Geometry of the Kirkwood-Dirac-positive states

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The Kirkwood-Dirac (KD) distribution is a quasiprobability distribution, meaning that it has all the properties of a probability distribution except that it can take complex values. It has arisen recently as an adaptable tool in many quantum domains such as quantum thermodynamics, quantum foundations, and quantum thermodynamics [1, 5, 3, 2]. Given two observables, one can associate to each quantum state its KD distribution. We say a quantum state is KD nonpositive if its associated KD distribution is not a genuine probability distribution. As KD-nonpositive states have recently been linked to quantum phenomena in quantum metrology [5], it is of interest to identify such states. The complementary task is to identify the so called KD-positive states for which the KD distribution is a genuine probability distribution. This task is already known to be difficult for the Wigner function [6]. In this talk, we will study the geometry of the set of KD-positive states E+ KD. We will see that the simplest geometry, where the bases states are the only extreme points, occurs with probability 1 in dimension 2 and 3. In prime dimensions, this is also true for the Discrete Fourier Transform. However, we construct an example in dimension 3, which can be associated with a spin 1 measurement, where the geometry is much more complicated as there exist extreme mixed states [4].

References

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