in z direction
- Strong splitting in x direction
- Postselection in x direction (or spin state)
- Recording of the z deflection

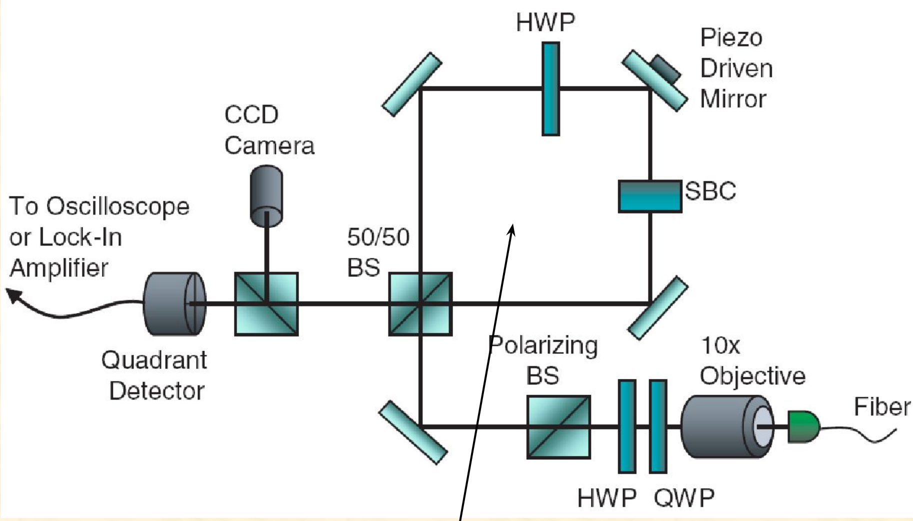
After Postselection

$$\frac{\langle \psi_{out} | A | \psi_{in} \rangle}{\langle \psi_{out} || \psi_{in} \rangle}$$



(Optical) Realization of the weak value





Polarization based

Interference based

Rev. Mod. Phys. 86, 307 (2014)
Justin Dressel, Mehul Malik, Filippo M. Miatto, Andrew N. Jordan, Robert W. Boyd

Naturally related to Kirkwood-Dirac quasi-distribution

$$\hat{\rho} = \sum_{i,j} \hat{\Lambda}_{a_i,b_j} Q_{i,j}(\hat{\rho}) .$$

$$Q_{i,j}(\hat{\rho}) = \langle b_j | a_i \rangle \langle a_i | \hat{\rho} | b_j \rangle .$$

$$\hat{A} = \sum_i a_i \, |a_i\rangle\langle a_i| ,$$

$$\hat{B} = \sum_j b_j \, |b_j\rangle\langle b_j| ,$$

$$\hat{\Lambda}_{a_i,b_j} = |a_i\rangle \langle b_j| / \langle b_j | a_i \rangle$$

$$A_w(\psi, b_{j^\star}) = \sum_i a_i \frac{Q_{i,j^\star}(\psi)}{P(b_{j^\star}|\psi)} = \sum_i a_i \tilde{Q}_{i|j^\star}(\psi).$$

Thus, the weak value is an average of the eigenvalues of A under a conditional quasiprobability distribution -> must have negativity for it to exceed the eigenvalues. ANJ+JD+DAS+et al, in preparation.

Naturally related to Kirkwood-Dirac quasi-distribution

Moreover, the full KD distributional representation for a quantum state can be decomposed into conditional quasiprobabilities,

$$Q_{i,j}(\psi) = \tilde{Q}_{i|j}(\psi) P(b_j|\psi),$$

… and ensemble averages of weak values recover expectation values.

$$\sum_j P(b_{j^\star}|\psi)\, A_w(\psi, b_{j^\star}) = \sum_j \langle\psi|b_{j^\star}\rangle\langle b_{j^\star}|\, \hat{A}\, |\psi\rangle = \langle\psi|\, \hat{A}\, |\psi\rangle\,,$$

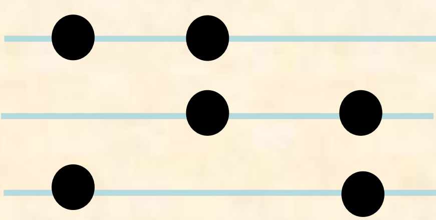
Connection to the Leggett-Garg inequality

An interesting argument testing the limits of macroscopic coherence and notions of “quantumness" was formulated by Leggett and Garg (1985).

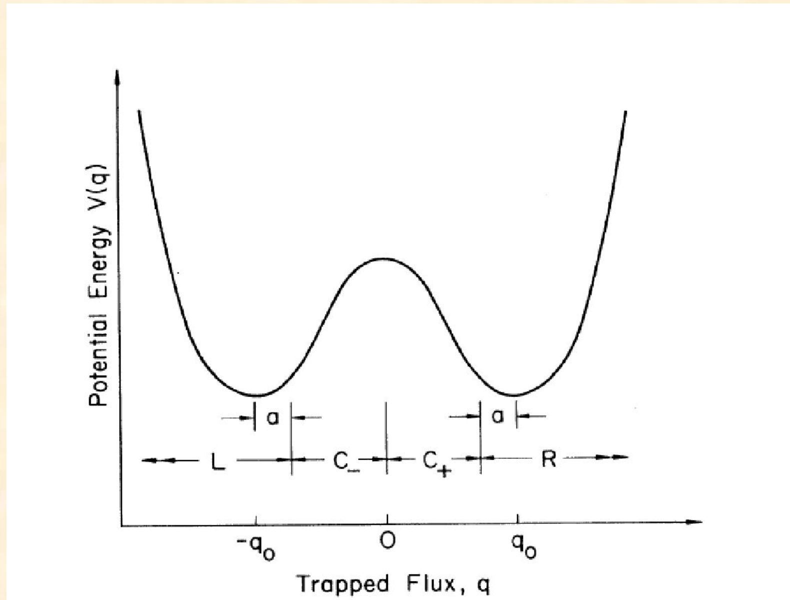
- Quantum Mechanics versus Macroscopic Realism: Is the Flux There when Nobody Looks?
- (A1) Macroscopic realism: A macroscopic system with two or more macroscopically distinct states available to it will at all times be in one or the other of these states.
- (A2) Noninvasive measurability at the macroscopic level: It is possible, in principle, to determine the state of the system with arbitrarily small perturbation on its subsequent dynamics.
- Idea – try and find test of these assumptions.

Proposed Experiment – SQUID loop

We define a quantity Q, which equals + 1 (-1) if the system is observed to be in region R (L).



Time 1 Time 2 Time 3



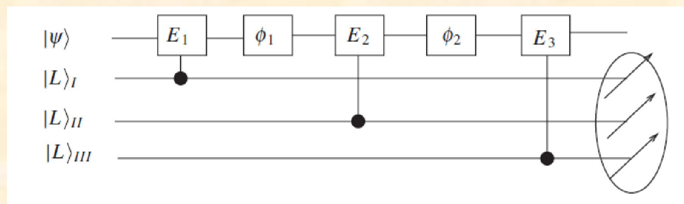
$$1 + K_{12} + K_{23} + K_{13} \geq 0,$$

$$|K_{12} + K_{23}| + K_{14} - K_{24}| \leq 2.$$

We can define (i) joint probability densities p(Q1,Q2), p(Q1,Q2,Q3), etc. for Q to have the values Q1 at times t1 (we take t0 < t1 < t2 • • .) , (ii) correlation functions $K_{ij} = < Q_i Q_j >$. Applying the assumptions A1 and A2 give inequalities.

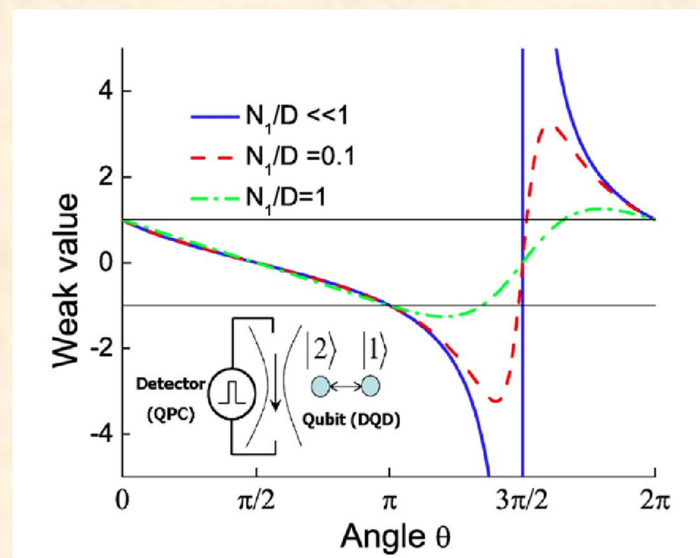
New idea – ANJ et al.

- Do all the measurements at once with weak measurements. PRL 97, 026805 (2006)

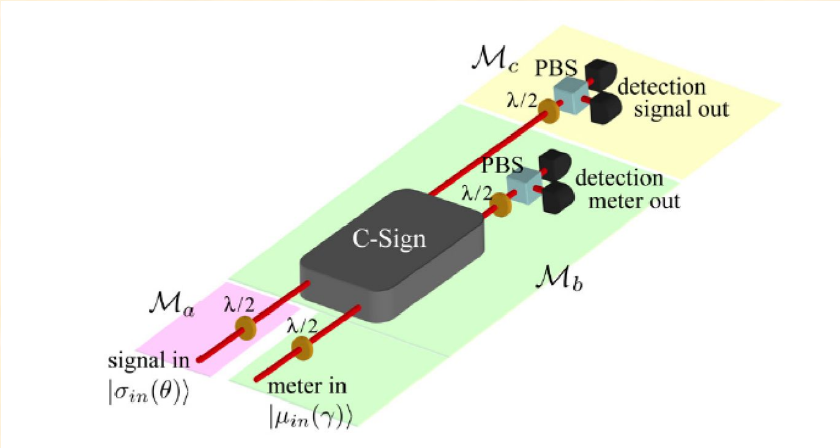


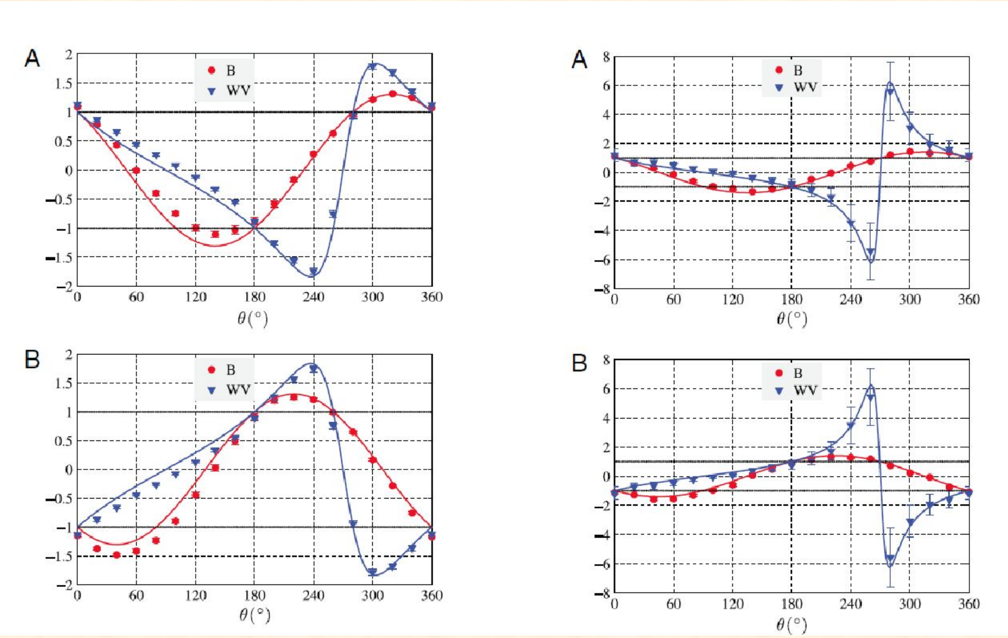
- Violation of generalized LGIs the same as strange weak values. PRL 100, 026804 (2008).

$$B = \langle \mathscr{M}_a \mathscr{M}_b \rangle + \langle \mathscr{M}_b \mathscr{M}_c \rangle - \langle \mathscr{M}_a \mathscr{M}_c \rangle,$$



Experiments

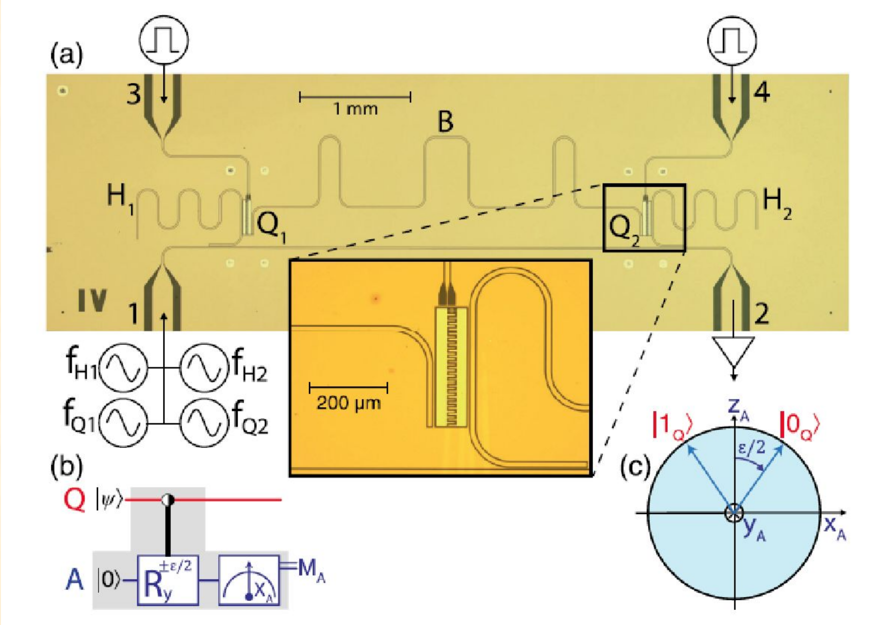


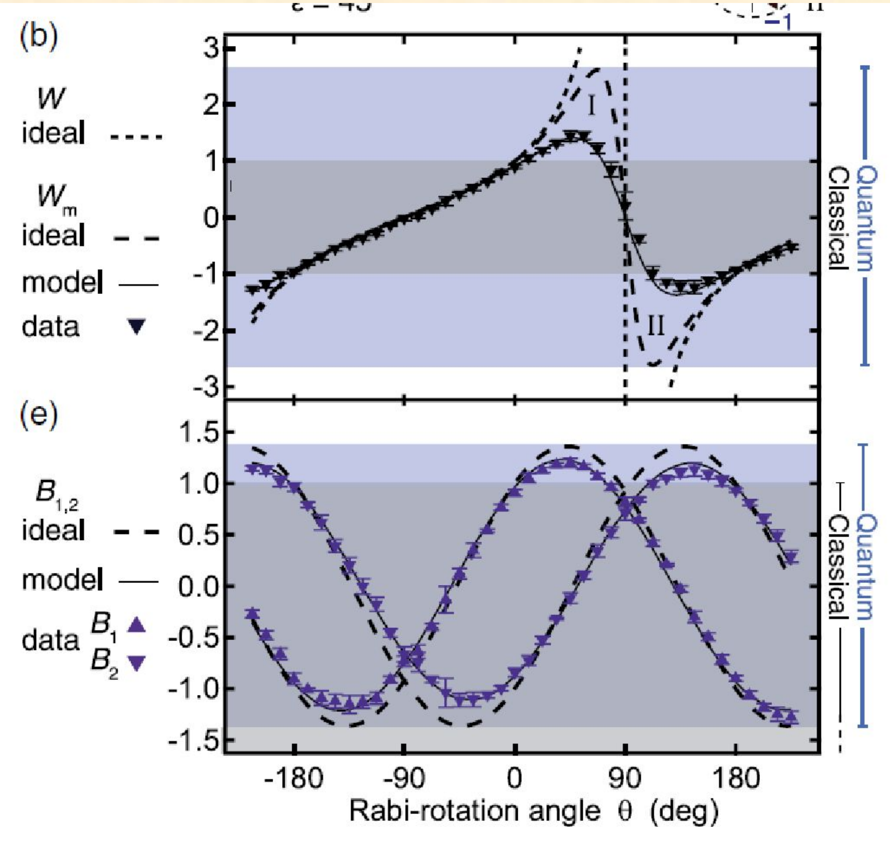


Goggin et al., PNAS (2010)

Superconductors

Groen et al., PRL 111, 090506 (2013)





Can reexamine this effect as a manifestation of a quasi-distribution

The three outcomes are r_1; r_2; r_3, and
the correlation functions $K_{ij} = \langle r_i r_j \rangle$ are considered.

$$\hat{A} = |a_1\rangle\langle a_1| - |a_2\rangle\langle a_2|,\ \hat{B} = |b_1\rangle\langle b_1| - |b_2\rangle\langle b_2| \text{ and } \hat{C} = |c_1\rangle\langle c_1| - |c_2\rangle\langle c_2|$$

3 Dichotomic variables with eigenvalues +1, -1 each.
Generalized Leggett-Garg inequality:

$$\mathcal{L} = K_{12} + K_{23} - K_{13} < 1.$$

Use three observable KD distribution, Q= $\langle c_k|b_j\rangle\langle b_j|a_i\rangle\langle a_i|c_k\rangle$

Probability P of finding outcome a_i = $P(a_i|\psi) = |\langle a_i|\psi\rangle|^2$

Can reexamine this effect as a manifestation of a quasi-distribution

$$\mathcal{L} = K_{12} + K_{23} - K_{13} < 1.$$

$$\mathcal{L} = \sum_{i,j,k} Q_{j,k}(|a_i\rangle\langle a_i|)P(a_i|\psi)[a_i b_j + b_j c_k - a_i c_k].$$

The term in brackets has an upper bound of 1, so once again we see that in order for the right-hand side to exceed the upper bound of 1, as observed experimentally, **Q must exhibit negativity**.

Connect to Weak Value

$$\mathcal{L} = \sum_{i,k} P(c_k|a_i)P(a_i|\psi)[(a_i + c_k)B_w(a_i, c_k) - a_i c_k].$$

Conditional probability

Weak value of B.

We see that the LGI must be bounded by +1 if a and c both take +1, and B is its best “classical value”, +1. So one can prove in this case that the LGI is violated if and only if the weak value takes anomalous values.

Thus, in order to violate the Leggett-Garg inequality's upper bound of 1 as observed experimentally, at least one of two specic KD quasiprobabilities, must become negative, each case causing a corresponding weak value, to violate its eigenvalue bounds.

Intermediate Conclusions

- The weak value becoming anomalous can be seen as a manifestation of the negativity of a KD pseudo-distribution
- The violation of the generalized LGI can also be seen as a manifestation of the negativity of the KD pseudo-distribution.
- In the case we have discussed, the LGI is violated at the same points where the weak value becomes anomalous.

- Additional assumptions/argumentation needed to rule out classical models (invasive detectors, clumsiness, etc.)

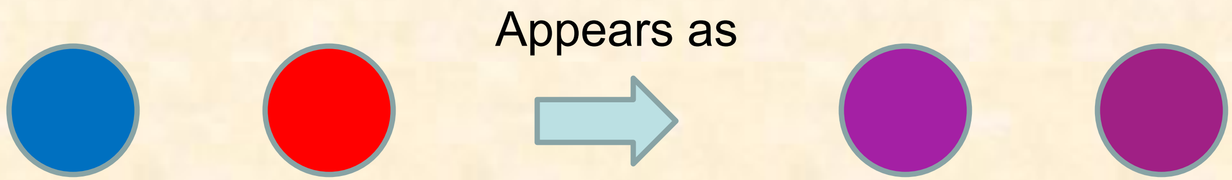
Another way to do all of this with true probabilities!

Contextual values

- Different theoretical approach to weak values – *contextual values (or generalized eigenvalues)*.
- Go beyond thinking of an observable in terms of its eigenvalues, and interpret the measurement results within their own context.
- We derive a generalization of the AAV formula applicable to arbitrary strength measurements, mixed states, and POVM postselections in terms of weighted averages of the contextual values.
- Resolves many of the paradoxical features of WVs.
- Recovers other known specific results in literature.
- Closer connection of WVs to POVM formalism.

J. Dressel, S. Agarwal, A. N. Jordan, Phys. Rev. Lett. 104, 240401 (2010)

Contextual values – basic idea



Find the color distribution of a jar of marbles, if you are nearly colorblind.

You know that you guess blue (b) correctly 51% of the time, and red (r) correctly 51% of the time. Write “thumbs up” (u) if you think it is blue and “thumbs down” (d) if you think it is red.

Assign numbers: r $\Rightarrow$ -1, b $\Rightarrow$ 1, but different numerical values for u $\Rightarrow$ a and d $\Rightarrow$ b. **Which?**

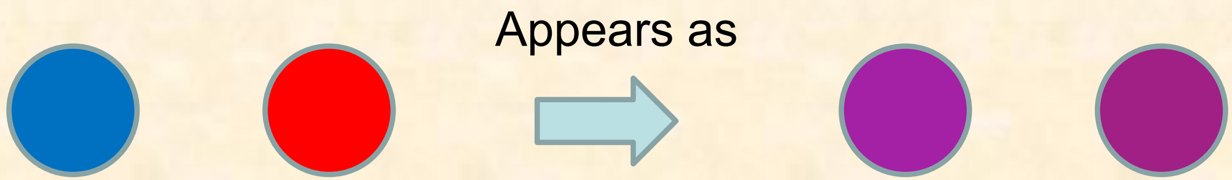
We can find the color average 1 P(b) – 1 P(r) = a P(u) + b P(d).

$$\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} .51 & .49 \\ .49 & .51 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix}$$

Works regardless what P(r) and P(b) are!!! Thus, we can choose a = 50, b = -50 to get the right average. Amplification of eigenvalue range!

Context is the precise degree of color-blindness; **Contextual values** for color are 50, -50. Multiple ways to measure same observable!

Contextual values – basic idea



We can now consider averages conditioned on a later event f:

${}_f\langle \text{color} \rangle = a\,P(u|f) + b\,P(d|f)$,

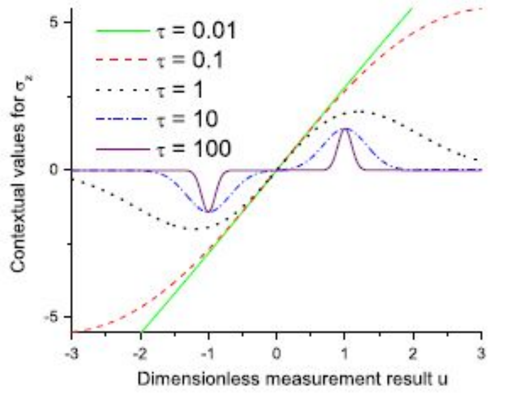
In the usual way – no funny negative probabilities. P(u|f) and P(d|f) are simply conditional probabilities of actual events you have access to.

In classical physics, this turns out to be between -1, 1 – whereas in QM, the analogous calculations can have interference, enhancing one CV over to other to exceed the eigenvalue range.

Contextual Values – more

A measurement context is given by the kind of experiment that is being done, represented formally by a Positive operator valued measure (POVM)

$$\mathfrak{E} = \{\hat{\boldsymbol{E}}_j = \hat{\boldsymbol{M}}_j^\dagger \hat{\boldsymbol{M}}_j\}$$



Together with the density matrix, this gives the probabilities of different things happening.

$$P_j = \mathrm{Tr}[\hat{\boldsymbol{E}}_j \hat{\boldsymbol{\rho}}]$$

The key idea is to expand the measured observable in terms of the POVM elements, with coefficients that generalize the eigenvalues of A.

$$\hat{\boldsymbol{A}} = \sum_j \alpha_j \hat{\boldsymbol{E}}_j = \sum_k a_k \hat{\boldsymbol{\Pi}}_k$$

From this, it is possible to reconstruct averages of the operator moments, or to find conditioned averages in the weak limit:

$$A_w = \mathrm{Tr}[\hat{\boldsymbol{E}}_f^{(2)} \{\hat{\boldsymbol{A}}, \hat{\boldsymbol{\rho}}\}] \,/\, 2\mathrm{Tr}[\hat{\boldsymbol{E}}_f^{(2)} \hat{\boldsymbol{\rho}}],$$

Contextual Values…

- Show how any measurable conditioned average is always real.
- Resolves the negative probability “problem” by expanding the eigenvalue range to the contextual value range.
- Allows a proof of the uniqueness of a WV under certain reasonable conditions (in the weak limit).
- Allows a principled generalization of conditioned averages to mixed initial state, post-selection on generalized measurements and arbitrary strength measurements.

Long paper: Dressel and Jordan, PRA, 85, 022123, (2012)

Conclusions

- Weak measurements
- Weak values
- Pseudo-Distributions
- Leggett-Garg inequality
- Contextual values

- See forthcoming review on the subject of the KD pseudo-distribution with David AS + Justin + many people in the room.