Phase-space inequalities: quantum correlations in phase space

Elizabeth Agudelo

$$\hat{\rho} = \int d^{2n}\boldsymbol{\alpha}\; P(\boldsymbol{\alpha})\, |\boldsymbol{\alpha}\rangle\langle\boldsymbol{\alpha}|$$

- PRL 124, 133601 (2020).
- Quantum 4, 343 (2020).
- PRL 126, 023605 (2021).

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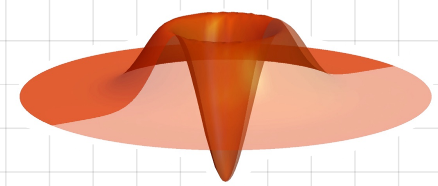
Coherent states

probability distribution
Classical

attain negativities!
Nonclassical



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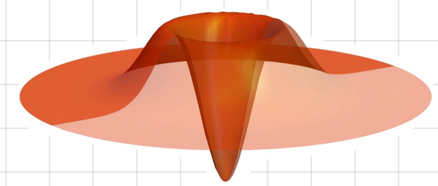

PROPERTIES

- $\int d^2\alpha \, P(\alpha) = 1$
- $\int d^2\alpha \, P(\alpha) \, f(\alpha, \alpha^*) = \langle : f(\hat{a}, \hat{a}^\dagger) : \rangle$

S-PARAMETRIZED QUASIPROBABILITIES

$$P(\alpha, s) = \frac{2}{\pi(1-s)} \int d^2\gamma \, P(\gamma) \, e^{-\frac{2|\alpha-\gamma|^2}{(1-s)}}$$

Husimi — Wigner — Glauber-Sudarshan

s: -1, 0, 1

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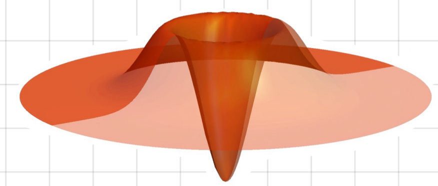

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Husimi: $s = -1$; Wigner: $s = 0$; Glauber-Sudarshan: $s = 1$

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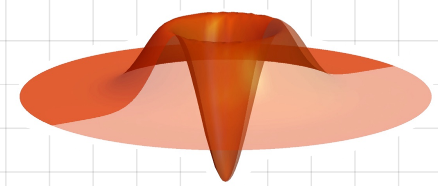

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Husimi | Wigner | Glauber-Sudarshan

-1 | 0 | 1 | s

$$\langle : \hat{f}^\dagger \hat{f} : \rangle = \int d^2\alpha \; P(\alpha) \, \overbrace{|f(\alpha,\alpha^*)|^2}^{\text{nonnegative}}$$

$$\boxed{\langle : \hat{f}^\dagger \hat{f} : \rangle < 0}$$

NCL certified!

NCL witness

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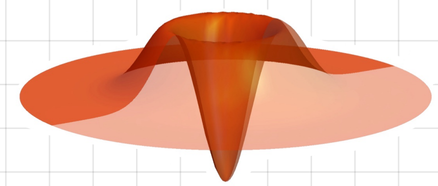

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Husimi, Wigner, Glauber-Sudarshan

s: -1 (Husimi), 0 (Wigner), 1 (Glauber-Sudarshan)

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$$\hat{f} = \sum_i c_i \, e^{-\frac{2\hat{n}_i(\alpha_i)}{(1-s_i)}}$$

infinite hierarchy of inequalities

- $\frac{\pi}{2\sigma} P(\alpha; 2\sigma) < 0$
- $P(\alpha, 2\sigma) - \frac{2\pi}{\sigma} [P(\alpha; \sigma)]^2 < 0$
- $P(\alpha_1; 2\sigma_1) P(\alpha_2; 2\sigma_2) - \frac{4\tilde{\sigma}}{\Sigma} \left[e^{-\tilde{\sigma}|\alpha_2 - \alpha_1|^2} P(A; \Sigma) \right]^2 < 0$
- + Multimode

$* \; \sigma = \frac{2}{1-s}$

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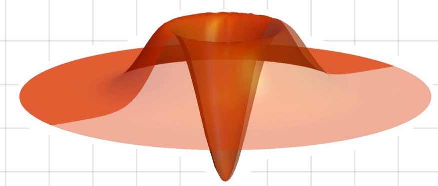

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infinite hierarchy of inequalities

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Original definition !!

- $\frac{\pi}{2\sigma} P(\alpha; 2\sigma) < 0$
- $P(\alpha, 2\sigma) - \frac{2\pi}{\sigma} [P(\alpha;\sigma)]^2 < 0$
- $P(\alpha_1; 2\sigma_1) P(\alpha_2; 2\sigma_2) - \frac{4\tilde{\sigma}}{\Sigma} \left[e^{-\tilde{\sigma}|\alpha_2-\alpha_1|^2} P(A;\Sigma) \right]^2 < 0$
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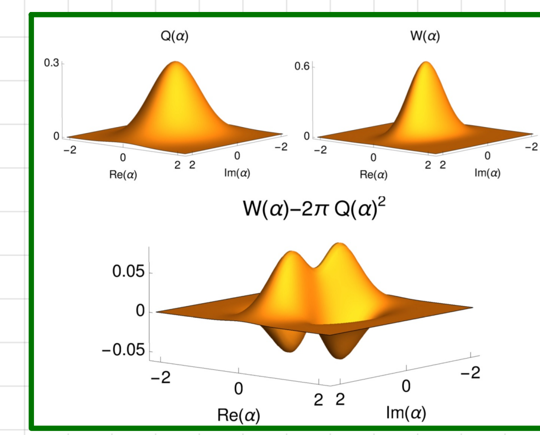
SQUEEZED STATES



$$W(\alpha) - 2\pi Q(\alpha)^2 < 0$$

$$W(\alpha_1) W(\alpha_2) - e^{-|\alpha_2-\alpha_1|^2} [W(\alpha_1+\alpha_2)/2]^2 < 0$$

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