Entanglement is among the most fundamental - and at the same time puzzling - properties of quantum physics. Its modern description relies on a resource-theoretical approach, which treats any pair of entangled systems as a means to enable or accelerate certain informational tasks. Hence, it is of paramount importance to determine whether - and how - different entangled states can be converted into each other under free operations (those which do not create entanglement from nothing).

Here, we show that the majorization lattice provides an efficient framework in order to characterize the allowed transformations of pure entangled states under local operations and classical communication. The underlying notions of meet $\wedge$ and join $\vee$ in the majorization lattice lead us to define, respectively, the optimal common resource and optimal common product states. Based on these two states, we introduce two optimal probabilistic protocols for the (single-copy) conversion of incomparable bipartite pure states, which we name greedy and thrifty. Both protocols reduce to Vidal’s protocol [2] if the initial and final states are comparable, but otherwise the thrifty protocol can be shown to be superior to the greedy protocol as it yields a more entangled residual state when it fails (they both yield the same entangled state with the same optimal probability when they succeed). Finally, we consider the generalization of these protocols to entanglement transformations involving multiple possible initial or final states.

References
