Explicit and efficient estimators for Gaussian state parameters

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In this study, we employ the method of moments to construct explicit estimators for the parameters characterizing single-mode Gaussian states. Our approach harnesses homodyne data acquired through continuous phase scanning of the local oscillator, enabling efficient parameter estimation. We demonstrate that our estimators reach the Cramer-Rao bound for homodyne measurements, obviating the necessity for computationally intensive methods like maximum likelihood estimation. Hence, this fast estimation of the state parameters can be done during the process of measurement.

The noise of the homodyne measurement of the single-mode Gaussian state can be expressed as

$$A(\psi, \vec{\theta}) = \mathrm{Tr}\left[\hat{X}_\psi 2\hat{\rho}(\vec{\theta})\right] = \kappa\left(s\cos^2[\phi_s - \psi] + s^{-1}\sin^2[\phi_s - \psi]\right) , \tag{1}$$

where $P = 1/\kappa$ represents the purity of the state, sss and ϕ_s correspond to the degree and direction of squeezing, and ψ denotes the phase of the local oscillator. To estimate these three parameters, denoted as $\vec{\theta} = (s, \kappa, \phi_s)$, one can sample the squared quadratures A_j with an oscilloscope for a set of phases ψ_j of the local oscillator. Starting with an initial guess $\vec{\theta}_0$ regarding the parameters of interest, optimal linear combinations Y_k of the measured data can be constructed as

$$Y_k = \frac{1}{N_\psi}\sum_j c_k(\psi_j, \vec{\theta}_0) , \quad \text{with} \quad c_k(\psi_j, \vec{\theta}_0) = \left(\frac{1}{2A^2(\psi_j, \vec{\theta})}\frac{\partial A(\psi_j, \vec{\theta})}{\partial\theta_k}\right)_{|\vec{\theta}=\vec{\theta}_0} . \tag{2}$$

From these linear combinations, one can estimate the parameters as

$$(\tilde{s}, \tilde{\kappa}, \tilde{\phi}_s) = \left(\sqrt{\left|\frac{Y_1 s_0(1+s_0) + Y_2\kappa_0}{Y_1(1+s_0) - Y_2\kappa_0}\right|}, 2\kappa_0\sqrt{\left|\frac{Y_1 s_0(1+s_0) + Y_2\kappa_0}{(Y_1(1+s_0) - Y_2\kappa_0)^{-1}}\right|}, \phi_{s_0} - \frac{Y_3}{Y_1(1-s_0^2)}\right) . \tag{3}$$

This procedure can be iteratively repeated multiple times, updating the initial guess $\vec{\theta}_0$. This approach eliminates the necessity for any prior information about the parameters. The method of moments not only yields estimators but also provides their respective variances. We analytically demonstrate that the accuracy of these estimators reaches the Cramer-Rao bound. Figure 1 showcases the results of the simulated data processing.

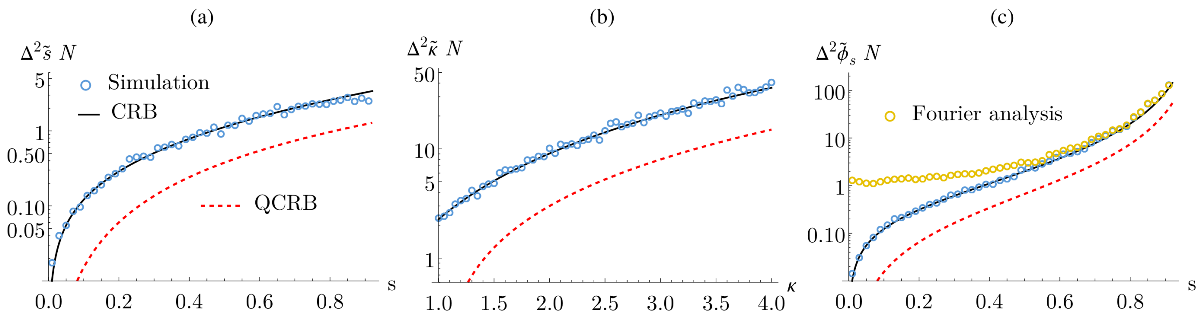

Figure 1: Square error of the estimators for the parameters s (a), κ (b) and ϕ_s (c). Points correspond to the square error of the simulated data reconstruction (reconstructed 500 data samples with $N = 1000$, $s = 0.6$, $\kappa = 1.4$, $\phi_s = 0.4$, unless another value is given on the plot axes). Black lines correspond to the Cramer-Rao bound, dashed red line to the quantum Cramer-Rao bound.