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Nonclassicality and genuine non-Gaussianity of photon-added/subtracted Gaussian states

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in collaboration with S. De Bièvre

National Research Council of Canada

Introduction

- We need non-Gaussian states to perform certain quantum information tasks.

- Non-Gaussianity is however not always enough
  $\Rightarrow$ Nonclassicality or the stronger property of genuine non-Gaussianity are needed

- One way of creating non-Gaussian states: photon addition/subtraction

- We will analyse their nonclassicality and genuine non-Gaussianity

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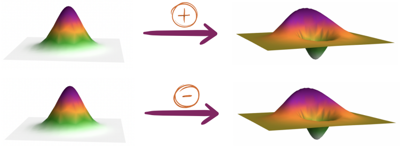

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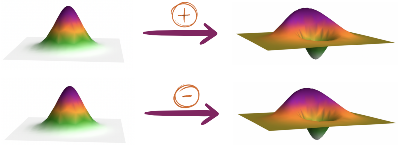

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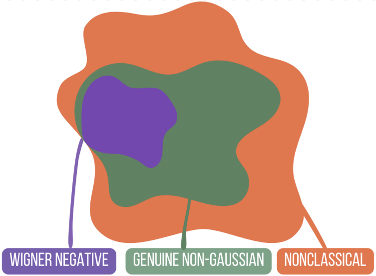
Definitions



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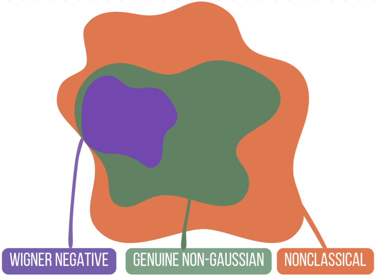

A state ρ of a bosonic field is classical if it is a mixture of coherent states $|\alpha\rangle$ (P = Glauber-Sudarshan P-function):

$$\rho = \int_{\mathbb{C}} P(\alpha)|\alpha\rangle\langle\alpha|\mathrm{d}\alpha, \quad \text{with} \quad P(\alpha) \geq 0, \int_{\mathbb{C}} P(\alpha)\mathrm{d}\alpha = 1.$$

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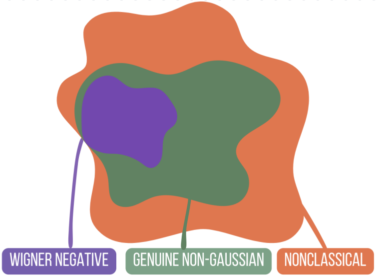

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A state ρ of a bosonic field is genuine non-Gaussian if it cannot be written as a mixture of Gaussian states ρ^G, i.e. if

$$\rho \notin \mathcal{G} \quad \text{where} \quad \mathcal{G} := \left\{ \rho \mid \rho = \int F(\gamma)\rho^G(\gamma)\mathrm{d}\gamma \right\}$$

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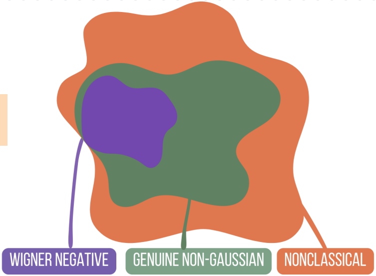
For example: $\frac{1}{2}\left(|0\rangle\langle 0| + |\alpha\rangle\langle\alpha|\right)$ is non Gaussian but in $\mathcal{G}$



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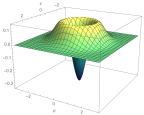
For **Nonclassicality**

- Quadrature Coherence Scale
- Wigner negative volume



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General: $\mathcal{C}^2(\rho) = \frac{1}{4n}\frac{\|\nabla W\|_2^2}{\|W\|_2^2}$

Pure state: $\mathcal{C}^2(|\psi\rangle) = \frac{1}{n}\sum_{i=1}^{n}((\Delta X_i)^2 + (\Delta P_i)^2)$

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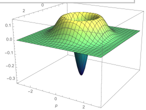

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Witness: $\mathcal{C}^2(\rho) > 1$ $\Rightarrow$ ρ is nonclassical

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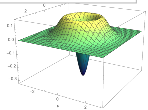

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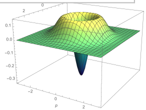

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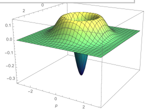

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- if $W(0) \leq \frac{2}{\pi} \mathrm{e}^{-2\bar{n}(1+\bar{n})}$

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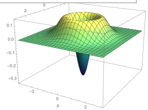

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NOTE: all this can be computed if one knows $W(\alpha)$.

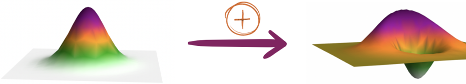
⇒ We provide explicit and general expressions for $\chi(\xi)$ and $W(\alpha)$ of photon added/subtracted Gaussian states

The questions

To which extent photon added/subtracted states are nonclassical and/or genuine non-Gaussian ?

How does their nonclassicality/ genuine non-Gaussianity compare to the one of the original Gaussian state?

Result: photon-addition



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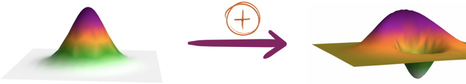

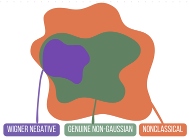

Any Gaussian state (classical or not) transforms into a Wigner negative state and hence nonclassical and quantum non-Gaussian.

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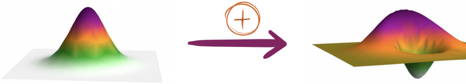

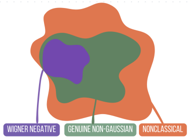

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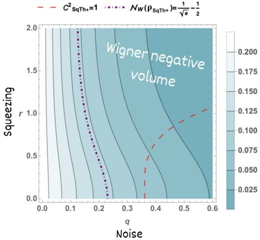

Wigner negative volume

→ is highest for $a^\dagger S(z)|0\rangle$.

→ is sensitive to noise

→ decreases with increased squeezing

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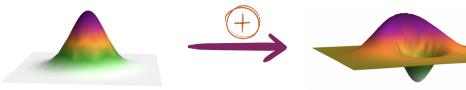

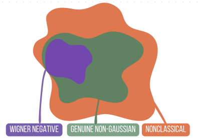

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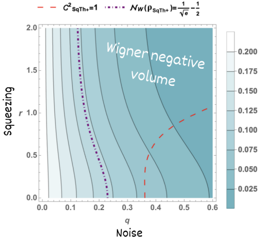

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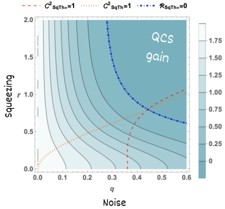
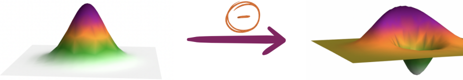


QCS gives more info about nonclassicality, but what is really interesting is the gain

- ✓ QCS can increase up to 200%
- × but become very sensitive to decoherence

Result: photon-subtraction



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- We proved: photon subtraction always transforms a Gaussian nonclassical state into a nonclassical state.

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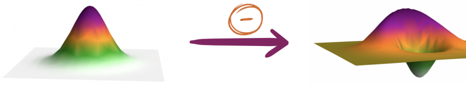

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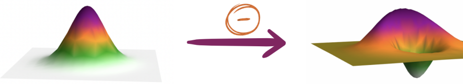

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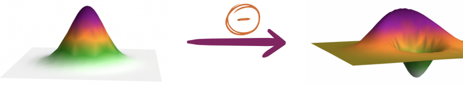

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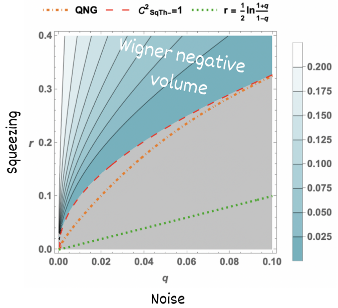

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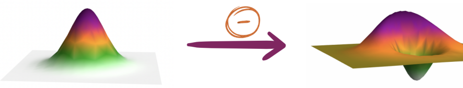

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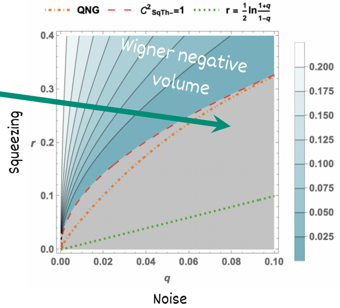

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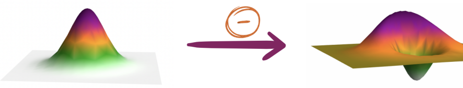

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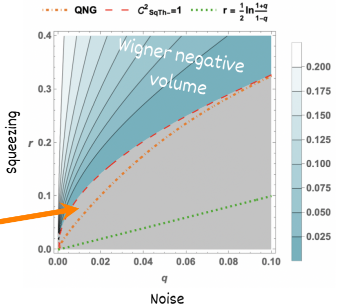

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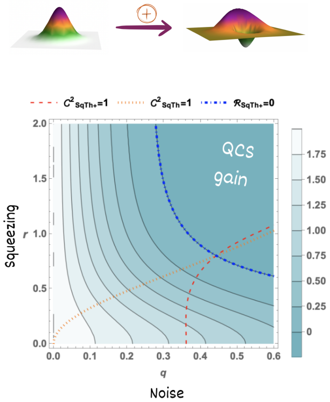

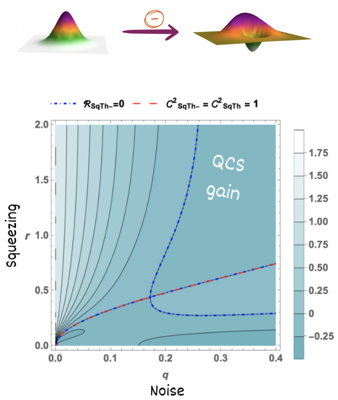

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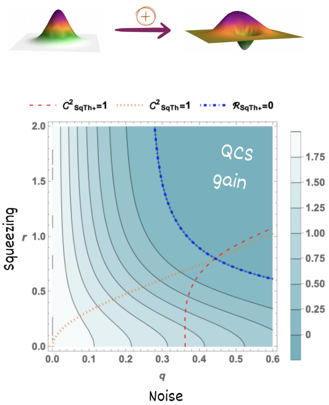

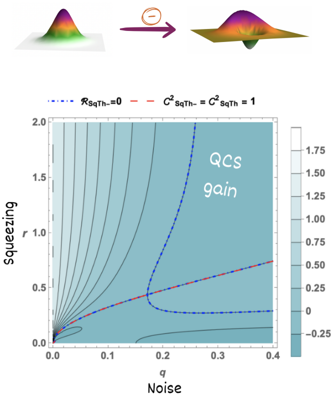

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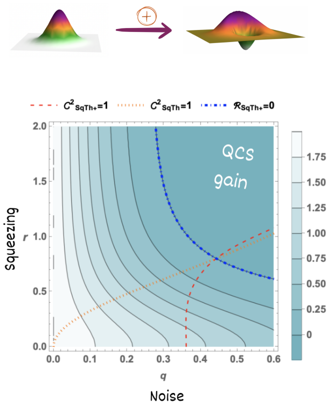

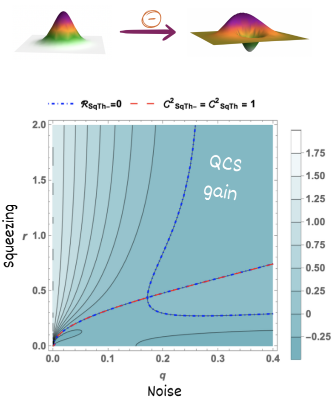

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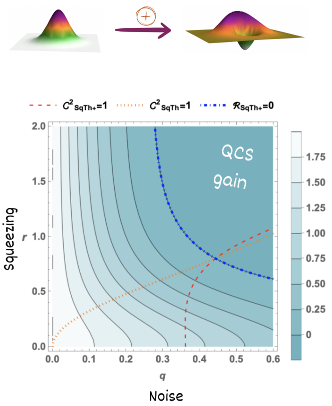

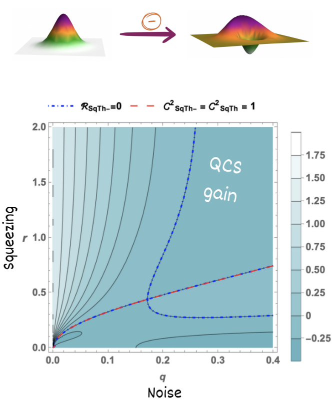

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$\Rightarrow$ The computation of $\mathcal{C}^2(\rho) = \frac{1}{4n}\frac{\|\nabla W\|_2^2}{\|W\|_2^2}$ is done without the need of integration. Result simply written in terms of c_m and parameters of the Gaussian functions.

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$$G_{\boldsymbol{\mu}_m,\gamma_m}(\boldsymbol{r}) = \frac{\exp[-\frac{1}{2}(\boldsymbol{r}-\boldsymbol{\mu}_m)^T\gamma_m^{-1}(\boldsymbol{r}-\boldsymbol{\mu}_m)]}{\sqrt{\det(2\pi\gamma_m)}}.$$

can also be a complex (normalized) function

$\Rightarrow$ The computation of $\mathcal{C}^2(\rho) = \frac{1}{4n}\frac{\|\nabla W\|_2^2}{\|W\|_2^2}$ is done without the need of integration. Result simply written in terms of c_m and parameters of the Gaussian functions.

We can then compute the QCS for

- GKP states
- States in a loss channel
- Output state of a breeding protocol
- etc.

Teaser

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For more details

Go check our paper

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Decoherence and nonclassicality of photon-added and photon-subtracted multimode Gaussian states

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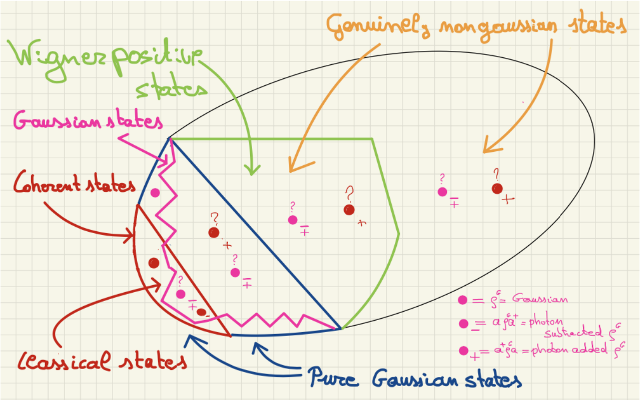
Wigner positive states
Genuinely nongaussian states
Gaussian states
Coherent states
Classical states
Pure Gaussian states
= ρ^G = Gaussian
$_-$ = $a\rho^G a^+$ = photon subtracted ρ^G
$_+$ = $a^+\rho^G a$ = photon added ρ^G