Wigner entropy and majorization in quantum phase space

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I will argue that the continuous version of majorization theory provides a natural approach to exploring the uncertainty of positive Wigner functions in phase space. After identifying all Gaussian pure states as equivalent, in the sense of continuous majorization, I will conjecture a fundamental majorization relation: any positive Wigner function is majorized by the vacuum state. As a consequence, any Schur-concave function of the Wigner function of a quantum state is lower bounded by its value for the vacuum state. In turn, this implies a fundamental entropic conjecture, namely that the Wigner entropy of any positive Wigner function is lower bounded by its value for the vacuum state. I will overview partial proofs of these conjectures and then discuss implications in the context of entropic uncertainty relations. Finally, I will examine the difficulty in finding an entropy-like functional in phase space that extends to negative Wigner functions and then advocate the merits of defining a complex-valued Wigner entropy via the analytical continuation of Shannon's differential entropy in the complex plane. I will show that it enjoys interesting properties, first of all both its real and imaginary parts are invariant under Gaussian unitaries. Its real part is physically relevant when considering the evolution of the Wigner function under a Gaussian convolution (via an extended de Bruijn's identity), while its imaginary part is simply proportional to the negative volume of the Wigner function.